An Effective Operators Analysis of CP violation

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• Overview
• Pure leptonic case, 4-Lepton Operators
• RGE
• The EDM of charged lepton.
• Conclusion
Beyond SM

Standard Model is an extremely successful and profound theory which describes our world. But it can only be regarded as a low energy effective theory.

- Neutrino Physics, Flavor Physics
- Stability of the Higgs sector
- Origin of CP violation, Baryogenesis, Leptogenesis?
- WMAP makes SM even more embarrassing.

Dark Energy
\sim 73\%

??

Dark Matter
\sim 23\%

SM
\sim 4\%

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Overview and Introduction

• In SM, the sole CP violating source is the phase in the CKM matrix and the resulting charged fermion EDM is extremely small:

$$d_e < 10^{-38} \text{ e cm}$$

Therefore EDM, if seen, will be a clean signal of new CPV source.

• In the past few decades, we have built countless of models. Can we have some model independent statement for the CP violation beyond SM?

![Diagram of New Phys with SM, SU(2)\textsubscript{L} \times U(1)\textsubscript{Y}, EW, RGE, Probed by LC, EDM, Michel Decays]
**Operators**

- Dim-6, $SU(2)_L \times U(1)_Y$ invariant 4-lepton operator. $a, b$: $SU(2)$, $i, j, k, l$: flavor.

\[
\mathcal{L}_{\text{new}} = \frac{1}{\Lambda^2} \left[ c_{LL}^{ijkl} (\bar{L}_{ia} \gamma^\mu L_{ja})(\bar{L}_{kb} \gamma^\mu L_{lb}) \\
+ d_{LL}^{ijkl} (\bar{L}_{ia} \gamma^\mu L_{jb})(\bar{L}_{kb} \gamma^\mu L_{la}) \\
+ c_{LR}^{ijkl} (\bar{L}_{ia} \gamma^\mu L_{ja})(\bar{e}_k \gamma^\mu e_l) \\
+ c_{RR}^{ijkl} (\bar{e}_i \gamma^\mu e_j)(\bar{e}_k \gamma^\mu e_l) \\
+ c_{S}^{ijkl} (\bar{L}_{ia} e_j)(\bar{e}_k L_{la}) \right] + H.c.
\]
Some remarks

• Note, there is no $SU(2)_L \times U(1)_Y$ invariant tensor operator!

\[ [\bar{L}\sigma^{\mu\nu}e][\bar{e}\sigma_{\mu\nu}L] = 0 \]

• It’s easy to include but we do not consider the dim-5 neutrino mass operator.
• We have done the analysis of semileptonic cases.
• It’s also straightforward to include all the dim-6 operators which involve the Higgs or gauge bosons.
The Coefficients

• The Hermiticity of the Lagrangian requires that

\[
\begin{align*}
(c_{ijkl})^* &= c_{lkji}, & (c_{ijkl})^* &= c_{jilk}
\end{align*}
\]

For \( V_{LL} \) and \( V_{RR} \), they have more ordering symmetry:

\[
\begin{align*}
(c_{ijkl})_{LL} &= (c_{klij})_{LL}, & (c_{ijkl})_{LL}^* &= c_{jilk}_{LL}, & d_{ijkl} &= d_{klij}_{LL}, & d_{ijkl}_{LL}^* &= d_{jilk}_{LL}
\end{align*}
\]

\[
\begin{align*}
(c_{ijkl})_{RR} &= (c_{klij})_{RR}, & (c_{ijkl})_{RR}^* &= c_{jilk}_{RR}, & (c_{ijkl})_{RR}^* &= c_{jilk}_{RR}
\end{align*}
\]

• Many flavor crossing terms potentially contain CP violating phases beyond SM. Also, in short, the complex phases can’t be totally rotated away.

• There are altogether 324 parameters and a general analysis is not meaningful.

