Toward precision measurements in solar neutrinos

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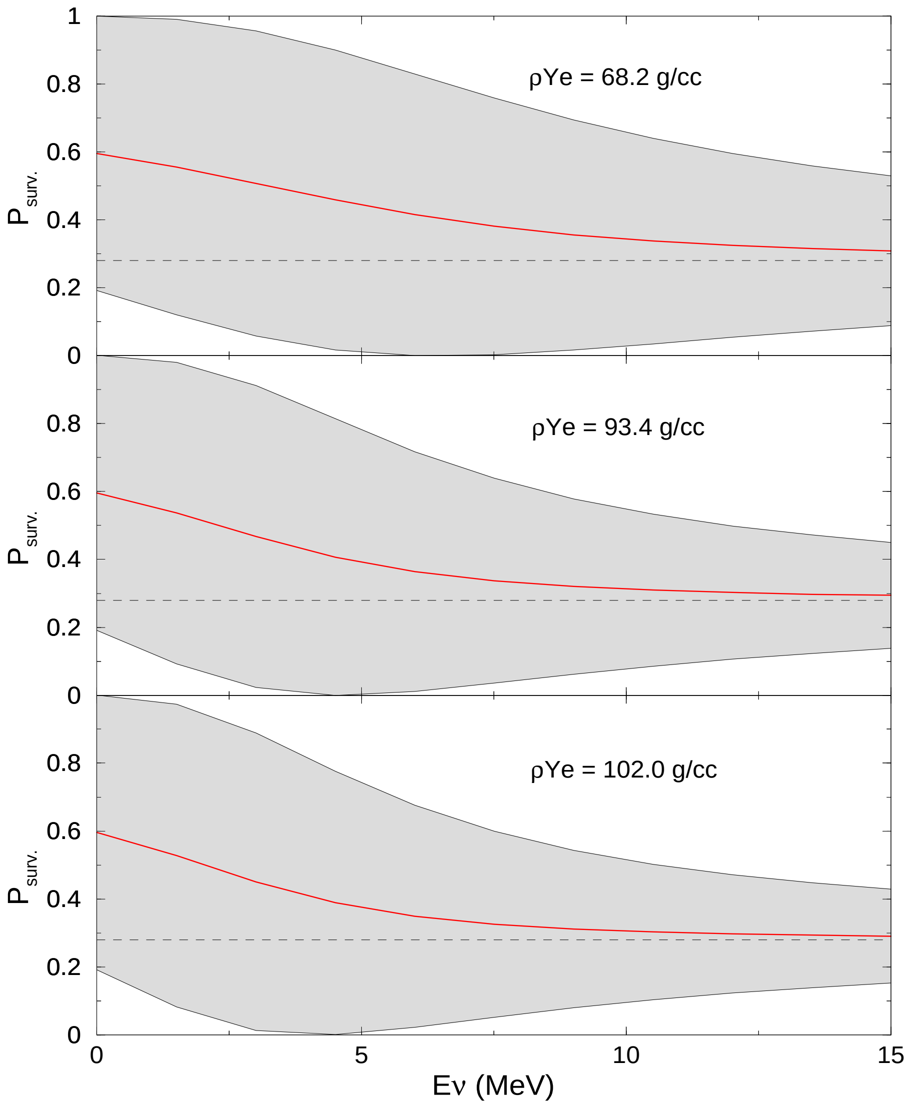
LMA MSW solution established

Super-K, SNO, KamLAND
Homestake, Sage, Gallex/GNO

LMA MSW solution:

$$\Delta m^2 = (4-10)\times 10^{-5}\text{eV}^2,$$
$$\tan^2\theta = 0.32 - 0.47$$

- $L_{osc} = \frac{4\pi E}{\Delta m^2} \sim$ a few hundreds km.
- For $E \gtrsim 7$ MeV, $P_{ee} = \frac{1}{2}(1-\cos 2\theta) \approx 0.3;$
- for $E \lesssim 1$ MeV, $P_{ee} = \frac{1}{2}(1+\cos^2 2\theta) \approx 0.6.$
- Matter effect small in Earth.



