An Effective Operators Analysis of CP violation

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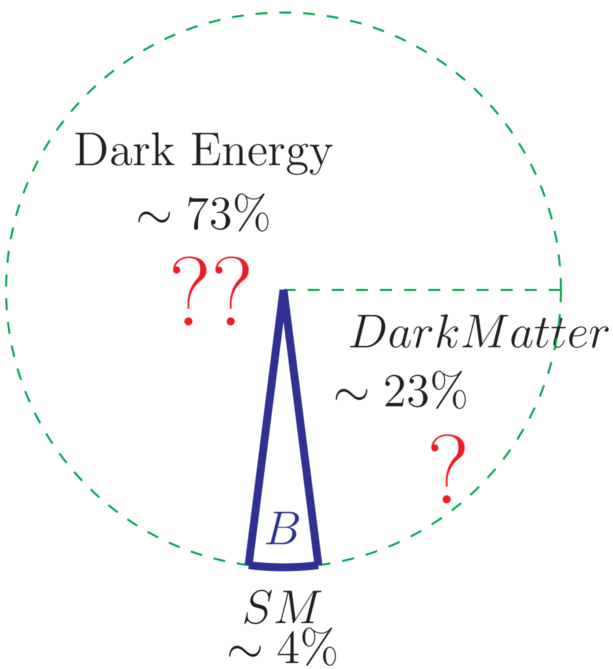
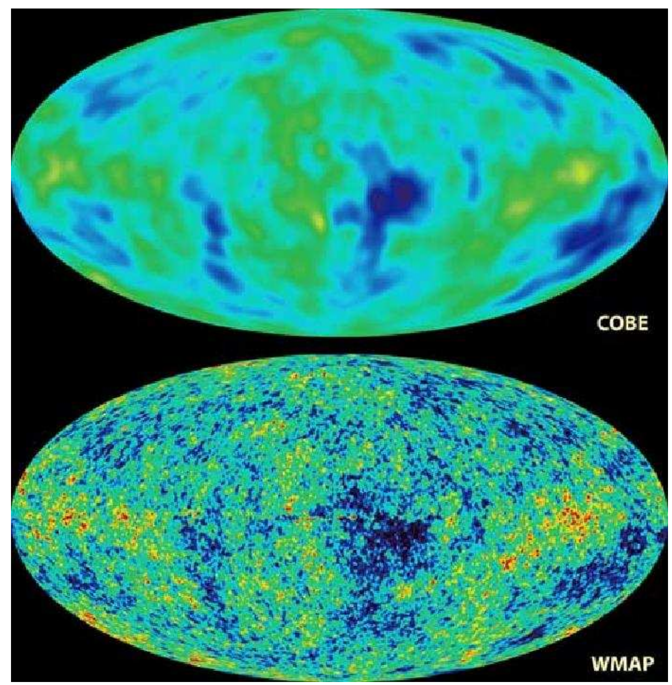
- Overview
- Pure leptonic case, 4-Lepton Operators
- RGE
- The EDM of charged lepton.
- Conclusion

Beyond SM

Standard Model is an extremely successful and profound theory which describes our world. But it can only be regarded as a low energy effective theory.

- Neutrino Physics, Flavor Physics
- Stability of the Higgs sector
- Origin of CP violation, Baryogenesis, Leptogenesis?
- WMAP makes SM even more embarrassing.





