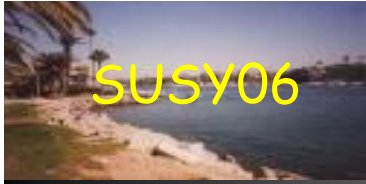


SUSY06 – Irvine, June 12 – June 17, 2006

Unitarity of the lepton mixing matrix

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Based on a collaboration with:
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J. López Pavón and C. Peña-Garay



Motivations

- ν masses and mixing \rightarrow evidence of Physics Beyond the SM
- Typical explanations (see-saw) involve NP with $\Lambda > \nu$
- This NP often induces deviations from **unitarity** at low energy



We will analyze the present constraints on the mixing matrix
without assuming **unitarity**

The general idea...

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23} - s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$



$$N = \begin{pmatrix} N_{e1} & N_{e2} & N_{e3} \\ N_{\mu1} & N_{\mu2} & N_{\mu3} \\ N_{\tau1} & N_{\tau2} & N_{\tau3} \end{pmatrix}$$

Effective Lagrangian

- 3 light ν
- all unitarity violation from NP with $\Lambda > v$
- flavour universality

$$L = i\bar{\nu}_\alpha \not{\partial} K_{\alpha\beta} \nu_\beta + \bar{\nu}_\alpha M_{\alpha\beta} \nu_\beta - \frac{g}{\sqrt{2}} (W_\mu^+ \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.) - \frac{g}{\cos\theta_W} (Z_\mu \bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.) + \dots$$

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Diagonal mass and canonical kinetic terms

$$L = i \bar{\nu}_i \partial \nu_i + \bar{\nu}_i m_{ii} \nu_i - \frac{g}{\sqrt{2}} (W_\mu^+ \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i} \nu_i + h.c.) - \frac{g}{\cos \theta_W} (Z_\mu \bar{\nu}_i \gamma^\mu P_L (N^+ N)_{ij} \nu_j + h.c.) + \dots$$

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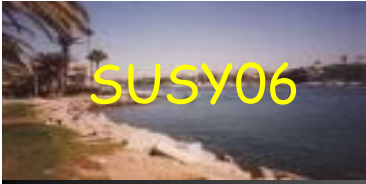
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$$\nu_\alpha = N_{\alpha i} \nu_i$$

N is not unitary



Effective Lagrangian

- 3 light ν
- all unitarity violation from NP with $\Lambda > \nu$
- flavour universality

$$L = i \bar{\nu}_\alpha \partial K_{\alpha\beta} \nu_\beta + \bar{\nu}_\alpha M_{\alpha\beta} \nu_\beta - \frac{g}{\sqrt{2}} (W_\mu^+ \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.) - \frac{g}{\cos \theta_W} (Z_\mu \bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.) + \dots$$

Diagonal mass and canonical kinetic terms

$$L = \underbrace{i \bar{\nu}_i \partial \nu_i + \bar{\nu}_i m_{ii} \nu_i}_{\text{unchanged}} - \frac{g}{\sqrt{2}} (W_\mu^+ \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i} \nu_i + h.c.) - \frac{g}{\cos \theta_W} (Z_\mu \bar{\nu}_i \gamma^\mu P_L (N^+ N)_{ij} \nu_j + h.c.) + \dots$$

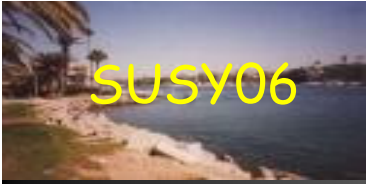
unchanged

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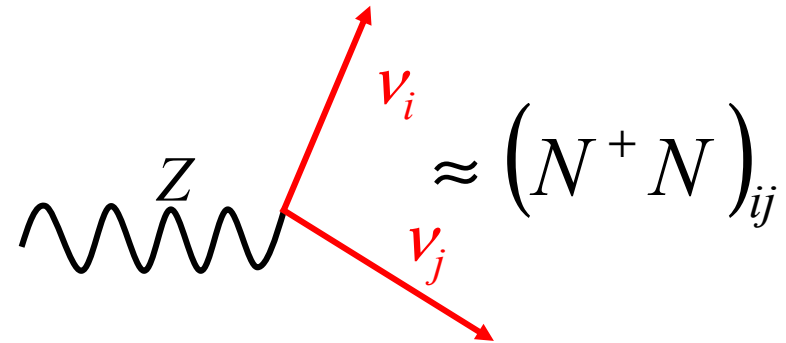
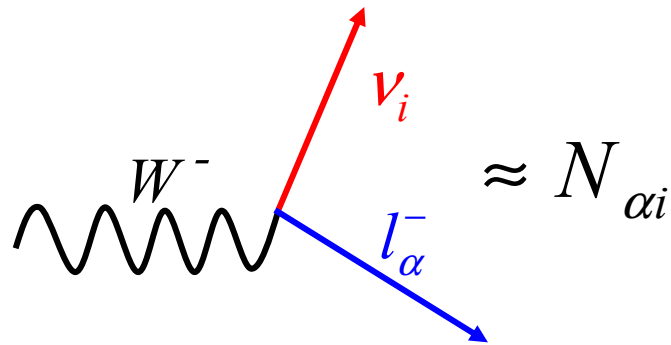


$$\langle \nu_i | \nu_j \rangle = \delta_{ij}$$



The effects of non-unitarity...