Future precision measurement

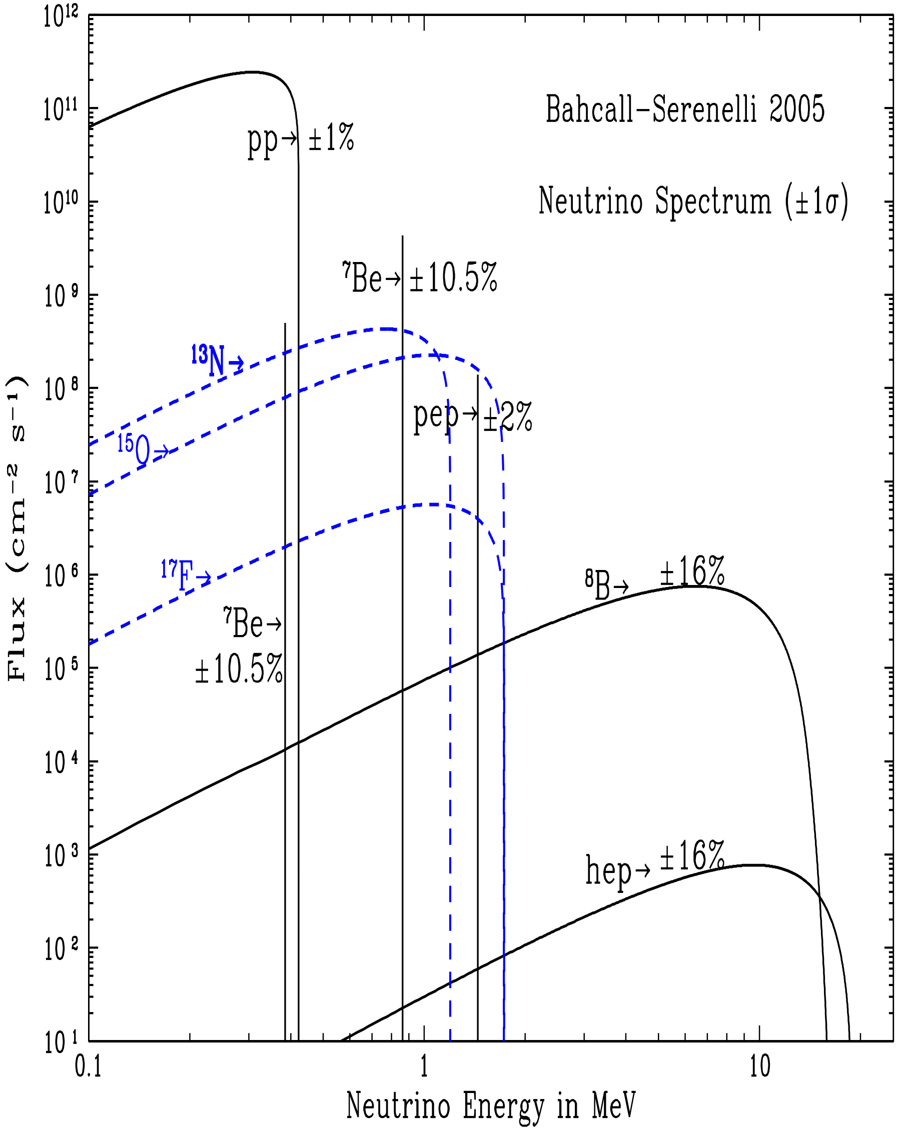
Future experiments include Borexino, Hyper-K

What to measure?

- spectrum distortion

$$f_{measured} \sim f_\nu \times P_{ee},$$
$$P_{ee} = \frac{1}{2}(1 + \cos 2\theta_m^0 \cos 2\theta).$$

$\theta_m^0 = \theta_m^0(E)$: mixing angle in matter at ν production point.



Future precision measurement

Earth Matter effect:

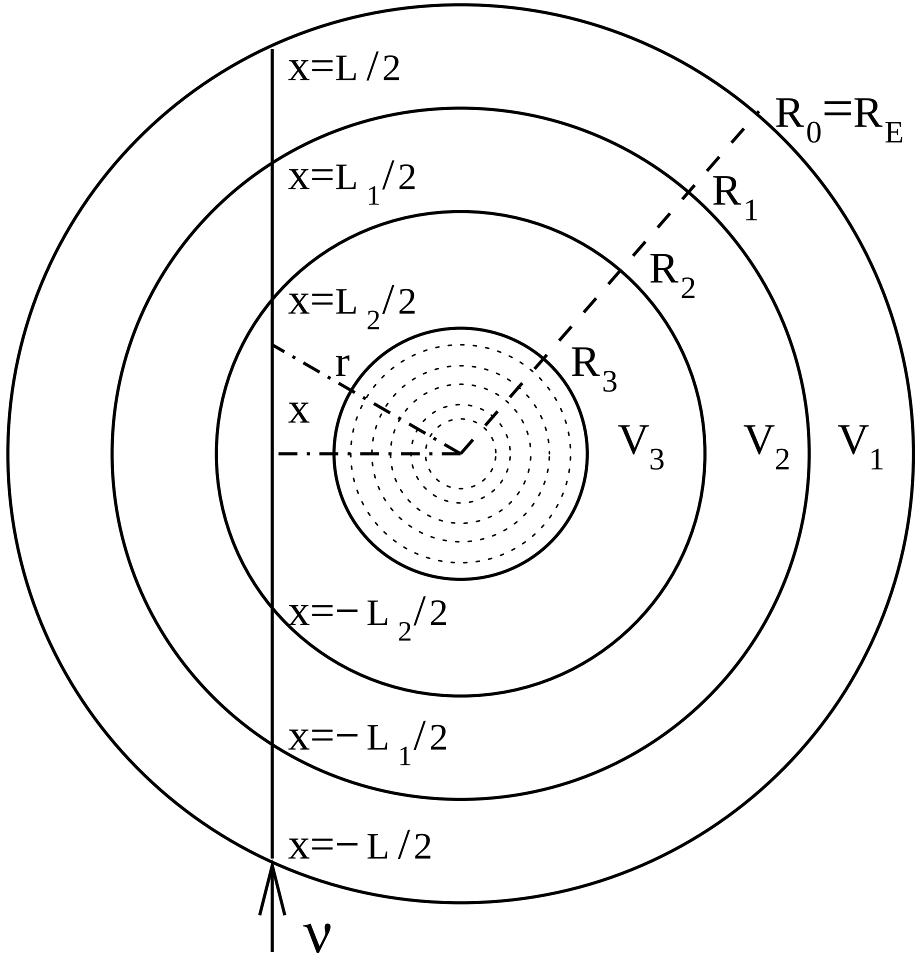
- Day-Night asymmetry.

$$A_{D-N} = \frac{Day - Night}{Day + Night}.$$

$$P_{ee} = \frac{1}{2}(1 + \cos 2\theta_m^0 \cos 2\theta) - \cos 2\theta_m^0 f_{reg}.$$

The regeneration factor f_{reg}:

$P(\nu_2 \to \nu_e) = \sin^2\theta + f_{reg}$.

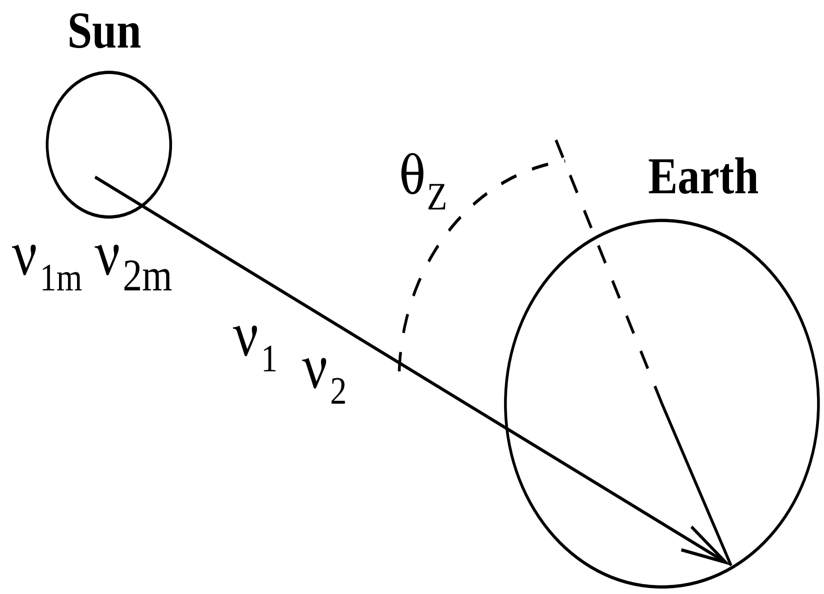


Toward precision measurement

- Future experiments (e.g. Borexino,Hyper-K) will reach precision $\lesssim 1\%$.
- Computations were done numerically and physics is far from completely understood.
- Theorists are challenged to obtain precise analytic formula directly related to observables.

Problems include

- non-adiabaticity in Sun;
- effect of averaging over distribution of ν source in Sun;
- non-adiabaticity in the Earth;
- uncertainties of small structures in the Earth;
- effect of averaging over energy range.



Adiabatic perturbation

$$i\frac{d}{dx}\begin{pmatrix}\psi_{m1}\\ \psi_{m2}\end{pmatrix} = \begin{pmatrix}-\Delta(x)/4E & -i\dot{\theta}_m\\ i\dot{\theta}_m & \Delta(x)/4E\end{pmatrix}\begin{pmatrix}\psi_{m1}\\ \psi_{m2}\end{pmatrix}.$$

$\Delta(x)$: the mass squared difference in matter. $\dot{\theta}_m = d\theta_m/dx$.