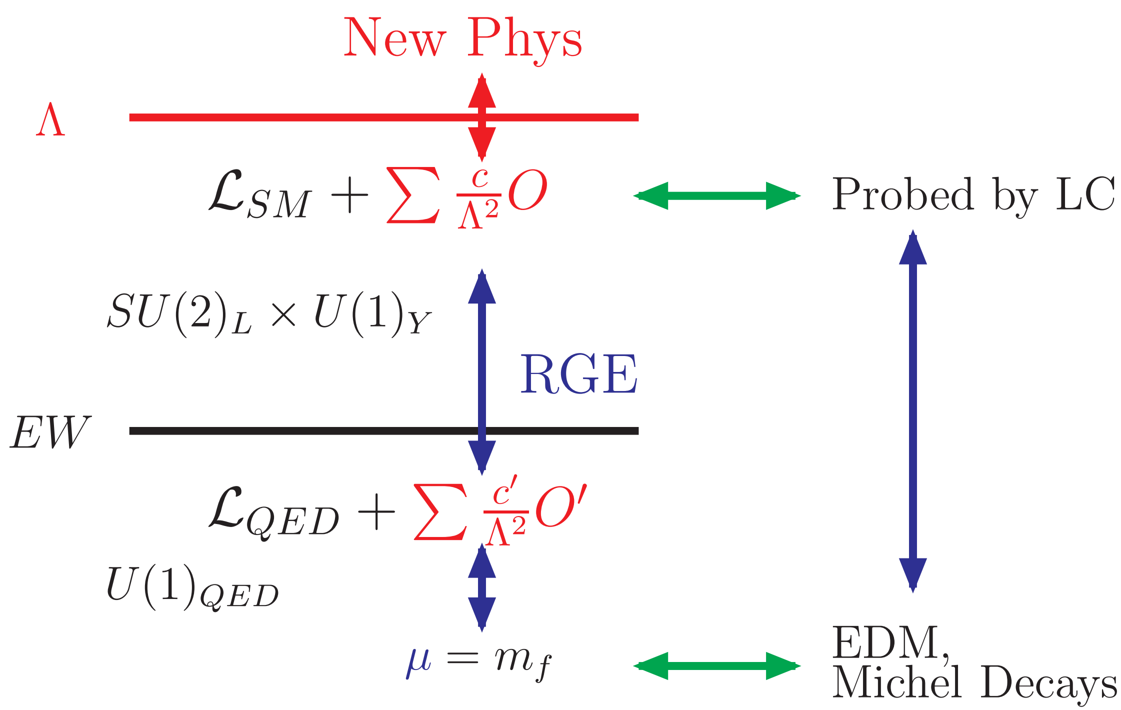
Overview and Introduction

- In SM, the sole CP violating source is the phase in the CKM matrix and the resulting charged fermion EDM is extremely small:

$$d_e < 10^{-38} \text{ e cm}$$

Therefore EDM, if seen, will be a clean signal of new CPV source.

- In the past few decades, we have built countless of models. Can we have some model independent statement for the CP violation beyond SM?



Operators

- Dim-6, $SU(2)_L \times U(1)_Y$ invariant 4-lepton operator. a, b: $SU(2)$, i, j, k, l: flavor.

$$\mathcal{L}_{new} = \frac{1}{\Lambda^2}\Big[\quad c_{LL}^{ijkl}(\bar{L}_{ia}\gamma^{\mu}L_{ja})(\bar{L}_{kb}\gamma_{\mu}L_{lb})$$

$$+ \; d_{LL}^{ijkl}\left(\bar{L}_{ia}\gamma^{\mu}L_{jb}\right)\left(\bar{L}_{kb}\gamma_{\mu}L_{la}\right)$$

$$+ \; c_{LR}^{ijkl}(\bar{L}_{ia}\gamma^{\mu}L_{ja})(\bar{e}_{k}\gamma_{\mu}e_{l})$$

$$+ \; c_{RR}^{ijkl}(\bar{e}_{i}\gamma^{\mu}e_{j})(\bar{e}_{k}\gamma_{\mu}e_{l})$$

$$+ \; c_{S}^{ijkl}(\bar{L}_{ia}e_{j})(\bar{e}_{k}L_{la}) \quad \Big] + H.c$$

Some remarks

- Note, there is no $SU(2)_L \times U(1)_Y$ invariant tensor operator!

$$[\bar{L}\sigma^{\mu\nu} e][\bar{e}\sigma_{\mu\nu} L] = 0$$

- It's easy to include but we do not consider the dim-5 neutrino mass operator.
- We have done the analysis of semileptonic cases.
- It's also straightforward to include all the dim-6 operators which involve the Higgs or gauge bosons.