... appear in the interactions



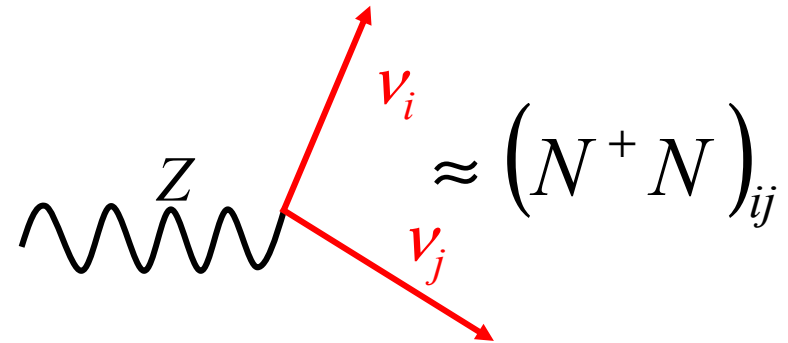
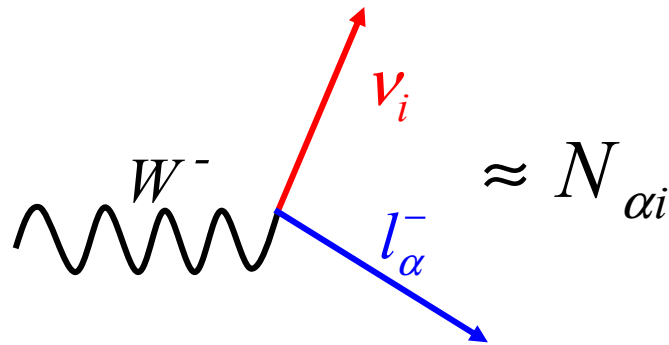
This affects **weak decays**...

$$\Gamma = \Gamma_{SM} \sum_i |N_{\alpha i}|^2 = \Gamma_{SM} (NN^+)_{\alpha\alpha}$$

$$\Gamma = \Gamma_{SM} \sum_{ij} |(N^+ N)_{ij}|^2$$

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This affects **weak decays**...

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$$\Gamma = \Gamma_{SM} \sum_{ij} |(N^+ N)_{ij}|^2$$

... and **oscillation probabilities**...

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{\sum_i |N_{\alpha i}|^2}} \sum_i N_{\alpha i}^* |\nu_i\rangle \equiv \sum_i \tilde{N}_{\alpha i}^* |\nu_i\rangle$$

$$\langle \nu_\beta | \nu_\alpha \rangle = (\tilde{N}^* \tilde{N}^t)_{\alpha\beta} \neq \delta_{\alpha\beta}$$

ν oscillation in vacuum

- mass basis
$$i \frac{d}{dt} |\nu_i\rangle = \hat{H}^{free} |\nu_i\rangle = \sum_j H_{ij}^{free} |\nu_j\rangle = E_i |\nu_i\rangle \quad |\nu_i(t)\rangle = e^{-iE_i t} |\nu_i^0\rangle$$

ν oscillation in vacuum

• mass basis $i \frac{d}{dt} |\nu_i\rangle = \hat{H}^{free} |\nu_i\rangle = \sum_j H_{ij}^{free} |\nu_j\rangle = E_i |\nu_i\rangle \quad |\nu_i(t)\rangle = e^{-iE_i t} |\nu_i^0\rangle$

• flavour basis $i \frac{d}{dt} |\nu_\alpha\rangle = \hat{H}^{free} |\nu_\alpha\rangle = \sum_\beta E_{\alpha\beta}^{free} |\nu_\beta\rangle$

with $E_{\alpha\beta}^{free} = \tilde{N}_{\alpha i}^* H_{ij}^{free} (\tilde{N}^*)_{j\beta}^{-1} \neq H_{\alpha\beta}^{free} = \tilde{N}_{\alpha i}^* H_{ij}^{free} \tilde{N}_{j\beta}^t$

ν oscillation in vacuum

• mass basis
$$i \frac{d}{dt} |\nu_i\rangle = \hat{H}^{free} |\nu_i\rangle = \sum_j H_{ij}^{free} |\nu_j\rangle = E_i |\nu_i\rangle \quad |\nu_i(t)\rangle = e^{-iE_i t} |\nu_i^0\rangle$$

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with
$$E_{\alpha\beta}^{free} = \tilde{N}_{\alpha i}^* H_{ij}^{free} (\tilde{N}^*)_{j\beta}^{-1} \neq H_{\alpha\beta}^{free} = \tilde{N}_{\alpha i}^* H_{ij}^{free} \tilde{N}_{j\beta}^t$$

$$P(\nu_\alpha \rightarrow \nu_\beta; L) = \frac{\left| \sum_i N_{\alpha i}^* e^{iP_i L} N_{\beta i} \right|^2}{\sum_j |N_{\alpha j}|^2 \sum_k |N_{\beta k}|^2}$$