We search solution of the following form

$$\begin{aligned}\begin{pmatrix}\psi_{1m}(x)\\ \psi_{2m}(x)\end{pmatrix} &= \begin{pmatrix}e^{i\Phi(x)} & c(x)e^{-i\Phi(x)}\\ -c^*(x)e^{i\Phi(x)} & e^{-i\Phi(x)}\end{pmatrix}\begin{pmatrix}\psi_{1m}(x_0)\\ \psi_{2m}(x_0)\end{pmatrix},\\ \Phi(x) &\equiv \int_{x_0}^{x} dx' \,\frac{\Delta(x')}{4E}.\end{aligned}$$

Φ is the adiabatic phase, $|c(x)| \ll 1$ is supposed to hold everywhere along the neutrino trajectory. We get

$$c(x) = -\int_{x_0}^{x} dx' \frac{d\theta_m(x')}{dx'} \exp\left[-i\int_{x}^{x'} dx'' \frac{\Delta(x'')}{2E}\right].$$

Results in 2-2 analysis can be directly inserted into 3-3 analysis.

Non-adiabaticity in Sun

Including the non-adiabatic correction, the survival probability is

$$P_{ee} = \frac{1}{2}[1 + (1 - 2P_c)\cos 2\theta_m(x_0)\cos 2\theta].$$

$P_c = P(\nu_{m1} \leftrightarrow \nu_{m2})$ is the level crossing probability.

In Sun $P_c = |c(x_f)|^2$ is estimated to be

$$P_c = \frac{\gamma^2(x_0)}{4} = \frac{1}{16\pi^2}\frac{l_{osc}^2(x_0)}{h^2(x_0)}\left[\frac{2EV(x_0)\Delta m^2 \sin 2\theta}{\Delta(x_0)^2}\right]^2,$$

$\gamma \equiv 4E|\dot{\theta}_m|/\Delta$.

- $P_c = (10^{-9} - 10^{-7})\,(E/10\ \mathrm{MeV})^2$. P_c in the Sun can be safely neglected;
- $P_{ee} = P_{ee}(x_0)$ has no dependence on trajectory and is simply

$$P_{ee} = \frac{1}{2}(1 + \cos 2\theta_m(x_0)\cos 2\theta).$$

Averaging over ν source

Since $P_{ee} = P_{ee}(x_0)$, averaging is simple:

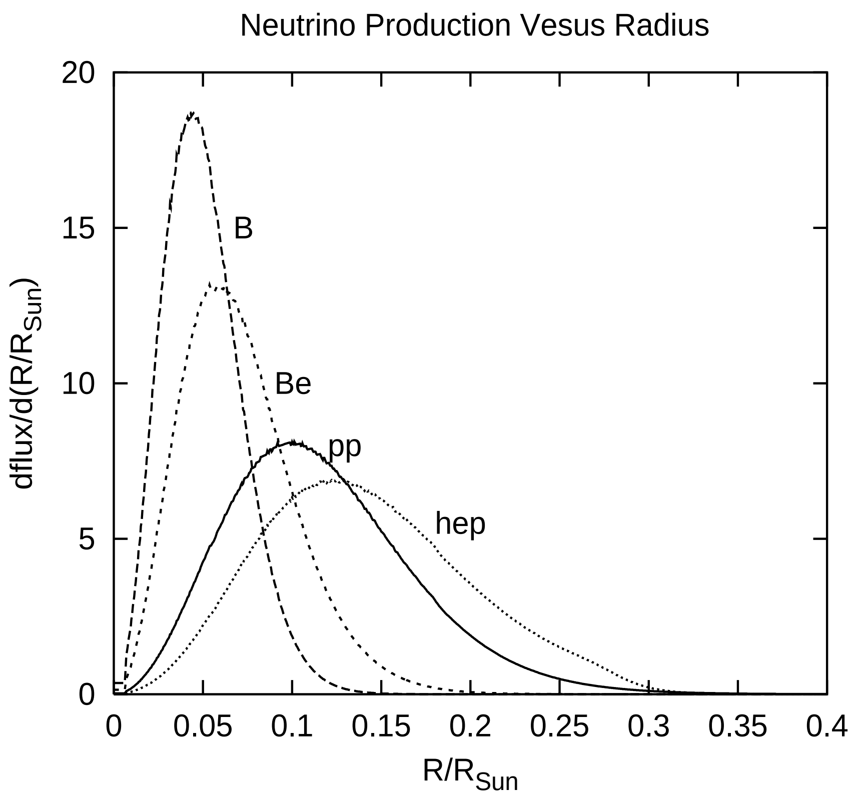
$$P_K = \frac{\int dr\ G_K(r) P_{ee}(r)}{\int dr\ G_K(r)},$$

where $K = pp, pep, Be, N, O, F, B, hep$.
G_K is the distribution function.