The Coeffcients

- The Hermiticity of the Lagrangian requires that

$$\left(c_S^{ijkl}\right)^* = c_S^{lkji}\,,\ \left(c_{LR}^{ijkl}\right)^* = c_{LR}^{jilk}$$

For V_{LL} and V_{RR}, they have more ordering symmetry:

$$c_{LL}^{ijkl} = c_{LL}^{klij}\,,\ \left(c_{LL}^{ijkl}\right)^* = c_{LL}^{jilk}\,,\ d_{LL}^{ijkl} = d_{LL}^{klij}\,,\ \left(d_{LL}^{ijkl}\right)^* = d_{LL}^{jilk}$$

$$c_{RR}^{ijkl} = c_{RR}^{klij} = c_{RR}^{ilkj}\,,\ \left(c_{RR}^{ijkl}\right)^* = c_{RR}^{jilk}$$

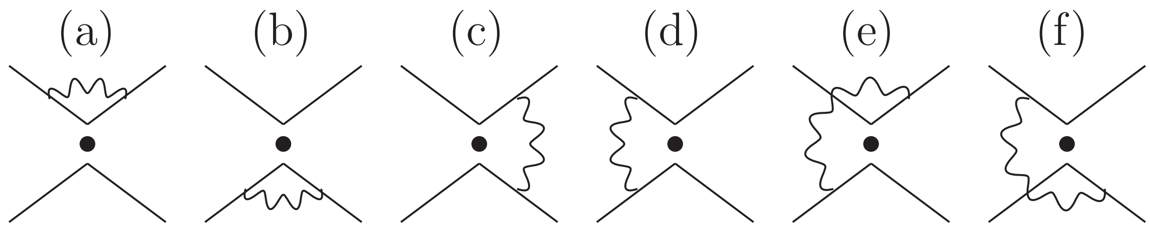
- Many flavor crossing terms potentially contain CP violating phases beyond SM. Also, in short, the complex phases can't be totally rotated away.
- There are altogether 324 parameters and a general analysis is not meaningful.
- So far, the search of lepton flavor violation gives null result. This phenomenological fact can be implemented by assuming the vector coefficients are flavor diagonal

$$\Rightarrow c_V^{ij,kl} = c_V \delta^{ij}\delta^{kl}$$

- With this simplification, all CP violation BSM are carried by the Wilson coefficient of the scalar operator, C_S.

RGE for the effective operators

- Here are the 1-loop Feynman diagrams we need to consider



- Need to include the fermion wave function renormalization as well.
- The anomalous dimension can be obtained through standard calculation. We found that

$$
\begin{aligned}
\gamma_S = \gamma_{LR} &= -12 Y_L Y_e \frac{\alpha_1}{4\pi} \\
\gamma_{RR} &= +12 Y_e^2 \frac{\alpha_1}{4\pi} \\
\gamma_{LL} &= \frac{1}{4\pi} \begin{pmatrix} 12 Y_L^2 \alpha_1 - 3\alpha_2 & 8\alpha_2 \\ 6\alpha_2 & 12 Y_L^2 \alpha_1 - 5\alpha_2 \end{pmatrix}
\end{aligned}
$$