Zero-distance effect:

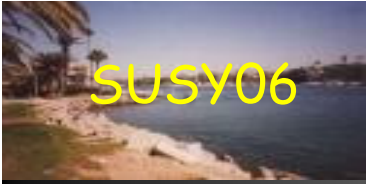
$$P(\nu_\alpha \rightarrow \nu_\beta; 0) \propto \left| \sum_i N_{\alpha i}^* N_{\beta i} \right|^2 \neq \delta_{\alpha\beta}$$

ν oscillation in matter

$$-L^{\text{int}} = \underbrace{\sqrt{2}G_F n_e \bar{\nu}_e \gamma^0 \nu_e}_{V_{CC}} - \underbrace{\frac{1}{\sqrt{2}}G_F n_n \sum_{\alpha} \bar{\nu}_{\alpha} \gamma^0 \nu_{\alpha}}_{V_{NC}}$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix} = \left[U^* \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^t + \begin{pmatrix} V_{CC} & -V_{NC} \\ 0 & -V_{NC} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix}$$

2 families



ν oscillation in matter

$$-L^{\text{int}} = \underbrace{\sqrt{2}G_F n_e \bar{\nu}_e \gamma^0 \nu_e}_{V_{CC}} - \frac{1}{\sqrt{2}} G_F n_n \sum_{\alpha} \underbrace{\bar{\nu}_{\alpha} \gamma^0 \nu_{\alpha}}_{V_{NC}}$$

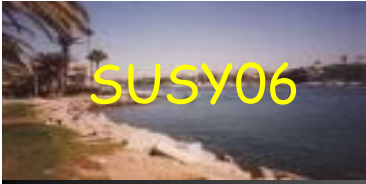
2 families

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix} = \left[U^* \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^t + \begin{pmatrix} V_{CC} & -V_{NC} & 0 \\ 0 & -V_{NC} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix}$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix} = N^* \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} (N^*)^{-1} + \begin{pmatrix} (V_{CC} - V_{NC}) \sum_i |N_{ei}|^2 & -V_{NC} \sqrt{\frac{\sum_i |N_{\mu i}|^2}{\sum_i |N_{ei}|^2}} \sum_i N_{ei}^* N_{\mu i} \\ (V_{CC} - V_{NC}) \sqrt{\frac{\sum_i |N_{ei}|^2}{\sum_i |N_{\mu i}|^2}} \sum_i N_{ei}^* N_{\mu i} & -V_{NC} \sum_i |N_{\mu i}|^2 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix}$$

1. non-diagonal elements

2. NC effects do not disappear



N elements from oscillations: e -row

- Only disappearance exps \rightarrow informations only on $|N_{\alpha i}|^2$

CHOOZ: $\Delta_{12} \approx 0$

$$\Delta_{ij} = \Delta m_{ij}^2 L / 2E$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \cong \left(|N_{e1}|^2 + |N_{e2}|^2 \right)^2 + |N_{e3}|^4 + 2 \left(|N_{e1}|^2 + |N_{e2}|^2 \right) |N_{e3}|^2 \cos(\Delta_{23})$$

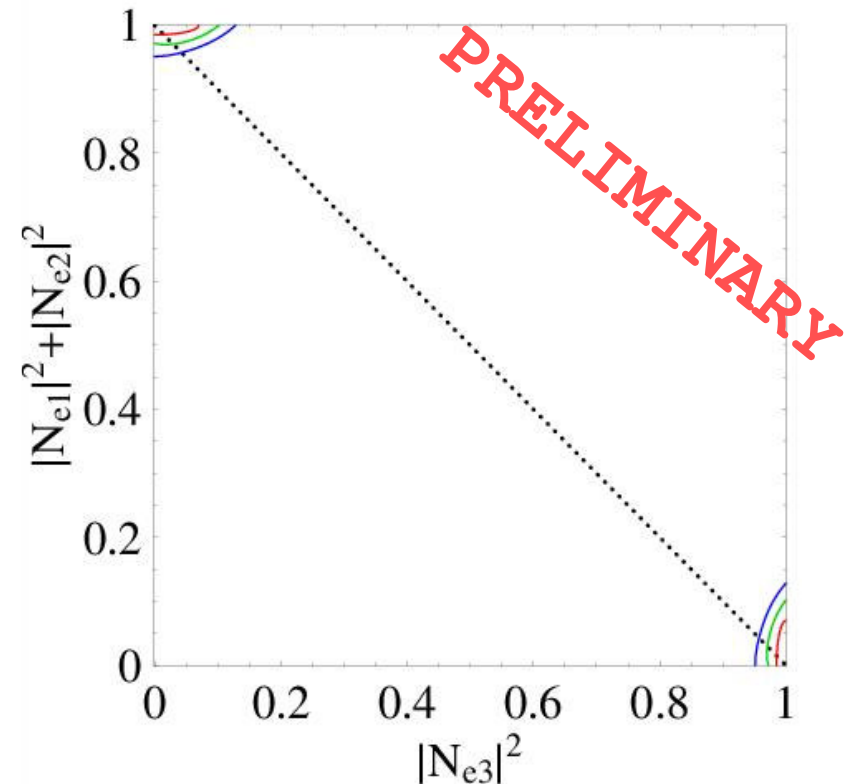
K2K ($\nu_\mu \rightarrow \nu_\mu$): Δ_{23}

