MSSM Radiative Corrections

to

Neutrino-nucleon Deep-inelastic Scattering

Oliver Brein

Institute of Particle Physics Phenomenology,
University of Durham

in collaboration with Wolfgang Hollik and Benjamin Koch

e-mail: Oliver.Brein@durham.ac.uk
outline :

- Introduction
  - deep inelastic $\nu N$ scattering at NuTeV
  - possible explanations
- MSSM radiative corrections to $\nu_\mu N$ DIS
  - definition of $\delta R^{\nu,\bar{\nu}} = R^{\nu,\bar{\nu}}_{\text{MSSM}} - R^{\nu,\bar{\nu}}_{\text{SM}}$
  - superpartner loop corrections
  - MSSM-SM Higgs loop difference
- Results for $\delta R^{\nu}, \delta R^{\bar{\nu}}$
  - how to scan over MSSM parameters?
  - MSSM parameter scan
Introduction

– deep inelastic $\nu N$ scattering at NuTeV

In the SM neutral (NC) and charged current (CC) neutrino nucleon scattering are described in LO by $t$-channel $W$ and $Z$ exchange.

At NuTeV $\nu_\mu$ and $\bar{\nu}_\mu$ beams of a mean energy of 125 GeV were scattered off a target detector and the ratios

$$R^\nu = \frac{\sigma^\nu_{\text{NC}}}{\sigma^\nu_{\text{CC}}}$$

$$R^{\bar{\nu}} = \frac{\sigma^{\bar{\nu}}_{\text{NC}}}{\sigma^{\bar{\nu}}_{\text{CC}}}$$

were measured.
NuTeV measured also the weak mixing angle [NuTeV '02].

\[
\sin^2 \theta_w^{\text{on-shell}} = 0.2277 \pm 0.0013 \pm 0.0009
\]

→ This is about 3\(\sigma\) below the SM prediction!

But the measurement is indirect, using the measurements of \(R^\nu, R^{\bar{\nu}}\) and making use of the Paschos-Wolfenstein relation

\[
R^- = \frac{R^\nu - r R^{\bar{\nu}}}{1 - r} = \frac{1}{2} - \sin^2 \theta_w + \cdots , \quad r = \frac{\sigma^{\bar{\nu}}_{\text{CC}}}{\sigma^\nu_{\text{CC}}} \approx \frac{1}{2}.
\]
More precisely, ratios of counting rates

\[ R_{\text{exp}}^{\nu} = \frac{\text{\# of NC-like } \nu \text{ events}}{\text{\# of CC-like } \nu \text{ events}} \approx R^{\nu}, \quad R_{\text{exp}}^{\bar{\nu}} = \frac{\text{\# of NC-like } \bar{\nu} \text{ events}}{\text{\# of CC-like } \bar{\nu} \text{ events}} \approx R^{\bar{\nu}} \]

are measured.

\( R_{\text{exp}}^{\nu}, R_{\text{exp}}^{\bar{\nu}} \) can be related to \( R^{\nu}, R^{\bar{\nu}} \) by a detailed MC physics simulation.

The deviation from the SM in terms of \( R_{\text{exp}}^{\nu}, R_{\text{exp}}^{\bar{\nu}} \) are \( \text{[NuTeV '02]}: \)

\[ \Delta R^{\nu} = R_{\text{exp}}^{\nu} - R_{\text{exp}}^{\nu}(SM) = -0.0032 \pm 0.0013, \]
\[ \Delta R^{\bar{\nu}} = R_{\text{exp}}^{\bar{\nu}} - R_{\text{exp}}^{\bar{\nu}}(SM) = -0.0016 \pm 0.0028. \]

\( \rightarrow \Delta R^{\nu,\bar{\nu}} : \) simple starting point for studying MSSM radiative corrections
- possible explanations

- **statistical fluctuation, errors underestimated?**
  → re-analyses of EW rad. corr.
  [Diener, Dittmaier, Hollik '03 & '05; Arbuzov, Bardin, Kalinovskaya '03]

- **relevant SM effects neglected?**
  → asymmetry of strange sea-quarks in the nucleon ($s \neq \bar{s}$)
  → isospin violation ($u_p \neq d_n$)
  → nuclear effects
  → etc. ... 

- **new physics?**
  → modified gauge boson interactions (e.g. in extra dimensions)
  → non-renormalizable operators (suppressed by powers of $\Lambda_{\text{new physics}}^{-1}$)
  → leptoquarks (e.g. $R$ parity violating SUSY)
  → SUSY loop effects (e.g. in MSSM)
  → etc. ...
Although the NuTeV "anomaly" is far from being settled, it is interesting, if the MSSM could account for such an effect.