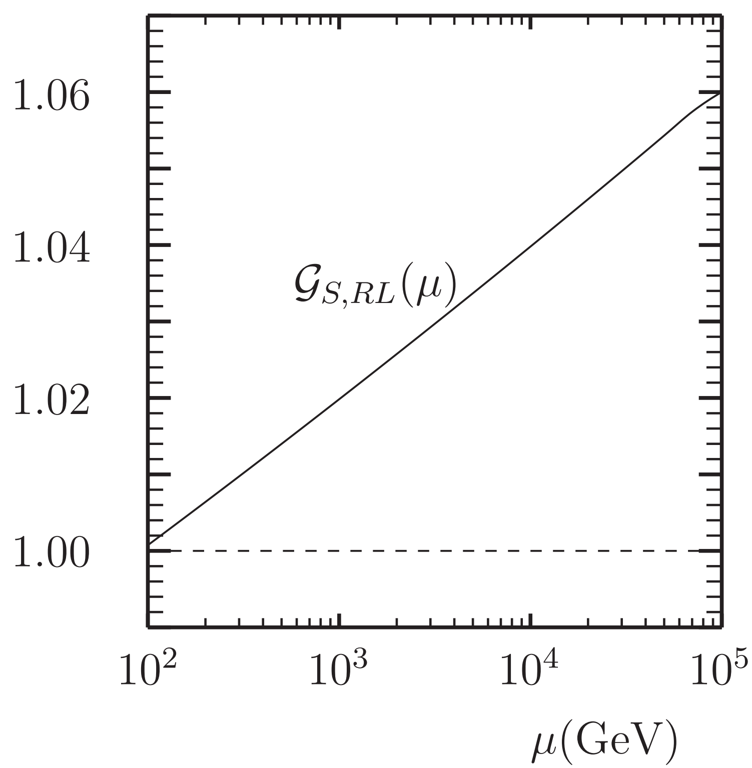
- At EWSB, do the matching and go to the mass basis.
- Below EWSB, QED controls the running.

The RG running

- The RGE for the scalar(LR) coefficient can be solved to be:

$$c_S(\mu) = c_S(M_Z)\left(\frac{\alpha_1(M_Z)}{\alpha_1(\mu)}\right)^{-\frac{30}{41}}$$

- As expected, the $U(1)$ effect about EWSB is not very big. ($\mathcal{G} \equiv C_S(\mu)/C_S(M_Z)$)



Tree-level EDM

- Since we are interested in charged lepton EDM, we should first check the $SU(2) \times U(1)$ invariant dim-6 dipole operator:

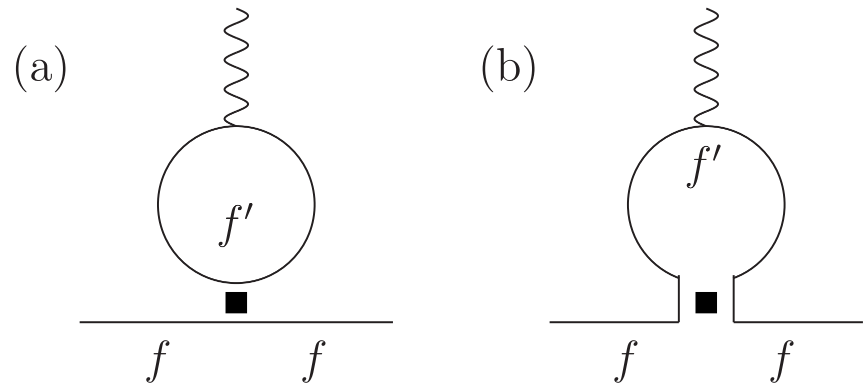
$$\mathcal{L} \supset \frac{c_d^{ij}}{\Lambda^2} H \bar{L}_i \sigma^{\mu\nu} e_j F_{\mu\nu} + H.c.$$