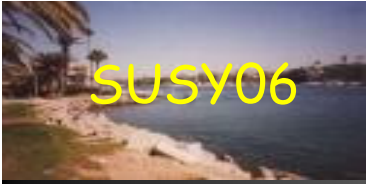
1. Degeneracy

$$|N_{e1}|^2 + |N_{e2}|^2 \leftrightarrow |N_{e3}|^2$$

2. $|N_{e1}|^2, |N_{e2}|^2$

cannot be disentangled





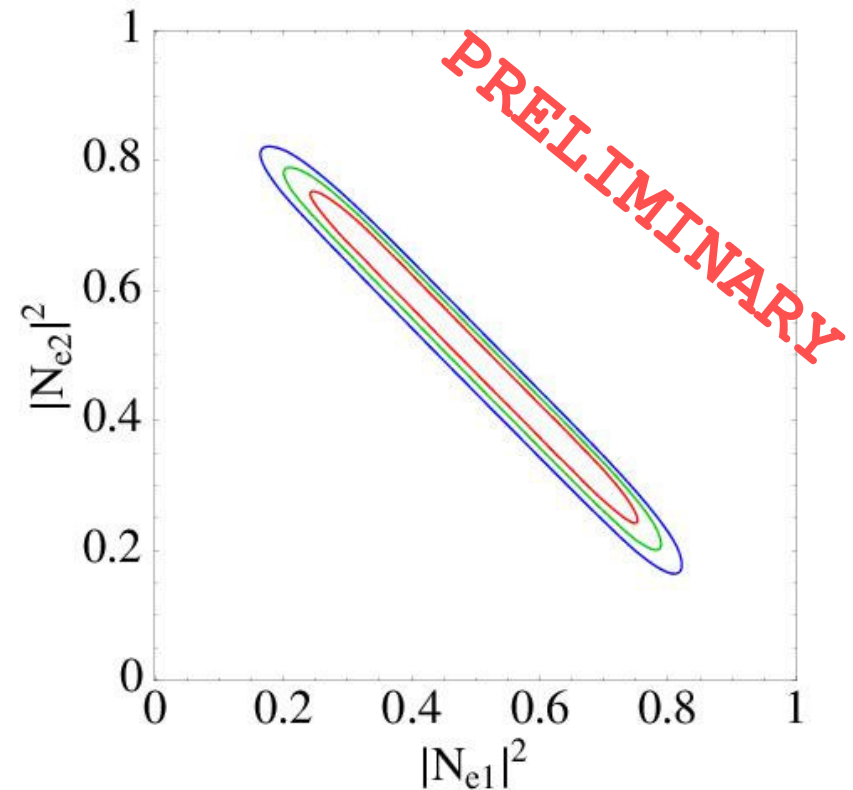
N elements from oscillations: e -row

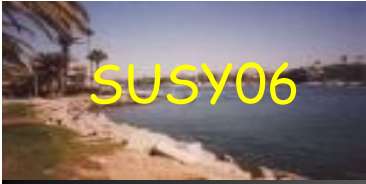
KamLAND: $\Delta_{23} \gg 1$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \cong |N_{e1}|^4 + |N_{e2}|^4 + |N_{e3}|^4 + 2|N_{e1}|^2|N_{e2}|^2 \cos(\Delta_{12})$$

$$\begin{cases} |N_{e1}|^2 + |N_{e2}|^2 \approx 1 \\ |N_{e3}|^2 \approx 0 \end{cases}$$

→ first degeneracy solved





N elements from oscillations: e -row

KamLAND: $\Delta_{23} \gg 1$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \cong |N_{e1}|^4 + |N_{e2}|^4 + |N_{e3}|^4 + 2|N_{e1}|^2|N_{e2}|^2 \cos(\Delta_{12})$$