We expand $P_{ee}(V_e)$ in series of

$$\delta V_e = V_e - \bar{V}_e.$$

One obtains



$$P_K \approx \frac{1}{2} + \frac{1}{2}(1 - \delta_K)\cos 2\theta_m(\bar{V}_K)\cos 2\theta,$$

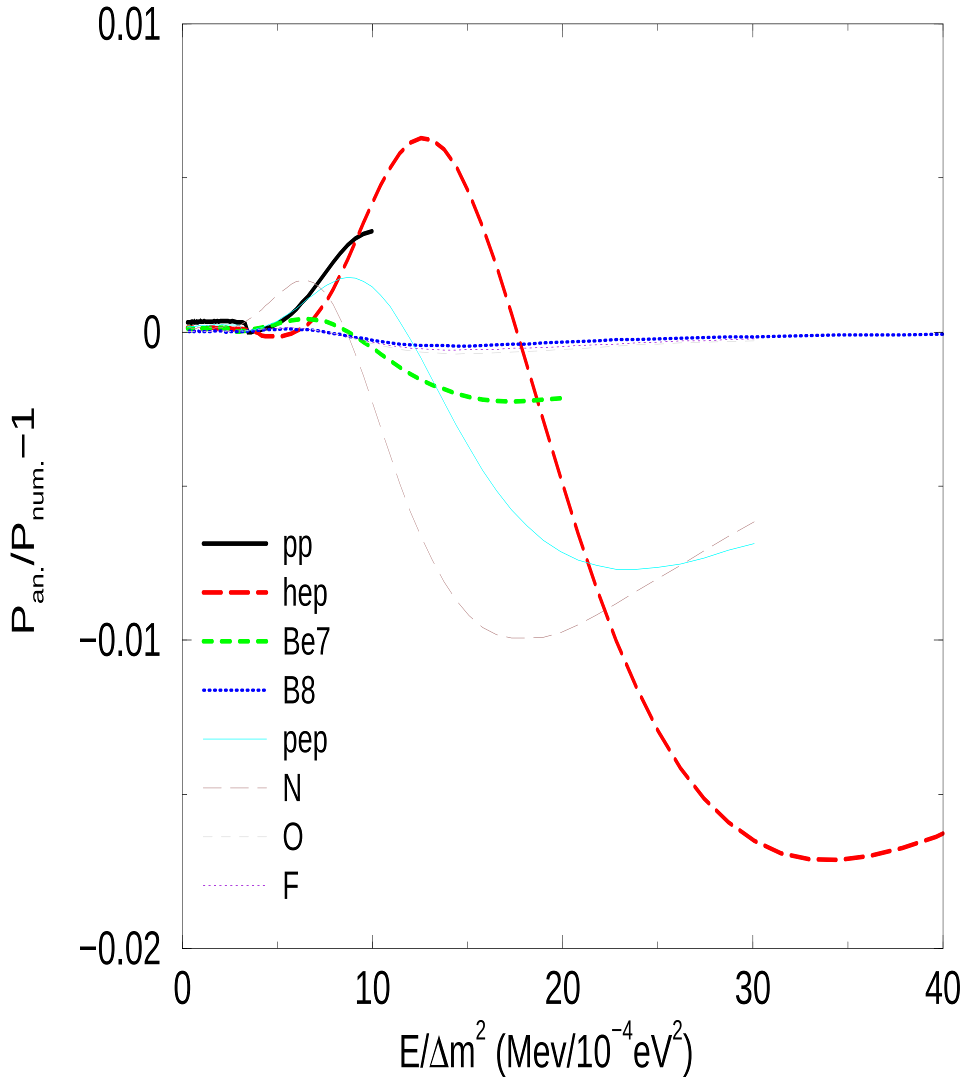
$$\delta_K = \frac{3}{2}\frac{(2E\bar{V}_K/\Delta m^2)^2 \sin^2 2\theta}{[(\cos 2\theta - 2E\bar{V}_K/\Delta m^2)^2 + \sin^2 2\theta]^2}\frac{\Delta V_K^2}{\bar{V}_K^2},$$

$$\bar{V}_K \equiv \frac{\int dr\ G_K(r)V(r)}{\int dr\ G_K(r)}, \quad \Delta V_K^2 \equiv \frac{\int dr\ G_K(r)(V(r) - \bar{V}_K)^2}{\int dr\ G_K(r)}.$$

Averaging over ν source

Comparison with the numerical computation shows:

- analytic formula reproduces numerical result with accuracy 10^{-3} for small and large values of $E/\Delta m^2$;
- maximal deviation, 1.8%, happens to $K = hep$ at around $E/\Delta m^2 = 34 \times 10^{10}$ eV^{-1}.