Earlier Studies:
- **Davidson et al. ['02]**
  - rough study in terms of oblique corrections
    (i.e. momentum transfer $q = 0$)
  - no Parton Distribution Functions (PDFs) used

- **Kurylov, Ramsey-Musolf, Su ['04]**:
  - detailed parameter dependence studied
  - momentum transfer $q = 0$ approximation
  - no PDFs used

results so far: MSSM not responsible (size ok, but wrong sign)

- **our calculation: try to include kinematic effects**
  - full $q^2$-dependence
  - use PDFs
  - use NuTeV cuts on hadronic Energy in final state
  - use mean neutrino beam energy (125 GeV)
MSSM radiative corrections to $\nu_\mu N$ DIS

- definition of $\delta R^{\nu,\bar{\nu}} = R^{\nu,\bar{\nu}}_{\text{MSSM}} - R^{\nu,\bar{\nu}}_{\text{SM}}$

The difference between MSSM and SM prediction, $\delta R^n = R^n_{\text{MSSM}} - R^n_{\text{SM}}$ with $R^n = \sigma^n_{\text{NC}} / \sigma^n_{\text{CC}} (n = \nu, \bar{\nu})$, using

$$(\sigma^n_{\text{NC}})_{\text{NLO}} = (\sigma^n_{\text{NC}})_{\text{LO}} + \delta \sigma^n_{\text{NC}} \quad (n = \nu, \bar{\nu})$$

$$(\sigma^n_{\text{CC}})_{\text{NLO}} = (\sigma^n_{\text{CC}})_{\text{LO}} + \delta \sigma^n_{\text{CC}} \quad (n = \nu, \bar{\nu})$$

can be expanded in $\delta \sigma^n_{\text{NC}}$ and $\delta \sigma^n_{\text{CC}}$

$$\delta R^n = \left(\frac{\sigma^n_{\text{NC}}}{\sigma^n_{\text{CC}}}\right)_{\text{LO}} \left(\frac{(\delta \sigma^n_{\text{NC}})_{\text{MSSM}} - (\delta \sigma^n_{\text{NC}})_{\text{SM}}}{\sigma^n_{\text{NC}}}_{\text{LO}} - \frac{(\delta \sigma^n_{\text{CC}})_{\text{MSSM}} - (\delta \sigma^n_{\text{CC}})_{\text{SM}}}{\sigma^n_{\text{CC}}}_{\text{LO}}\right).$$

→ Only differences between MSSM and SM radiative corrections and LO cross sections appear in $\delta R^n$. 
Because of $R$ parity conservation in the MSSM:

- Born cross section: $\text{SM} = \text{MSSM}$ (very good approx.)

- Real photon emission corrections: $\text{SM} = \text{MSSM}$ (very good approx.)

- $\text{SM} = \text{MSSM}$ for SM-like 1-loop graphs without virtual Higgs

Thus:

$$\delta\sigma_{\text{MSSM}} - \delta\sigma_{\text{SM}} = \text{const.} \times (\text{[superpartner loops]}$$

$$+ \text{[Higgs graphs MSSM} - \text{Higgs graphs SM]}).$$
- superpartner loop corrections

**CC self energy insertions**

\[
\begin{align*}
&\nu_\mu & \mu \\
&W & W & f \\
&d & u
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & W
\end{align*}
\]

\[
\begin{align*}
&\tilde{\nu}_i & \tilde{\epsilon}_i \\
&W & W
\end{align*}
\]

\[
\begin{align*}
&\tilde{u}_i & \tilde{d}_i \\
&W & W
\end{align*}
\]

**CC vertex corrections**

\[
\begin{align*}
&\nu_\mu & \mu \\
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&d & \tilde{u}^s & u \\
&d^s & W
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & W & \tilde{\nu}_i \\
&\tilde{d} & \tilde{u}^t
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & \tilde{d} & \tilde{u}^t
\end{align*}
\]

\[
\begin{align*}
&\tilde{g} & \tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & W & \tilde{\nu}_i & \tilde{\mu}^s
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & W & \tilde{\nu}_i & \tilde{\mu}^s
\end{align*}
\]

**CC box corrections**

\[
\begin{align*}
&\nu_\mu & \tilde{\mu}^s & \mu \\
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&d & \tilde{u}^t & u \\
&d^s & \tilde{u}^s
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & \tilde{\nu}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&\tilde{d} & \tilde{u}^t
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & W & \tilde{\nu}_i & \tilde{\mu}^s
\end{align*}
\]

\[
\begin{align*}
&\tilde{\chi}_i & \tilde{\chi}_i^0 & \tilde{\chi}_i \\
&W & W & \tilde{\nu}_i & \tilde{\mu}^s
\end{align*}
\]

