Update on $B \rightarrow X_s \gamma$ Decay and Implications for New Physics

Matthias Neubert
Cornell University

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+ work by many others
Introduction

- Radiative decays $B \to X_{s,d} \gamma$ are prototype FCNC processes
- Precision test of flavor sector in and beyond SM

- New Physics scales probed in rare $B$ decays are often higher than those accessible at colliders

[Borzumati, Greub (1998)] [Gambino, Misiak (2001)]
Introduction

- Constraints on New Physics crucially depend on theoretical accuracy of SM prediction

- Theoretical results at NLO:
  
  - $B_{SM}(E_\gamma > 1.6 \text{ GeV}) = (3.21 \pm 0.32) \cdot 10^{-4}$ \quad [Kagan, MN (1998)]
  
  - $B_{SM}(E_\gamma > 1.6 \text{ GeV}) = (3.60 \pm 0.30) \cdot 10^{-4}$ \quad [Gambino, Misiak (2001)]
  
  - $B_{SM}(E_\gamma > 1.6 \text{ GeV}) = (3.44 \pm 0.29 \pm 0.27) \cdot 10^{-4}$ \quad [MN (2004)]

  - $F(E_\gamma > 1.6 \text{ GeV}) = 0.93 \pm 0.07$

- World-average experimental result, extrapolated to $E_\gamma > 1.6 \text{ GeV}$: \quad [HFAG (2006)]

  - $B_{\text{exp}}(E_\gamma > 1.6 \text{ GeV}) = (3.55 \pm 0.24 \pm 0.09 \pm 0.03) \cdot 10^{-4}$
Introduction

- Two most important areas of theoretical improvements (ongoing)
  - Complete calculation of NNLO corrections to total $B \to X_s \gamma$ decay rate in SM [~ 75% done!]
  - Complete model-independent analysis of photon-energy cut effect at NNLO [~ 95% done!]

$$B_{SM}(E_\gamma > E_0) = B_{SM}(\text{tot}) \cdot F(E_0)$$

- total rate
- event fraction
Total decay rate at NNLO

- Effective weak Hamiltonian:

\[ \langle X_s \gamma | \mathcal{H}_{\text{eff}} | B \rangle = \sum_{i=1, \ldots, 8} C_i(\mu) \langle X_s \gamma | Q_i(\mu) | B \rangle \]

- RG evolution of Wilson coefficients:

\[ C_i(\mu) = \sum_j C_j(M_W, m_t) U_{ji}(M_W, \mu) \]

  matching \hspace{1cm} \text{running (anom. dims.)}
Total decay rate at NNLO

- Matching conditions for effective Hamiltonian
  - 2-loop corrections to $C_{1-6}(M_W)$ known [Bobeth, Misiak, Urban (1999)]
  - 3-loop corrections to $C_{7,8}(M_W)$ known [Misiak, Steinhauser (2004)]

- In many models, this is where New Physics would enter!

+ $O(1000)$ similar graphs
Total decay rate at NNLO

- Anomalous dimensions
  - 3-loop mixing in $Q_{1-6}$ sector completed [Gorbahn, Haisch (2004)]
  - 3-loop mixing in $Q_{7,8}$ sector completed [Gorbahn, Haisch, Misiak (2005)]
  - 4-loop mixing of $Q_{1-6}$ into $Q_7$ completed and mixing into $Q_8$ almost finished [Czakon et al.]
Total decay rate at NNLO

- **Matrix elements**
  - Large-$\beta_0$ limit of 2-loop matrix elements of $Q_{7,8}$ and 3-loop matrix elements of $Q_{1,2}$ completed; exact calculations in progress
  - Expect 6% → 3% reduction of $m_c$ uncertainty
- 2-loop matrix elements of dipole operator $Q_7$ completed

[Bieri, Greub, Steinhauser (2003)]

[Melnikov, Mitov (2005)]

[Blokland et al. (2005)]

[Asatrian et al. (2006)]
Theoretical error estimate (I)

- Expect that after completion of NNLO calculation the dominant uncertainties are reduced to:
  - Scales: ± 2%
  - \( m_c \) dependence: ± 3%
  - Others (\( \alpha_s \), CKM, \( m_t \), etc.): ± 2%
- Total theoretical uncertainty:
  \[ \Delta B_{SM}^{\text{tot}} \approx 4\% \]
Event fraction at NNLO

- Presence of photon-energy cut $E_\gamma > E_0 = 1.8$ GeV or larger leads to significant complications in theoretical analysis
- Sensitivity to low scale $\Delta = m_b - 2E_0 \approx 1$ GeV (barely perturbative)
- Effect controlled using multiscale OPE; expansion in $(\Lambda_{QCD}/\Delta)^n$ and $\alpha_s(\Delta)$

[BaBar 05: $E_0 = 1.9$ GeV]
[Belle 04: $E_0 = 1.8$ GeV]