Matter effect in the Earth

Regeneration by the Earth, $f_{reg} = P(\nu_2 \to \nu_e) - \sin^2\theta$ is found to be

$$P_{ee} = \frac{1}{2}(1 + \cos 2\theta_m(x_0)\cos 2\theta) - \cos 2\theta_m(x_0) f_{reg},$$
$$f_{reg} = \frac{2EV_R}{\Delta m^2}\sin^2 2\theta \sin^2 \Phi(x_f) + \sin 2\theta Re\{c(x_f)\};$$

$f_{reg} \sim \mathcal{O}(2EV_R/\Delta m^2) \sim 2\%$ **for** $E \sim 10$ **MeV;**
$c(x_f)$: non-adiabatic transition amplitude in the Earth;
V_R: potential at the surface of the Earth; $\Phi(x_f)$: total phase acquired in the Earth.

- non-adiabaticity within each layer is negligible ($\lesssim 0.1\%$, since density height $h \sim R_{Earth} \gg L_{osc}$);
- corrections of borders between neighboring layers are important. We approximate

$$\dot{\theta}_m(x) = \frac{E \sin 2\theta}{\Delta m^2} \sum_{i=1}^{n-1} \Delta V_i \left[\delta(x + \frac{L_i}{2}) - \delta(x - \frac{L_i}{2})\right].$$

$\pm\Delta V_i$: potential jumps at $x = \mp L_i/2$.