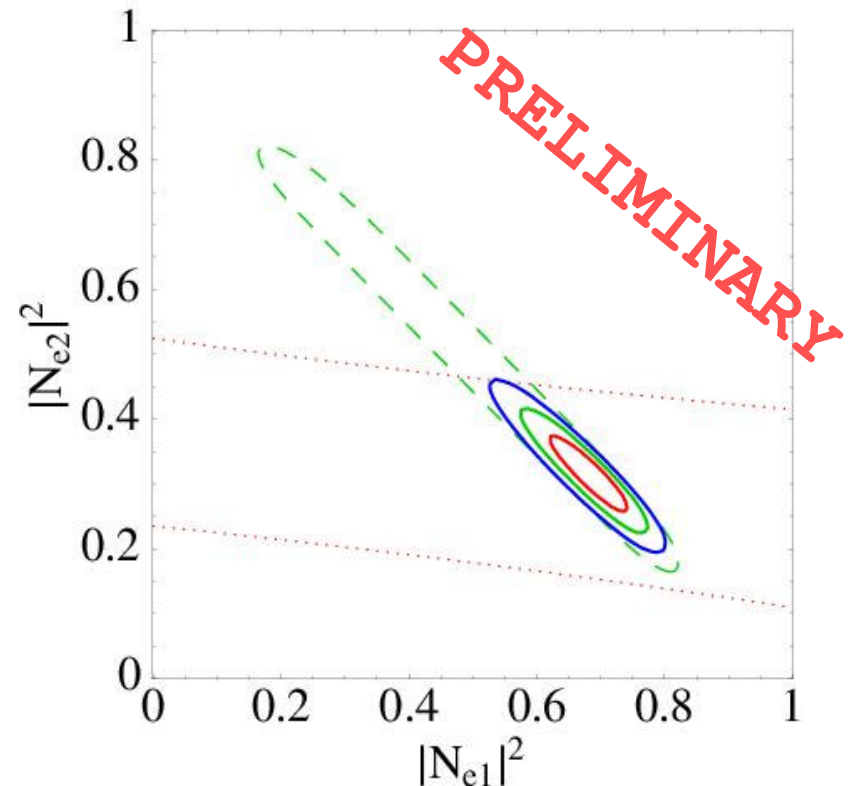
$$\begin{cases} |N_{e1}|^2 + |N_{e2}|^2 \approx 1 \\ |N_{e3}|^2 \approx 0 \end{cases}$$

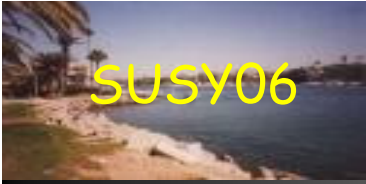
→ first degeneracy solved

SNO:

$$P(\nu_e \rightarrow \nu_e) \cong 0.1|N_{e1}|^2 + 0.9|N_{e2}|^2$$

→ all $|N_{ei}|^2$ determined





N elements from oscillations: μ -row

Atmospheric + K2K: $\Delta_{12} \approx 0$

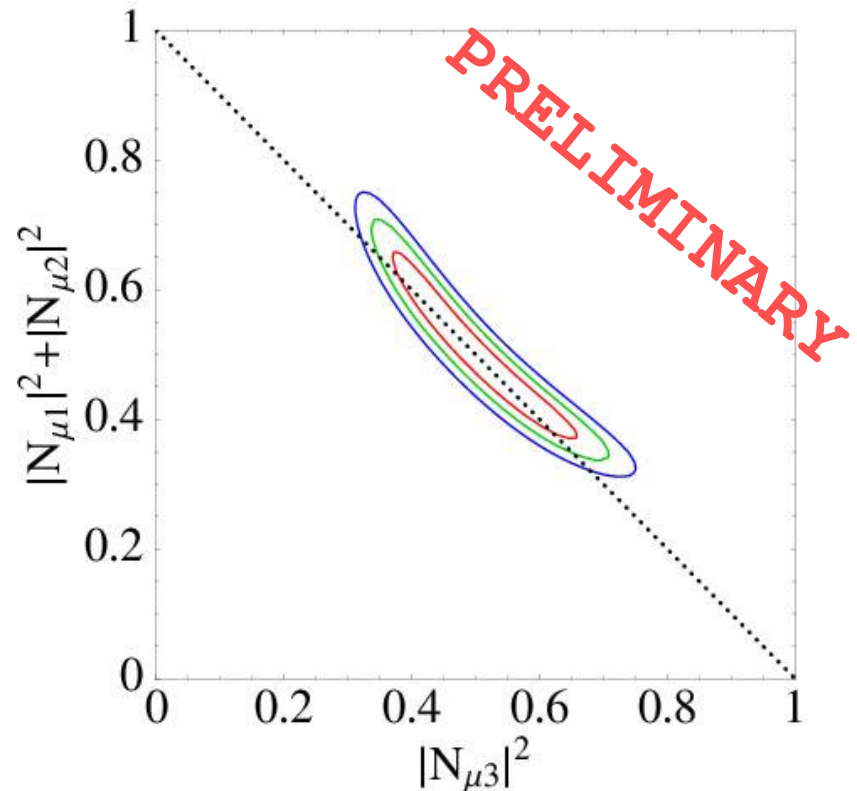
$$P(\nu_\mu \rightarrow \nu_\mu) \cong \left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2 \right)^2 + |N_{\mu 3}|^4 + 2 \left(|N_{\mu 1}|^2 + |N_{\mu 2}|^2 \right) |N_{\mu 3}|^2 \cos(\Delta_{23})$$

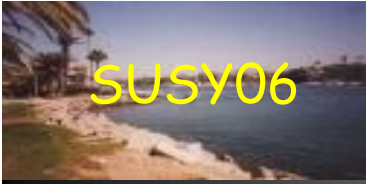
1. Degeneracy

$$|N_{\mu 1}|^2 + |N_{\mu 2}|^2 \leftrightarrow |N_{\mu 3}|^2$$

2. $|N_{\mu 1}|^2, |N_{\mu 2}|^2$

cannot be disentangled





N elements from oscillations only

with unitarity
OSCILLATIONS

$$|U|^2 = \begin{pmatrix} 0.62 - 0.77 & 0.22 - 0.37 & < 0.04 \\ 0.04 - 0.27 & 0.18 - 0.53 & 0.34 - 0.67 \\ 0.04 - 0.28 & 0.19 - 0.55 & 0.31 - 0.66 \end{pmatrix}$$