[MN (2004)]
Event fraction at NNLO

- Multi-scale OPE region
- Shape function region
- OPE region

Nonperturbative!
Event fraction at NNLO

- Exact all-order formula:

\[ F(E_0) = U_1(\mu_h, \mu_i) U_2(\mu_i, \mu_0) \left( \frac{m_b}{\mu_h} \right)^{-2a_\Gamma} \left( \frac{\Delta}{\mu_0} \right)^\eta \]

\[ \times h \left( \ln \frac{m_b}{\mu_h} \right) \tilde{j} \left( \ln \frac{m_b \Delta}{\mu_i^2} + \partial_\eta \right) \tilde{s} \left( \ln \frac{\Delta}{\mu_0} + \partial_\eta \right) \frac{e^{-\gamma_E \eta}}{\Gamma(1 + \eta)} \left[ 1 - \frac{\eta(1 - \eta)}{6} \frac{\mu_i^2}{\Delta^2} + \ldots \right] \]

Matching scales:

- \( \mu_h \sim m_b \)
- \( \mu_i \sim \sqrt{m_b \Delta} \)
- \( \mu_0 \sim \Delta = m_b - 2E_0 \)

RG functions:

- \( U_1, U_2, a_\Gamma, \eta \)
- \( \Gamma_c[\alpha_s(\mu)] \)

\[ \eta = 2 \int_{\mu_0}^{\mu_i} \frac{d\mu}{\mu} \Gamma_c[\alpha_s(\mu)] \]

Kinetic-energy parameter

[MN (2005)]

needed at 3-loop order

needed at 2-loop order
Event fraction at NNLO

- 2-loop soft and jet functions: \([\text{Becher, MN (2005, 2006)}]\)

\[
\tilde{s}(L, \mu) = 1 + \left(-0.873 - 0.424L - 0.424L^2\right) \alpha_s \\
+ \left(-0.603 + 0.750L + 0.471L^2 + 0.368L^3 + 0.090L^4\right) \alpha_s^2 + \ldots
\]

\[
\tilde{j}(L, \mu) = 1 + \left(0.045 - 0.318L + 0.212L^2\right) \alpha_s \\
+ \left(-0.185 + 0.145L + 0.301L^2 - 0.114L^3 + 0.023L^4\right) \alpha_s^2 + \ldots
\]
Event fraction at NNLO

- 2-loop hard function $h(L)$ can be extracted by combining effective field-theory results with 2-loop spectrum in QCD \cite{Melnikov, Mitov (2005)}
- After resummation, largest uncertainties probed by variation of low scale $\mu_0 \sim 1$ GeV
  - Large uncertainty at NLO
  - Good stability at NNLO
Theoretical error estimate (II)

- Expect that after completion of NNLO calculation the dominant uncertainties are reduced to:
  - Scales: ± 3%
  - Hadronic parameters: ± 3%
- Total theoretical uncertainty:
  \[ \Delta F(1.8 \text{ GeV}) \approx 4\% \]
Implications for New Physics

- After completion of NNLO analysis, expect SM prediction:

\[ B_{SM}(E_\gamma > 1.8 \text{ GeV}) = (3.30 \pm 0.13 \pm 0.13) \cdot 10^{-4} \]

- Assuming improved experimental error of 0.2\cdot10^{-4} (B factories), combined uncertainty in difference theory-experiment would be:

\[ \Delta B_{NP} = B_{exp} - B_{SM} = (0.?? \pm 0.27) \cdot 10^{-4} \]

- Present 2\sigma deviation would become a 3.5\sigma effect
Implications for New Physics

- Given present data, possibility of finding New Physics with $B \rightarrow X_s \gamma$ is essentially excluded (except for direct CP asymmetry) [Kagan, MN (1998)]

- However, reduced error will be very helpful in constraining parameters of New Physics models

- Discuss some examples …
Implications for New Physics: Type-II 2-Higgs-Doublet Model

Theory @ NLO

Theory @ NNLO

$\sigma(BR_\gamma) \times 10^4$

$BR_\gamma^{exp} \times 10^4$

$M_H > 200 \text{ GeV}$

[99% CL bounds on Higgs mass]

[Gambino, Misiak (2001); Becher, MN]
Implications for New Physics: Constrained MSSM

[Ellis, Heinemeyer, Olive, Weiglein (2004)]
Implications for New Physics: Universal extra dimensions

with KK modes (different pars.)

Theory @ NNLO

[Buras, Poschenrieder, Spranger, Weiler (2003)]
Conclusions

- $B \rightarrow X_s \gamma$ decay remains one of most sensitive probes of New Physics
- Strong motivation for NNLO calculation in Standard Model (many people involved!)
- Problem naturally splits up into two parts:
  - NNLO corrections to total decay rate
  - Effects of photon-energy cut at NNLO in multi-scale OPE
- Projected theoretical accuracy of 5-6% implies tight bounds on physics beyond SM