- Below EWSB, it induces tree-level EDM, anomalous magnetic moment, and radiative lepton flavor transition.
- However, the experimental upper bounds put stringent constraints on the dipole coefficients:

$$\begin{aligned}
d_e &: \quad |Im[c_d^{ee}]| < 2.4 \times 10^{-9} \left(\frac{\Lambda}{1\text{TeV}}\right)^2 \\
a_e &: \quad |Re[c_d^{ee}]| < 7.1 \times 10^{-6} \left(\frac{\Lambda}{1\text{TeV}}\right)^2 \\
a_\mu &: \quad |Re[c_d^{\mu\mu}]| < 8.8 \times 10^{-7} \left(\frac{\Lambda}{1\text{TeV}}\right)^2 \\
Br(\mu \to e + \gamma) &: \quad |c_d^{e\mu}| < 1.8 \times 10^{-10} \left(\frac{\Lambda}{1\text{TeV}}\right)^2
\end{aligned}$$

- The above bounds seem to suggest that the new physics beyond SM somehow suppress the dipole operator. And we will assume it disappear at tree-level in our study.

1-loop EDM?



- Vector type: LL, RR

$$(\bar{f}\gamma_{L/R,\mu} f)(\bar{f}'\gamma^{\mu}_{L/R} f')$$

  It's self-hermitian, No CP phase!

- Scalar type
  - The loop diagram (a) gives no dipole operator.
  - For diagram (b),

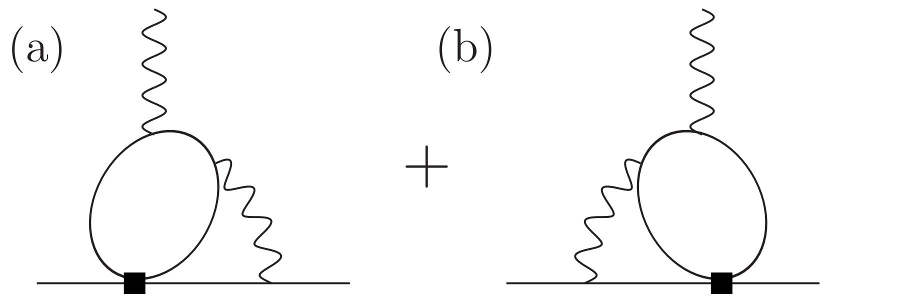
$$(\bar{f}\hat{R}f)(\bar{f}'\hat{L}f')$$

    carries no CP phase.

- To induce W-EDM, more than 3-loop is needed.

2-loop EDM

- The $1/\epsilon$ pole is absorbed by the dipole counter term, and the 2-loop EDM gives:



$$d_f = + \frac{e\alpha}{16\pi^3} \sum_i \frac{m_i \mathbf{Im} c_S^{f,i}}{\Lambda^2} \left(2 + \ln \frac{m_i^2}{M_W^2} \right)$$

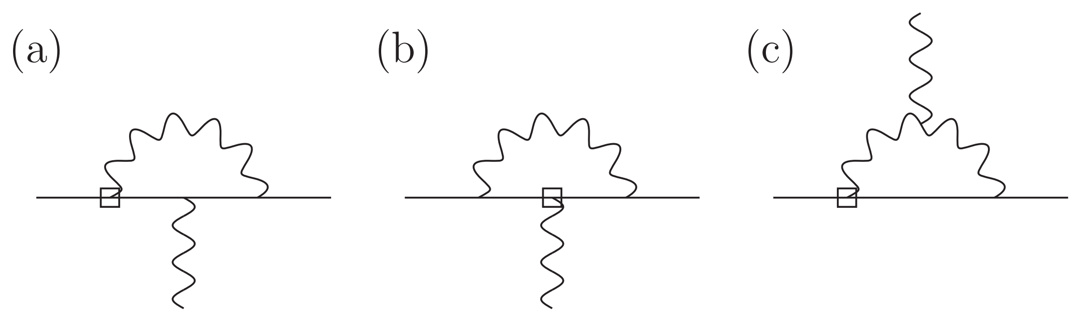
- Earlier references:
  - Multi-Higgs
    S.M. Barr and A. Zee, Phys.Rev.Lett.**65**, 21(1990)
  - Pseudoscalar in MSSM
    D. Chang, W.-Y. Keung, and A. Pilaftsis, Phys.Rev.Lett.**82**, 900(1999)
  - To rule out the MSSM EW Baryogenesis
    D. Chang, W.-F. Chang, W.-Y. Keung, Phys. Rev.**D66**,116008 (2002)
  - Split SUSY
    D. Chang, W.-F. Chang, W.-Y. Keung, Phys.Rev.**D71**,076006(2005)