M. C. Gonzalez Garcia hep-ph/0410030

3σ

without unitarity
OSCILLATIONS

$$|N|^2 = \begin{pmatrix} 0.58 - 0.80 & 0.20 - 0.44 & < 0.13 \\ [|N_{\mu 1}|^2 + |N_{\mu 2}|^2 = 0.32 - 0.73] & 0.32 - 0.73 \\ ? & ? & ? \end{pmatrix}$$

PRELIMINARY

(NN^+) from decays

- W decays

$$\rightarrow \frac{(NN^+)_{\alpha\alpha}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$$

- Universality tests

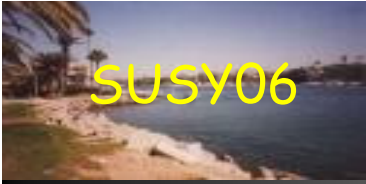
$$\rightarrow \frac{(NN^+)_{\alpha\alpha}}{(NN^+)_{\beta\beta}}$$

- Invisible Z

$$\rightarrow \frac{\sum_{ij} (N^+ N)_{ij}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$$

- Rare leptons decays

$$\rightarrow \frac{|(NN^+)_{\beta\alpha}|^2}{(NN^+)_{\alpha\alpha} (NN^+)_{\beta\beta}}$$



(NN^+) from decays

• W decays $\rightarrow \frac{(NN^+)_{\alpha\alpha}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$

• Universality tests $\rightarrow \frac{(NN^+)_{\alpha\alpha}}{(NN^+)_{\beta\beta}}$

• Invisible Z $\rightarrow \frac{\sum_{ij} (N^+ N)_{ij}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$

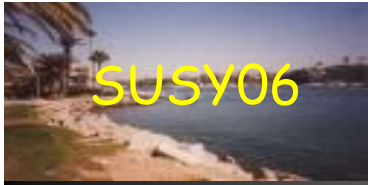
• Rare leptons decays $\rightarrow \frac{|(NN^+)_{\beta\alpha}|^2}{(NN^+)_{\alpha\alpha} (NN^+)_{\beta\beta}}$

→ Global fit

90% cl $|NN^+| \approx \begin{pmatrix} 1.002 \pm 0.005 & < 7.2 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.2 \cdot 10^{-5} & 1.003 \pm 0.005 & < 1.3 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.3 \cdot 10^{-2} & 1.003 \pm 0.005 \end{pmatrix}$

PRELIMINARY

→ N is unitary at % level



N elements from oscillations & decays

with unitarity
OSCILLATIONS

$$|U|^2 = \begin{pmatrix} 0.62 - 0.77 & 0.22 - 0.37 & < 0.04 \\ 0.04 - 0.27 & 0.18 - 0.53 & 0.34 - 0.67 \\ 0.04 - 0.28 & 0.19 - 0.55 & 0.31 - 0.66 \end{pmatrix}$$

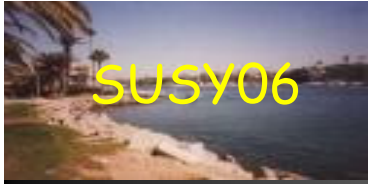
M. C. Gonzalez Garcia hep-ph/0410030

3σ

without unitarity
OSCILLATIONS
+DECAYS

$$|N|^2 = \begin{pmatrix} 0.57 - 0.79 & 0.21 - 0.43 & < 0.04 \\ 0.04 - 0.30 & 0.17 - 0.54 & 0.33 - 0.67 \\ 0.01 - 0.33 & 0.10 - 0.58 & 0.29 - 0.70 \end{pmatrix}$$

PRELIMINARY



In the future...

MEASUREMENT OF MATRIX ELEMENTS

- $|N_{e3}|^2$, μ -row \rightarrow MINOS, T2K, Super-Beams, ν Factories...
- τ -row \rightarrow high energies: ν Factories
- phases \rightarrow *appearance* experiments: Super-Beams, ν Factories, β -beams

TESTS OF UNITARITY



Rare leptons decays (present)

- $\mu \rightarrow e\gamma$ $|\sum_i N_{ei} N_{\mu i}^*|^2 < 7.2 \cdot 10^{-5}$

- $\tau \rightarrow e\gamma$ $|\sum_i N_{ei} N_{\tau i}^*|^2 < 0.016$

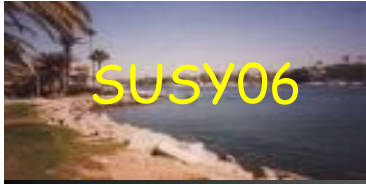
- $\tau \rightarrow \mu\gamma$ $|\sum_i N_{\mu i} N_{\tau i}^*|^2 < 0.013$