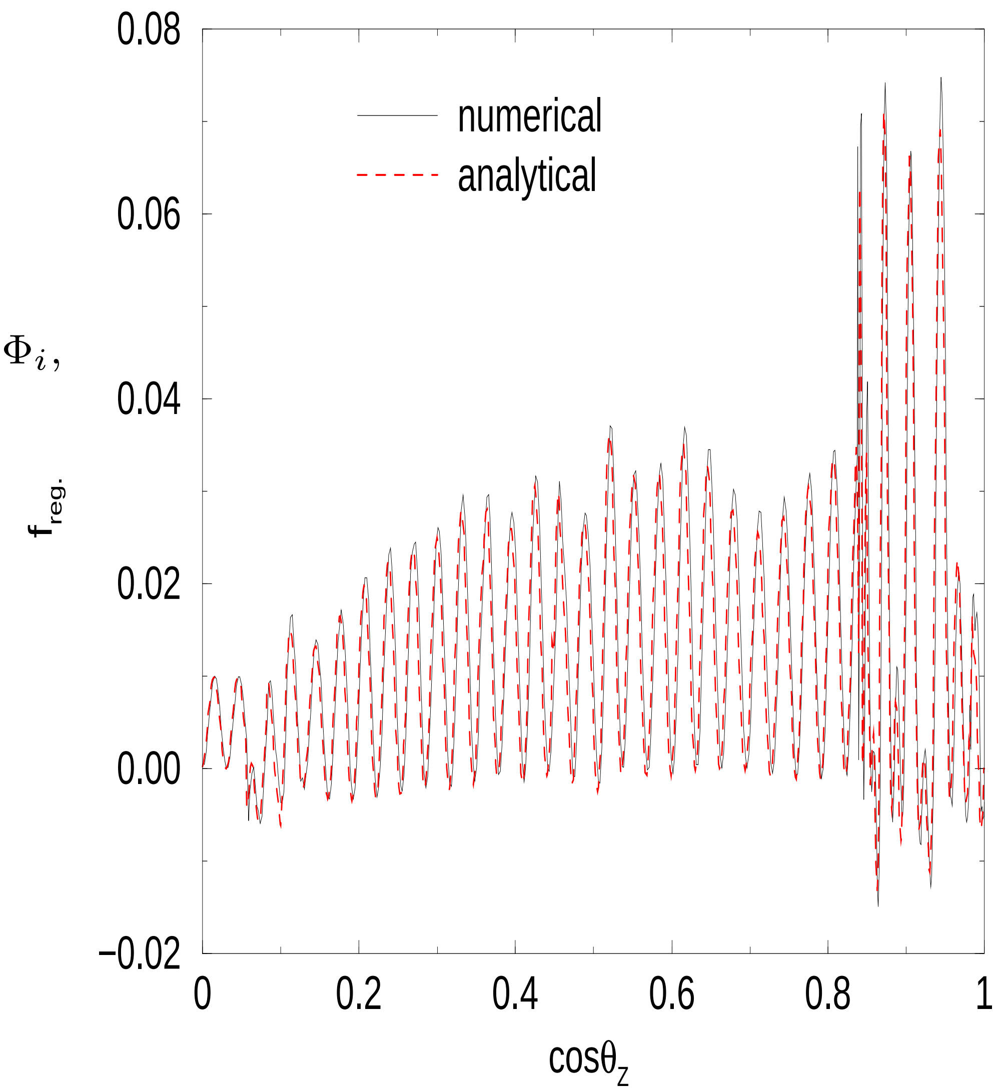
Matter effect in the Earth

f_{reg} is obtained as a sum of contributions of the borders of the shells in the Earth:

$$f_{reg} = \frac{2E\sin^2 2\theta}{\Delta m^2} \sin\Phi_0 \sum_{i=0}^{n-1} \Delta V_i \sin\Phi_i,$$
$$\Phi_i = \int_{-L_i/2}^{L_i/2} dx \frac{\Delta(x)}{4E}.$$

The analytic formula reproduces precisely the magnitude and the phase structure of f_{reg} as a function of the zenith angle θ_Z.

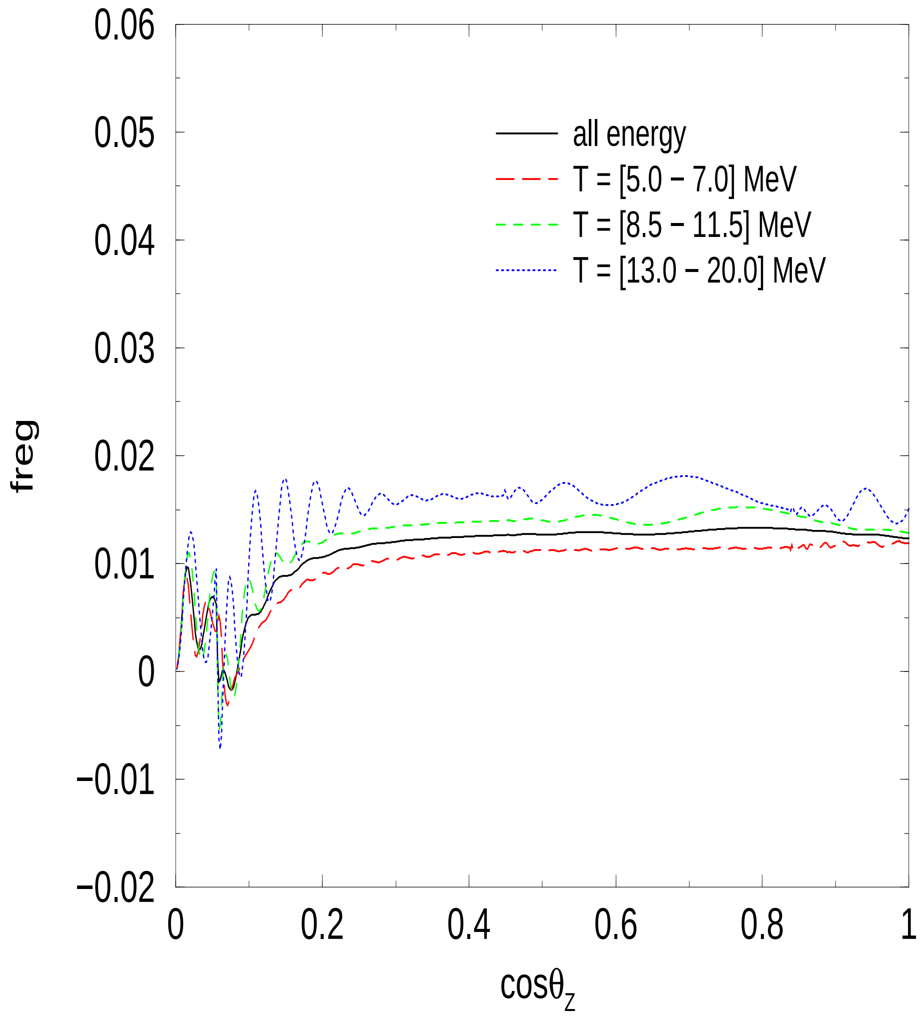
Large density jump in Earth gives jump of magnitude in f_{reg} as clearly seen in the plot.