RG for the dipole operator

- The dim-6 EDM operator:

$$\mathcal{O}_{EDM} = m(\,H\,)\bar{f}\sigma^{\mu\nu}\gamma_5 f F_{\mu\nu}(\,W_{\mu\nu}\,)\,,$$

- It mixes with other 4-fermion operators at 2-loop level.
- The 1-loop RG



- $\sim 10\%$ reduction form EW to charged lepton masses

$$d_f(\mu) \sim \left(1 - \frac{4\alpha}{\pi}\ln\frac{M_W}{\mu}\right) d_f(M_W)$$

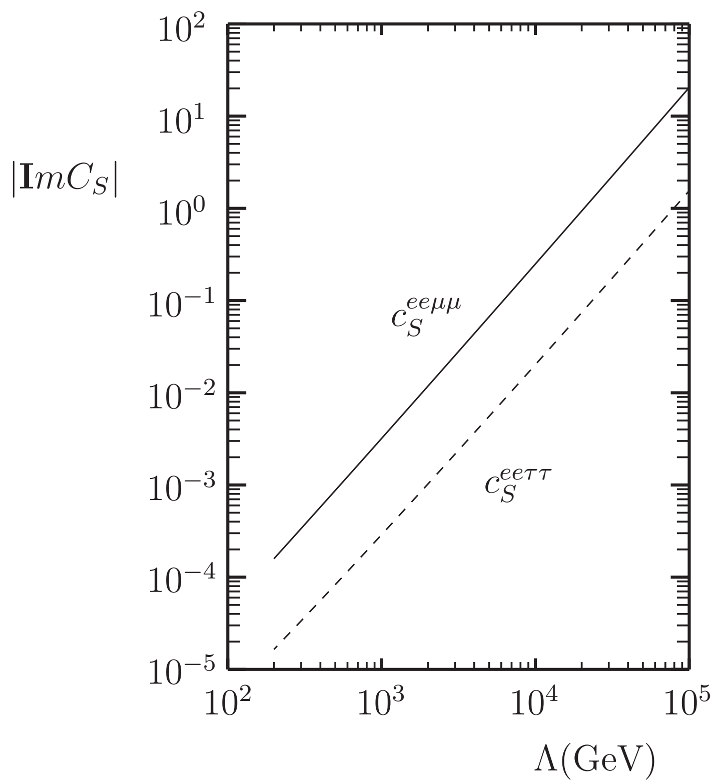
G.Degrassi and G.F. Giudice, Phys.Rev.**D58**,053007(1998).

Charged lepton EDM

- The electron EDM

$$d_e = \frac{e\alpha}{16\pi^3}\left\{ \frac{m_\tau}{\Lambda^2}\mathbf{I}m(c_S^{ee\tau\tau})\left(2+\ln\frac{m_\tau^2}{M_W^2}\right) + \frac{m_\mu}{\Lambda^2}\mathbf{I}m(c_S^{ee\mu\mu})\left(2+\ln\frac{m_\mu^2}{M_W^2}\right)\right\}$$

- Barring the unnatural cancellation, the current limit $|d_e| < 1.7\times10^{-27}$e cm gives