ZERO-DISTANCE EFFECT
Near detector at a ν factory

- $\nu_e \rightarrow \nu_\mu$ $|\sum_i N_{ei} N_{\mu i}^*|^2 < 2.3 \cdot 10^{-4}$

- $\nu_e \rightarrow \nu_\tau$ $|\sum_i N_{ei} N_{\tau i}^*|^2 < 2.9 \cdot 10^{-3}$

- $\nu_\mu \rightarrow \nu_\tau$ $|\sum_i N_{\mu i} N_{\tau i}^*|^2 < 2.6 \cdot 10^{-3}$



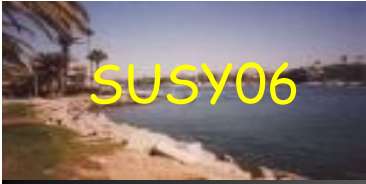
Conclusions

If we **don't assume unitarity** for the lepton mixing matrix

- Present **oscillation experiments** can only measure half the elements
- **EW decays** confirms **unitarity at % level**
- Combining oscillations and decays, bounds for **all** the elements can be found comparable with the ones obtained with the unitary analysis

Future experiments can:

- improve the present measurements on the e - and μ - rows
- give information on the τ -row and on phases (appearance exps)
- test unitarity by constraining the zero-distance effect with a near detector
- discriminate among different NP scenarios



Non-unitarity from see-saw

$$L = L_{SM} + i\bar{N}_R \not{\partial} N_R - Y_\nu (\bar{l}_L \tilde{H} N_R + \bar{N}_R \tilde{H}^+ l_L) - \frac{1}{2} M (\bar{N}_R^c N_R + \bar{N}_R N_R^c)$$

Integrate out N_R

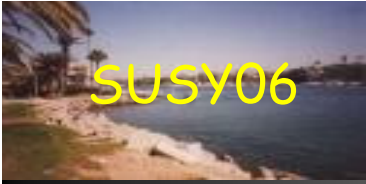
$$L^{eff} = L_{SM} + \frac{1}{\Lambda} L^{d=5} + \frac{1}{\Lambda^2} L^{d=6} + \dots$$

$$-\frac{1}{4} \left(\bar{l}_{L\alpha} \vec{\tau} \left[Y_\nu^* \frac{\eta}{M} Y_\nu^+ \right]_{\alpha\beta} l_{L\beta} \right) (\tilde{H}^+ \vec{\tau} H)$$

d=5 operator
it gives mass to ν

$$(\bar{l}_{L\alpha} \tilde{H}) i \not{\partial} \left[Y_\nu \frac{1}{M^2} Y_\nu^+ \right]_{\alpha\beta} (\tilde{H}^+ l_{L\beta})$$

d=6 operator
it renormalises kinetic energy



In a real experiment...

$$n_{ev} \sim \int dE \frac{d\Phi_\alpha}{dE} P_{\alpha\beta}(E) \sigma_\beta(E) \epsilon(E)$$

$$\nu \text{ produced in CC} \quad \frac{d\Phi_\alpha}{dE} \sim \sum_i |N_{\alpha i}|^2 \frac{d\Phi_\alpha^{SM}}{dE}$$

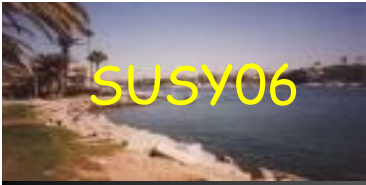
$$\nu \text{ detected in CC} \quad \sigma_\beta \sim \sum_i |N_{\beta i}|^2 \sigma_\beta^{SM}$$

$$n_{ev} \sim \int dE \frac{d\Phi_\alpha^{SM}}{dE} \underbrace{\sum_i |N_{\alpha i}|^2 P_{\alpha\beta}(E) \sum_i |N_{\beta i}|^2 \sigma_\beta^{SM}(E)}_{\hat{P}(\nu_\alpha \rightarrow \nu_\beta; L)} \epsilon(E)$$

$$\hat{P}(\nu_\alpha \rightarrow \nu_\beta; L) = \left| \sum_i N_{\alpha i}^* e^{iP_i L} N_{\beta i} \right|^2$$

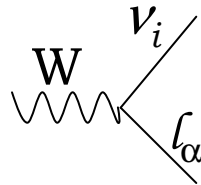
Exceptions:

- flux measured: no $\sum_i |N_{\alpha i}|^2$
- detection via NC: $\sum_i |N_{\beta i}|^2$ replaced by $\sum_{i,j} |N_{\alpha i}^* N_{\beta j}|^2$
- production via μ or τ decay: flux corrected by 2 factors $\sum_i |N_{\alpha i}|^2 \sum_j |N_{\beta j}|^2$



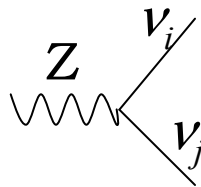
(NN^\dagger) from decays: G_F

- W decays



$$\rightarrow \frac{(NN^+)_{\alpha\alpha}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$$