• So far, the search of lepton flavor violation gives null result. This phenomenological fact can be implemented by assuming the vector coefficients are flavor diagonal

\[
\Rightarrow c_{V,ijkl} = c_V \delta_{ij} \delta_{kl}
\]

• With this simplification, all CP violation BSM are carried by the Wilson coefficient of the scalar operator, \( c_S \).
**RGE for the effective operators**

- Here are the 1-loop Feynman diagrams we need to consider

- ![Feynman diagrams](image)

- Need to include the fermion wave function renormalization as well.
- The anomalous dimension can be obtained through standard calculation. We found that

\[
\gamma_S = \gamma_{LR} = -12Y_L Y_e \frac{\alpha_1}{4\pi}
\]

\[
\gamma_{RR} = +12Y_e^2 \frac{\alpha_1}{4\pi}
\]

\[
\gamma_{LL} = \frac{1}{4\pi} \begin{pmatrix}
12Y_L^2 \alpha_1 - 3\alpha_2 & 8\alpha_2 \\
6\alpha_2 & 12Y_L^2 \alpha_1 - 5\alpha_2
\end{pmatrix}
\]

- At EWSB, do the matching and go to the mass basis.
- Below EWSB, QED controls the running.
**The RG running**

- The RGE for the scalar(LR) coefficient can be solved to be:

\[ c_S(\mu) = c_S(M_Z) \left( \frac{\alpha_1(M_Z)}{\alpha_1(\mu)} \right)^{-\frac{30}{41}} \]

- As expected, the \( U(1) \) effect about EWSB is not very big. ( \( G \equiv C_S(\mu)/C_S(M_Z) \) )

![Graph showing the RG running of the scalar coefficient](image-url)
Tree-level EDM

• Since we are interested in charged lepton EDM, we should first check the $SU(2) \times U(1)$ invariant dim-6 dipole operator:

$$\mathcal{L} \supset \frac{c_{d}^{ij}}{\Lambda^2} H \bar{L}_i \sigma_{\mu\nu} e_j F_{\mu\nu} + H.c.$$  

• Below EWSB, it induces tree-level EDM, anomalous magnetic moment, and radiative lepton flavor transition.

• However, the experimental upper bounds put stringent constraints on the dipole coefficients:

$$d_e : |Im[c_{d}^{ee}]| < 2.4 \times 10^{-9} \left(\frac{\Lambda}{1\text{ TeV}}\right)^2$$

$$a_e : |Re[c_{d}^{ee}]| < 7.1 \times 10^{-6} \left(\frac{\Lambda}{1\text{ TeV}}\right)^2$$

$$a_\mu : |Re[c_{d}^{\mu\mu}]| < 8.8 \times 10^{-7} \left(\frac{\Lambda}{1\text{ TeV}}\right)^2$$

$$Br(\mu \rightarrow e + \gamma) : |c_{d}^{e\mu}| < 1.8 \times 10^{-10} \left(\frac{\Lambda}{1\text{ TeV}}\right)^2$$

• The above bounds seem to suggest that the new physics beyond SM somehow suppress the dipole operator. And we will assume it disappear at tree-level in our study.
1-loop EDM?

- Vector type: LL, RR
  \[(\bar{f} \gamma_{L/R, \mu} f)(\bar{f'} \gamma^{\mu}_{L/R} f')\]
  It’s self-hermitian, No CP phase!

- Scalar type
  - The loop diagram (a) gives no dipole operator.
  - For diagram (b),
    \[(\bar{f} \hat{R} f)(\bar{f'} \hat{L} f')\]
    carries no CP phase.