Muon Michel Decay

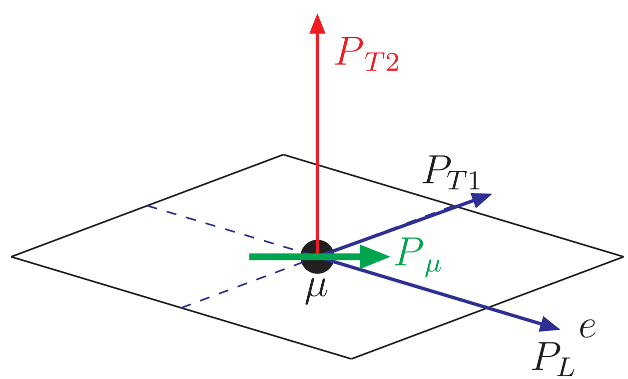
- Michel decay:

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu \text{ or } (\mu^+ \to e^+ \bar{\nu}_\mu \nu_e)$$

- If the electron polarization can be measured

$$\overrightarrow{P_e} = P_L \hat{z} + P_{T1} \frac{(\hat{z} \times \overrightarrow{P}_\mu) \times \hat{z}}{|(\hat{z} \times \overrightarrow{P_\mu}) \times \hat{z}|} + P_{T2} \frac{\hat{z} \times \overrightarrow{P}_\mu}{|\hat{z} \times \overrightarrow{P_\mu}|}$$

The P_{T2} is a T-violating observable.



Muon Michel Decay–P_{T2}

- The latest bound on P_{T2} was given by PSI two decades ago which can be translated into 2 standard CP-odd Michel parameters:

$$\alpha' = -0.003(69), \; \beta' = 0.024(101)$$

- Since 1994, a new experiment R-94-10 at PSI has been trying to push the precision one order of magnitude better.
- However, we predict that

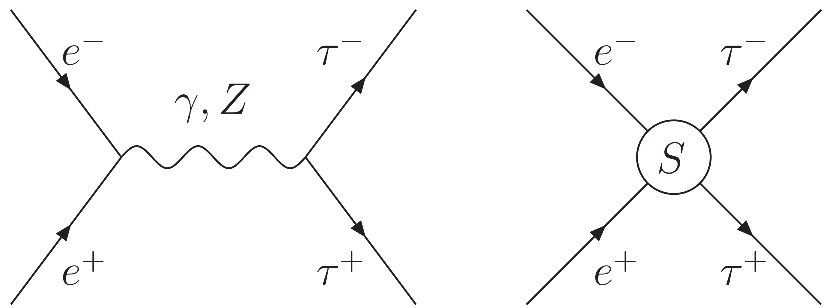
$$\alpha' = 0, \; \beta' = \frac{\sqrt{2}}{G_F \Lambda^2} Im C_S^{\mu\mu ee}$$

- From the present EDM constraint, we already have upper bound $\beta' < 4 \times 10^{-4}$ for $\Lambda = 1$TeV.

PSI (or TRIUMF) can not see it!

Triple product correlation in $e^+e^- \to \tau^+\tau^-$

- With the upper bound obtained from EDM we can now estimate the size of CP violating signatures in a purely leptonic flavor conservation reaction such as $e^+(p_+)e^-(p_-) \to \tau^+(k_+)\tau^-(k_-)$ where the 4-momentum of each particle are shown in the corresponding bracket.
- It is straightforward to calculate the T-odd amplitude square



and we obtain

$$|M_{TO}|^2 = \frac{s}{24e^2\Lambda^2} ImC_s^{ee,\tau\tau}(\hat{p}_- + \hat{k}_-) \cdot (\vec{s}_e \times \vec{s}_\tau)$$

where we have scaled by the strength of the QED term and s is the cm energy square.

Conclusion

- We study the leptonic CP violation by employing the complete set of dimension-six pure leptonic effective operators.
- Connection among the observable at different energy scales can be made by the running of the renormalization group equations.
- Explicitly, we study the charged lepton electric dipole moment, muon Michel decay, and the triple spin-momentum correlations at the Linear Collider.
- We found the electron electric dipole moment, which starts at 2-loop level, severely constrains the possibilities to detect the CP violating signatures in muon decay and at the linear colliders.