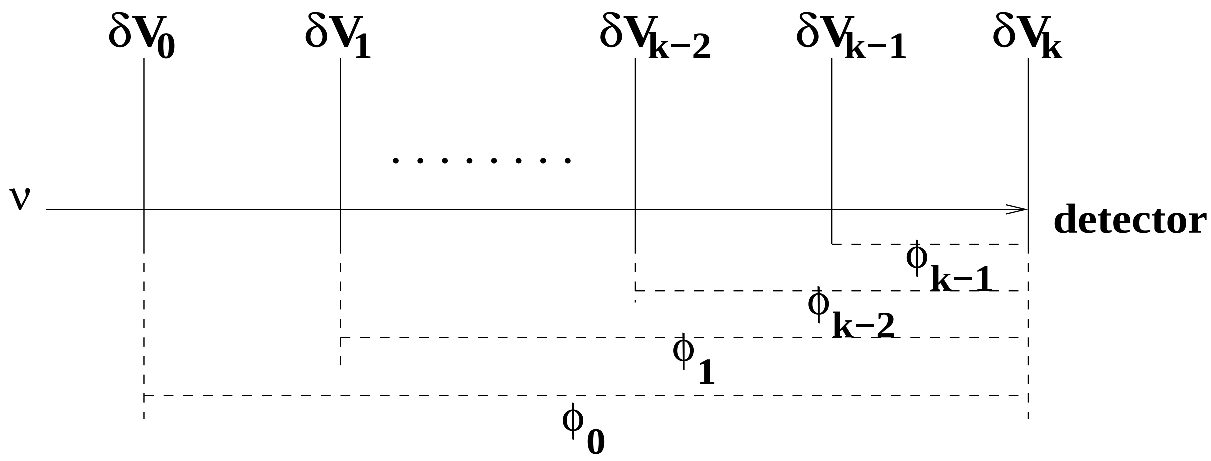
Averaging over energy range

Properties:

- Contributions of structures close to the surface of the Earth interfere constructively for $\cos\theta_Z \gtrsim 0.5$. This is the reason of the increase of the magnitude of f_{reg} as $\cos\theta_Z$ increases.
- For $\cos\theta_Z \lesssim 0.2$, f_{reg} is not averaged because the total phase is not large enough.
- **Averaging over energy makes contributions of structures far from the surface of the Earth significantly reduced (comparing last figure).**



Effect of small structures



Using again δ function to parametrize the density jumps we arrive at

$$f_{reg} = -\frac{E\sin^2 2\theta}{\Delta m^2}\sum_{i=0}^{k}\delta V_i \cos 2\phi_i, \quad \phi_i = \int_{x_i}^{x_k} dx\,\frac{\Delta(x)}{4E}, \quad i = 0, \cdots, k$$

Effects of averaging over energy interval is not sufficient for contributions of structures situated close to the detectors.

Main uncertainties in the Earth matter effects are from contributions of small structures close to the detectors. These uncertainties can not be reduced by averaging over energy.

Conclusion

The complete formula for the ν_e survival probability is obtained:

$$P_K = \frac{1}{2} + \frac{1}{2}(1-\delta_K)\cos 2\theta_m(\bar{V}_K)\cos 2\theta - (1-\delta_K)\cos 2\theta_m(\bar{V}_K) f_{reg}.$$

where $K = pp, pep, Be, N, O, F, B, hep$.

- The non-adiabatic transition in the Sun is quantitatively understood and can be safely neglected (about $10^{-9} - 10^{-7}$).
- Effect of averaging over distribution of ν sources is understood and precise analytic formula is obtained.
- The dependence of P_{ee} on solar model is understood. It is encoded in $\bar{V}_K$ and δ_K (computed using ΔV_K^2 and $\bar{V}_K$ shown below for BP2000 model).

K	pp	^8B	^{13}N	^{15}O	^{17}F	^7Be	pep	hep
$\bar{V}_K(10^{-12}$eV)	4.68	6.81	6.22	6.69	6.74	6.16	5.13	3.96
$\Delta V_K^2/\bar{V}_K^2$	0.109	0.010	0.054	0.013	0.012	0.029	0.076	0.165

Conclusion

- Matter effect in the Earth (regeneration effect) is understood and precise analytic formula for the regeneration is obtained for the realistic density profile.
- Averaging over energy can significantly reduce the contributions of structures situated far from the surface of the Earth.
- Contributions from small structures close to the detector are the main uncertainties in the Earth matter effects for the future precision measurements. These uncertainties are not reduced by avaraging over energy.
- To perform measurement on the Earth matter effects, we propose to concentrate on the night events far from horizontally coming to the detector and average energy range of neutrino as wide as possible.