- To induce W-EDM, more than 3-loop is needed.
2-loop EDM

- The $1/\epsilon$ pole is absorbed by the dipole counter term, and the 2-loop EDM gives:

\[
\begin{align*}
\frac{d_f}{d_f} &= + \frac{e\alpha}{16\pi^3} \sum_i \frac{m_i \text{Im} c_{f,i}}{\Lambda^2} + 2 \ln \frac{m_i^2}{M_W^2} \\
&\quad + \Lambda^2 \ln \frac{m_i^2}{\Lambda^2}
\end{align*}
\]

- Earlier references:
  - Multi-Higgs
  - Pseudoscalar in MSSM
  - To rule out the MSSM EW Baryogenesis
  - Split SUSY
**RG for the dipole operator**

- The dim-6 EDM operator:

\[ \mathcal{O}_{EDM} = m(H) \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu}(W_{\mu\nu}) , \]

- It mixes with other 4-fermion operators at 2-loop level.
- The 1-loop RG

\[ d_f(\mu) \sim \left( 1 - \frac{4\alpha}{\pi} \ln \frac{M_W}{\mu} \right) d_f(M_W) \]

Charged lepton EDM

- The electron EDM

\[
d_e = \frac{e\alpha}{16\pi^3} \left\{ \frac{m_\tau}{\Lambda^2} \text{Im}(c_{e\tau\tau}^S) \ 2 + \ln \frac{m_\tau^2}{M_W^2} + \frac{m_\mu}{\Lambda^2} \text{Im}(c_{e\mu\mu}^S) \ 2 + \ln \frac{m_\mu^2}{M_W^2} \right\}
\]

- Barring the unnatural cancellation, the current limit \(|d_e| < 1.7 \times 10^{-27} \text{e cm}\) gives
Muon Michel Decay

• Michel decay:
  \[ \mu^- \rightarrow e^- \bar{\nu}_e \nu_{\mu} \text{ or } (\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e) \]

• If the electron polarization can be measured

\[
\vec{P}_e = P_L \hat{z} + P_{T1} \frac{(\hat{z} \times \vec{P}_\mu) \times \hat{z}}{|(\hat{z} \times \vec{P}_\mu) \times \hat{z}|} + P_{T2} \frac{\hat{z} \times \vec{P}_\mu}{|\hat{z} \times \vec{P}_\mu|}
\]

The \( P_{T2} \) is a T-violating observable.
**Muon Michel Decay—$P_{T_2}$**

- The latest bound on $P_{T_2}$ was given by PSI two decades ago which can be translated into 2 standard CP-odd Michel parameters:

  \[
  \alpha' = -0.003(69), \ \beta' = 0.024(101)
  \]

- Since 1994, a new experiment R-94-10 at PSI has been trying to push the precision one order of magnitude better.

- However, we predict that

  \[
  \alpha' = 0, \ \beta' = \frac{\sqrt{2}}{G_F \Lambda^2} \text{Im} C_{S\mu\mu ee}^{\mu\mu}
  \]

- From the present EDM constraint, we already have upper bound $\beta' < 4 \times 10^{-4}$ for $\Lambda = 1$TeV.

**PSI (or TRIUMF) can not see it!**
**Triple product correlation in** \( e^+ e^- \rightarrow \tau^+ \tau^- \)**

- With the upper bound obtained from EDM we can now estimate the size of CP violating signatures in a purely leptonic flavor conservation reaction such as 
  \( e^+(p_+) e^-(p_-) \rightarrow \tau^+(k_+) \tau^-(k_-) \) where the 4-momentum of each particle are shown in the corresponding bracket.
- It is straightforward to calculate the T-odd amplitude square

\[
|M_{TO}|^2 = \frac{s}{24e^2 \Lambda^2} \text{Im} C_{s}^{ee, \tau \tau} (\hat{p}_- + \hat{k}_-) \cdot (\vec{s}_e \times \vec{s}_\tau)
\]

where we have scaled by the strength of the QED term and \( s \) is the cm energy square.
Conclusion

• We study the leptonic CP violation by employing the complete set of dimension-six pure leptonic effective operators.
• Connection among the observable at different energy scales can be made by the running of the renormalization group equations.
• Explicitly, we study the charged lepton electric dipole moment, muon Michel decay, and the triple spin-momentum correlations at the Linear Collider.
• We found the electron electric dipole moment, which starts at 2-loop level, severely constrains the possibilities to detect the CP violating signatures in muon decay and at the linear colliders.