mSUGRA Fits and Naturalness Priors

by

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Talk outline

• Constraints on SUSY models
• Implications
Constraints on SUSY Models

Shortcomings

• Really, would like to combine likelihoods from different measurements \(^a\)

• Typically only 2d scans, but in general we have \(\alpha_s(M_Z), m_t, m_b, m_0, M_{1/2}, A_0, \tan \beta\) to vary

• Effective 3d type scan done \(^b\) which parameterises a 2d surface of correct \(\Omega h^2\)

• Baltz et al managed to perform a 4d scan, but lost the likelihood interpretation. They used the impressive Markov Chain Monte Carlo technique.

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\(^a\) Done in 2d in Ellis et al, hep-ph/0310356

\(^b\) Ellis et al, hep-ph/0411218
**Likelihood**

\[ \mathcal{L} \equiv p(d|m) \] is pdf of reproducing data \( d \) assuming mSUGRA model \( m \) (which depends on parameters).

\[
p(m|d) = \frac{p(d|m)p(m)}{p(d)}
\]

\[
p(m_1|d) = \frac{p(d|m_1)p(m_1)}{p(d|m_2)p(m_2)}
\]

We will compare \( p(m_i) = 1 \) with a *naturalness* prior: \( 1/(m_i \text{ fine tuning}) \).
Naturalness

\[ M_Z^2 = \tan 2\beta \left[ m_{H_2}^2 \tan \beta - m_{H_1}^2 \cot \beta \right] - 2\mu^2 \]

Cancellation implied by sparticle mass bounds. Quantify by

\[ f = \max_x \left\{ \left\| \frac{d \ln M_Z^2}{d \ln x} \right\| \right\} \]

where \( x \in \{ M_{1/2}, m_0, A_0, \mu, B \} \). We will choose the prior to be \( 1/f \).
Markov-Chain Monte Carlo

Markov chain consists of list of parameter points $x^{(t)}$ and associated likelihoods $\mathcal{L}^{(t)}$

1. Pick a point at random for $x^{(1)}$

2. Pick a point around $x^{(t)}$ (say with a Gaussian width) as the potential new point.

3. If $\mathcal{L}^{(t+1)} > \mathcal{L}^{(t)}$, the new point is appended onto the chain. Otherwise, the proposed point is accepted with probability $\mathcal{L}^{(t+1)} / \mathcal{L}^{(t)}$. If not accepted, a copy of $x^{(t)}$ is added on to the chain.

Final density of $x$ points $\propto \mathcal{L}$. Required number of points goes \textit{linearly} with number of dimensions.
Implementation

Input parameters are: \( m_0, A_0, M_{1/2}, \tan \beta \)

- \( m_t = 172.7 \pm 2.9 \) GeV
- \( m_b(m_b)_{\overline{MS}} = 4.2 \pm 0.2 \) GeV,
- \( \alpha_s(M_Z)_{\overline{MS}} = 0.1187 \pm 0.002. \)

For the likelihood, we also use

- \( \Omega_{DM} h^2 = 0.1125^{+0.0081}_{-0.0091} \)
- \( \delta(g - 2)_{\mu}/2 = (19 \pm 8.4) \times 10^{-10} \)
- \( BR[b \to s\gamma] = (3.52 \pm 0.42) \times 10^{-5} \)

\[
\ln \mathcal{L} = -\frac{1}{2} \sum_i \frac{(p_i - m_i)^2}{2\sigma_i^2} + c
\]
Annihilation Mechanism

Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

<table>
<thead>
<tr>
<th>mechanism</th>
<th>flat prior</th>
<th>natural prior</th>
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</thead>
<tbody>
<tr>
<td>$h^0$—pole</td>
<td>0.025</td>
<td>0.07</td>
</tr>
<tr>
<td>$A^0$—pole</td>
<td>0.41</td>
<td>0.14</td>
</tr>
<tr>
<td>$\tilde{\tau}$—co-annihilation</td>
<td>0.26</td>
<td>0.18</td>
</tr>
<tr>
<td>rest</td>
<td>0.31</td>
<td>0.61</td>
</tr>
</tbody>
</table>
### 95% CL Upper Limits on Masses