[ MSSM radiative corrections to $\nu_\mu N$ DIS ]
NC self energy insertions

\[ \nu_\mu \to \nu_\mu \]

\[ \tilde{\chi}_i \to \tilde{\chi}_i \]

\[ Z \gamma q \to Z \gamma q \]

\[ f_c \circlearrowleft \tilde{f}_c \]

\[ \tilde{\chi}_i \circlearrowleft \tilde{\chi}_i \]

\[ Z \to f \]

\[ f \to \tilde{f} \]

\[ \tilde{\chi}_i \to \tilde{\chi}_j \]

\[ \tilde{\chi}_j \to \tilde{\chi}_i \]

NC vertex corrections

\[ \nu_\mu \to \nu_\mu \]

\[ \tilde{\chi}_i \to \tilde{\chi}_j \]

\[ u \to \tilde{u}^s \to u \]

\[ d^s \to d \]

\[ Z \to Z \]

\[ \tilde{u}^s \to \tilde{u}^t \]

\[ \tilde{d} \to \tilde{d}^s \]

\[ \tilde{\chi}_i \to \tilde{\chi}_j \]

\[ \tilde{\chi}_j \to \tilde{\chi}_i \]

\[ \bar{\nu}_\mu \to \bar{\mu}^s \to \bar{\mu}^t \]

\[ \bar{\nu}_\mu \to \bar{\nu}_\mu \]

\[ \tilde{\mu}^s \to \tilde{\mu} \]

\[ \tilde{\mu} \to \tilde{\mu}^s \]

\[ \tilde{\mu} \to \tilde{\mu}^t \]

\[ \tilde{\chi}_i \to \tilde{\chi}_j \]

\[ \tilde{\chi}_j \to \tilde{\chi}_i \]

NC box corrections

\[ \nu_\mu \to \bar{\nu}_\mu \to \nu_\mu \]

\[ \tilde{\chi}_i \to \tilde{\chi}_j \]

\[ u \to \tilde{u}^s \to u \]

\[ \tilde{d} \to \tilde{d}^s \]

\[ \tilde{\chi}_i \to \tilde{\chi}_j \]

\[ \tilde{\chi}_j \to \tilde{\chi}_i \]

\[ \bar{\nu}_\mu \to \bar{\mu}^s \to \bar{\mu}^t \]

\[ \bar{\nu}_\mu \to \bar{\nu}_\mu \]

\[ \tilde{\mu} \to \tilde{\mu}^s \to \tilde{\mu}^t \]

\[ \tilde{\mu} \to \tilde{\mu} \]
MSSM radiative corrections to $\nu_\mu N$ DIS

- MSSM-SM Higgs loop difference

+ CC MSSM Higgs loops

- CC SM Higgs loops

+ NC MSSM Higgs loops

- NC SM Higgs loops
The partonic processes were calculated using FeynArts/FormCalc.

[Küblbeck, Böhm, Denner'90],
[Eck '95], [Hahn, Perez-Victoria '99], [Hahn '01],
[Hahn, Schappacher '02]

see: www.feynarts.de
• Results for $\delta R^\nu, \delta R^\nu$,

how to scan over MSSM parameters?

goal: find regions of parameter space, where the MSSM might explain the NuTeV anomaly.

difficult, large dimensionality of MSSM parameter space

scanning strategy: ”adaptive scan” [OBr’04]:

exploit adaptive integration by importance sampling

method: calculate an approximation to the integral

$$ I = \int_{M_1^{\min}}^{M_1^{\max}} dM_1 \cdots \int_{d\tan \beta^{\min}}^{d\tan \beta^{\max}} d\tan \beta \ F(\delta R^\nu(\nu)(M_1, \ldots, \tan \beta), M_1, \ldots, \tan \beta) $$

with VEGAS and store the sampled parameter points.

automatically, the sample points will be enriched in the regions where $F(\delta R^\nu(\nu)(M_1, \ldots, \tan \beta), M_1, \ldots, \tan \beta)$ is large.
some sample choices of F:

- \( F = \begin{cases} 
  1 & \text{if parameters } (M_1, \ldots, \tan \beta) \text{ not excluded} \\
  0 & \text{elsewhere} 
\end{cases} \) 
  → enrich points in allowed region

- \( F = \begin{cases} 
  \delta R^\nu(\bar{\nu})(M_1, \ldots, \tan \beta) & \text{if parameters } (M_1, \ldots, \tan \beta) \text{ not excluded} \\
  0 & \text{elsewhere} 
\end{cases} \) 
  → enrich points where \(|\delta R^\nu(\bar{\nu})|\) is large in allowed region