- Invisible Z



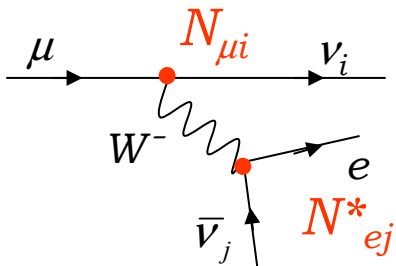
$$\rightarrow \frac{\sum_{ij} (N^+ N)_{ij}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$$

- Universality tests

$$\rightarrow \frac{(NN^+)_{\alpha\alpha}}{(NN^+)_{\beta\beta}}$$

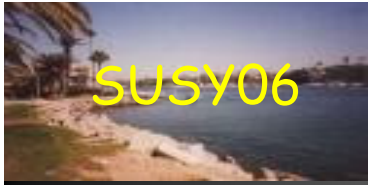
Infos on $(NN^\dagger)_{\alpha\alpha}$

G_F is measured in μ -decay



$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \sum_i |N_{\mu i}|^2 \sum_j |N_{e j}|^2$$

$$G_F^2 = \frac{G_{F,\text{exp}}^2}{\sum_i |N_{\mu i}|^2 \sum_j |N_{e j}|^2}$$



(NN^\dagger) and $(N^\dagger N)$ from decays

PRELIMINARY

$$|NN^\dagger| \approx \begin{pmatrix} 1.002 \pm 0.005 & < 7.2 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.2 \cdot 10^{-5} & 1.003 \pm 0.005 & < 1.3 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.3 \cdot 10^{-2} & 1.003 \pm 0.005 \end{pmatrix} \quad \text{Experimentally}$$

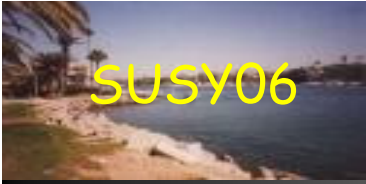
$$N = HV \quad NN^\dagger = H^2 = 1 + \varepsilon \quad \text{with } \varepsilon = \varepsilon^+$$

$$N^\dagger N = 1 + V^\dagger \varepsilon V = 1 + \varepsilon'$$

$$|\varepsilon'_{ij}| \leq \sqrt{\sum_{\alpha\beta} |\varepsilon_{\alpha\beta}|^2} \approx 0.03$$

$$|N^\dagger N| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix} \quad \begin{array}{l} \text{Estimation} \\ \text{(the most conservative)} \end{array}$$

→ N is unitary at % level

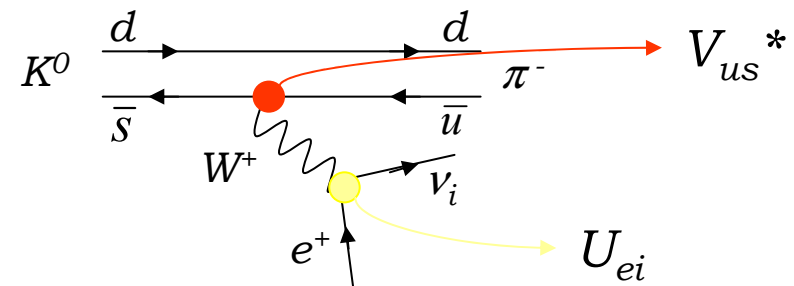


Unitarity in the quark sector

Quarks are detected as mass eigenstates

→ we can directly measure $|V_{ab}|$

ex: $|V_{us}|$ from $K^0 \rightarrow \pi^- e^+ \nu_e$

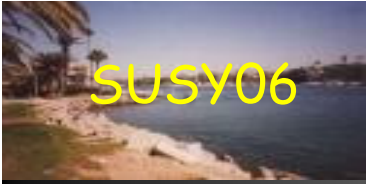


$$\rightarrow \sum_i |U_{ei}|^2 = 1 \quad \text{if } U \text{ unitary}$$

With V_{ab} we check unitarity conditions:

$$\text{ex: } |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008 \pm 0.0011$$

→ *Measurements of V_{CKM} elements relies on U_{PMNS} unitarity*

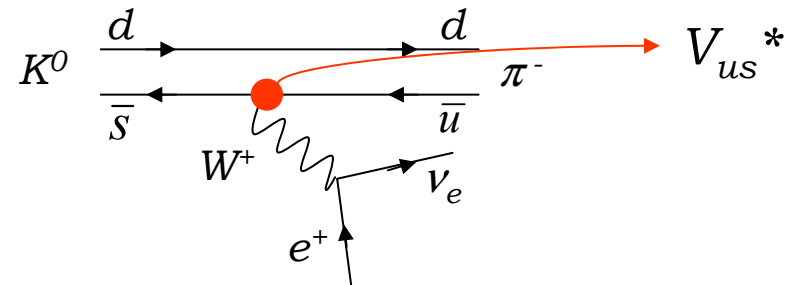


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→ Measurements of V_{CKM} elements relies on U_{PMNS} unitarity

- decays → only (NN^\dagger) and $(N^\dagger N)$
- With leptons: • N elements → **we need oscillations**
- to study the unitarity of N : no assumptions on V_{CKM}