<table>
<thead>
<tr>
<th>particle</th>
<th>flat prior</th>
<th>natural prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h^0$</td>
<td>0.123</td>
<td>0.120</td>
</tr>
<tr>
<td>$A^0$</td>
<td>1.45</td>
<td>1.50</td>
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<tr>
<td>$\chi^0_1$</td>
<td>0.65</td>
<td>0.45</td>
</tr>
<tr>
<td>$\chi^{\pm}_1$</td>
<td>1.20</td>
<td>0.85</td>
</tr>
<tr>
<td>$\tilde{g}$</td>
<td>3.25</td>
<td>2.30</td>
</tr>
<tr>
<td>$\tilde{e}_R$</td>
<td>1.90</td>
<td>1.90</td>
</tr>
<tr>
<td>$\tilde{q}_L$</td>
<td>3.20</td>
<td>2.45</td>
</tr>
<tr>
<td>$\tilde{t}_1$</td>
<td>2.45</td>
<td>1.80</td>
</tr>
</tbody>
</table>

P(500 GeV ILC$>\chi^0_1\chi^0_1, \chi^{\pm}_1\chi^{\pm}_1$) ILC=0.7,0.33

P(800 GeV ILC$>\chi^0_1\chi^0_1, \chi^{\pm}_1\chi^{\pm}_1$) ILC=0.93,0.58
Summary

- Markov chains bring out the multi-dimensionality of the space: is a lot less constrained than in 2d
- Still, current data is constraining
- Likelihood of LHC-friendly chain
  \[
  \tilde{q}_L \rightarrow \chi_2^0 \rightarrow \tilde{l}_R \rightarrow \chi_1^0 \text{ is } 24\pm4\%
  \]
- Tevatron has 32% chance of seeing $B_s \rightarrow \mu\mu$ rare decay.
- Good news for ILC: light gauginos
- Ruiz de Austri, Trotta, Roszkowski hep-ph/0602028 confirms and extends our study.
Supplementary Material
Convergence

We run $9 \times 1\,000\,000$ points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound $r$ on the expected variance decrease for infinite statistics.
SUSY Prediction of $\Omega h^2$

- Assume relic in thermal equilibrium with $n_{eq} \propto (MT)^{3/2} \exp(-M/T)$.
- Freeze-out with $T_f \sim M_f/25$ once interaction rate $< \expansion rate (t_{eq} \text{ critical})$
- We use microMEGAs $^a$: $\Omega h^2 \propto 1/\langle \sigma v \rangle$ to solve coupled Boltzmann equations
- Generate SUSY spectrum with SOFTSUSY $^b$ linked with SLHA $^c$

$^a$Belanger et al, CPC 149 (2002) 103
$^b$BCA, CPC 143 (2002) 305
$^c$BCA et al, JHEP0407 (2004) 036
Additional observables

\[ \delta \frac{(g - 2)\mu}{2} \sim 13 \times 10^{-10} \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta \]

\[ BR[b \rightarrow s\gamma] \propto \tan \beta \left( \frac{M_W}{M_{\text{SUSY}}} \right)^2 \]
mSUGRA Regions

After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

- **τ coannihilation**: small $m_0$, $m_{\tilde{\tau}_1} \approx m_{\chi^0_1}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi^0_1 \to \tau \gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \to \tau \tau$.

- **Higgs Funnel**: $\chi^0_1 \chi^0_1 \to A \to b\bar{b}/\tau\tau$ at large $\tan \beta$. Also via $h$ at large $m_0$ small $M_{1/2}$.

- **Focus region**: Higgsino LSP at large $m_0$: $\chi^0_1 \chi^0_1 \to WW/ZZ/Zh/\tau \tau$. 

- **t coannihilation**: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi^0_1}$. $\tilde{t}_1 \chi^0_1 \to gt$, $\tilde{t}\tilde{t} \to tt$
LHC SUSY Measurements

$\widetilde{q}_L, \chi^0_2, \widetilde{l}, \chi^0_1$

$m^2_{\ell\ell} = (p_{l_1} + p_{l_2})^2$

does not measure

$\sqrt{\frac{(m^2_{\chi_2^0} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\chi_1^0})}{m^2_{\tilde{l}}}}$

$^{a}$BCA, C Lester, A Parker, B Webber, JHEP 09 (2000) 004
Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$.

Run to $M_S$.

REWSB, iterative solution of $\mu$

$M_X$. Soft SUSY breaking BC.

Run to $M_S$. Calculate supersymmetric particle pole masses.

Run to $M_Z$

Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \rightarrow Z, h \rightarrow l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \rightarrow \tau + X$

Figure 1: Bulk/coannihilation region. Full: SoftSusy, dotted: SPheno.
Figure 2: Focus point region. Full: SoftSusy, dotted: SPheNo, dashed: SuSpect. Higgsino LSP annihilates into $ZZ/WW$
High tan $\beta$


Figure 3: High tan $\beta$ region. Full: SoftSusy, dotted: SPheno, dashed: SuSpect. Get annihilation into $A$. 