- \( F = \begin{cases} 
  \sqrt{(\delta R^\nu(\ldots))^2 + (\delta R^{\bar{\nu}}(\ldots))^2} & \text{if } \delta R^\nu \text{ and } \delta R^{\bar{\nu}} < 0 \\
  0 & \text{elsewhere} 
\end{cases} \) 
  → enrich points where \(\delta R^\nu, \delta R^{\bar{\nu}} < 0 \text{ and } \sqrt{\ldots} \text{ is large}\)
- MSSM parameter scan

restrictions taken into account:

- mass exclusion limits for Higgs bosons and superpartners
- $\Delta \rho$-constraint on sfermion mixing

quantities varied:

\[
\begin{align*}
M_1, M_2, M_{\text{gluino}} & : 10 \ldots 1000 \text{ GeV} & \mu & : -2000 \ldots 2000 \text{ GeV} \\
M_{\text{Sf.}} & : 10 \ldots 1000 \text{ GeV} & A_b, A_t, A_\tau & : -2000 \ldots 2000 \text{ GeV} \\
m_{A_0} & : 10 \ldots 1000 \text{ GeV} & \tan \beta & : 1 \ldots 50
\end{align*}
\]
Scan for large values of $|\delta R^\nu|$ and $|\delta R^\bar{\nu}|$ with parameter restrictions

$\delta R^\nu$ vs. $X_t$ [TeV]

$\delta R^\nu(\bar{\nu})$

- $0 - 1 \cdot 10^{-5}$
- $1 \cdot 10^{-5} - 5 \cdot 10^{-5}$
- $5 \cdot 10^{-5} - 1 \cdot 10^{-4}$
- $1 \cdot 10^{-4} - 3 \cdot 10^{-4}$
- $3 \cdot 10^{-4} - 8 \cdot 10^{-4}$
- $8 \cdot 10^{-4} - 1 \cdot 10^{-3}$
- $> 1 \cdot 10^{-3}$
Scan for negative values of $\delta R^\nu$
without parameter restrictions

$-\delta R^\nu$

0 – $1 \cdot 10^{-5}$
1 $\cdot 10^{-5}$ – 5 $\cdot 10^{-5}$
5 $\cdot 10^{-5}$ – 1 $\cdot 10^{-4}$
1 $\cdot 10^{-4}$ – 3 $\cdot 10^{-4}$
3 $\cdot 10^{-4}$ – 8 $\cdot 10^{-4}$
8 $\cdot 10^{-4}$ – 1 $\cdot 10^{-3}$
>$1 \cdot 10^{-3}$

$m_{\chi_1^0} [\text{GeV}]$
Scan for negative values of $\delta R^\nu$ without parameter restrictions

$-\delta R^\nu$

$0 - 1 \cdot 10^{-5}$
$1 \cdot 10^{-5} - 5 \cdot 10^{-5}$
$5 \cdot 10^{-5} - 1 \cdot 10^{-4}$
$1 \cdot 10^{-4} - 3 \cdot 10^{-4}$
$3 \cdot 10^{-4} - 8 \cdot 10^{-4}$
$8 \cdot 10^{-4} - 1 \cdot 10^{-3}$
$> 1 \cdot 10^{-3}$

$m_{\chi_1^0} > 94\,\text{GeV}$
$m_{\chi_1^+} > 46\,\text{GeV}$

$\rightarrow$ all interesting regions excluded
Scan for negative values of $\delta R^{\tilde{\nu}}$ without parameter restrictions

$-\delta R^{\tilde{\nu}}$

<table>
<thead>
<tr>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 - 1 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$1 \cdot 10^{-5} - 5 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$5 \cdot 10^{-5} - 1 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$1 \cdot 10^{-4} - 3 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$3 \cdot 10^{-4} - 8 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$8 \cdot 10^{-4} - 1 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$&gt; 1 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

$m_{\chi_1^+} > 94\text{GeV}$

$m_{\chi_1^0} > 46\text{GeV}$

$\rightarrow$ all interesting regions excluded
Results for $\delta R^\nu, \delta R^\nu$, MSSM parameter scan

[Diagram showing distributions for $\delta R^\nu$ and $\delta R^\nu$, with regions marked for no restrictions and NUTEV (+/- 2 sigma)].
Summary

- The NuTeV measurement of $\sin^2 \theta_w$ is intriguing but has to be further established (especially confirmation by other experiment(s) is desirable).

- Loop effects in the MSSM are not capable of explaining the NuTeV “anomaly”. (size can be right, but sign is wrong).

- If the “anomaly” was established, the MSSM would be in trouble.

Our result can be easily combined with the one-loop SM result.

→ The complete MSSM one-loop prediction for $\nu N$ scattering is available for future analyses.