Precision Observables M_W, $(g-2)_\mu$

Dominik Stöckinger

Edinburgh

SUSY 2006

main refs: [Heinemeyer, Hollik, DS, Weber, Weiglein, '06] (M_W)

[DS '06 (soon. . .)] (review on $(g-2)_\mu$ and SUSY)

Outline

1. Motivation
2. M_W
3. $(g-2)_\mu$

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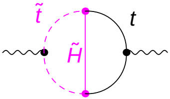
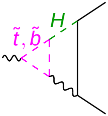
Precision Observables

Quantum effects from SUSY particles can have an influence on precision observables

- M_W



- $(g-2)_\mu$



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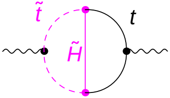
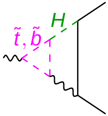
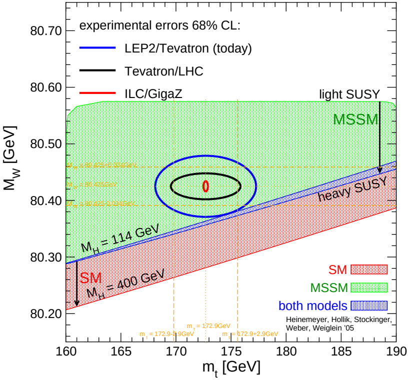


- $(g-2)_\mu$





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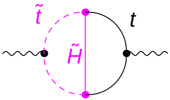
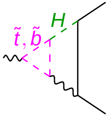
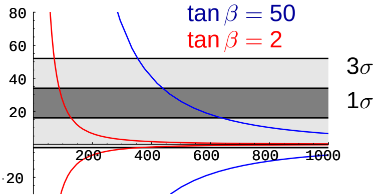


- $(g-2)_\mu$



$a_\mu(\exp - \text{SM}) = 25(9) \times 10^{-10}$

$a_\mu(\text{SUSY})$



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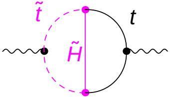

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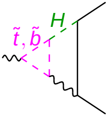

MSSM tends to agree better with experimental data than SM

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Muon-decay, M_W, and $\Delta\rho$

Muon lifetime ($\leftrightarrow G_\mu$) related to M_W:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}(1+\Delta r)$$

$\Rightarrow$ Prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_t, M_H, m_{\tilde{t}}, \ldots)$

Loop calculation:

$$\Delta r = -\frac{c_W^2}{s_W^2}\Delta\rho + \text{rest}$$

$\Delta\rho$: breaking of isospin invariance $\rightarrow t-b$-mass splitting ...

Muon-decay: Status

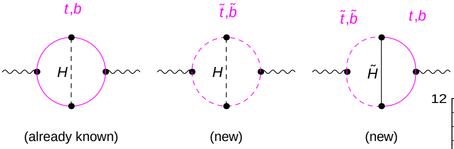
SM:	$\Delta\rho$:	3-loop, 4-loop	
	rest:	complete 2-loop	
MSSM:	$\Delta\rho$:	2-loop	[Haestier,Heinemeyer,DS,Weiglein'05]
	rest:	1-loop complete	[Heinemeyer,Hollik,DS,Weber,Weiglein'06]

↓

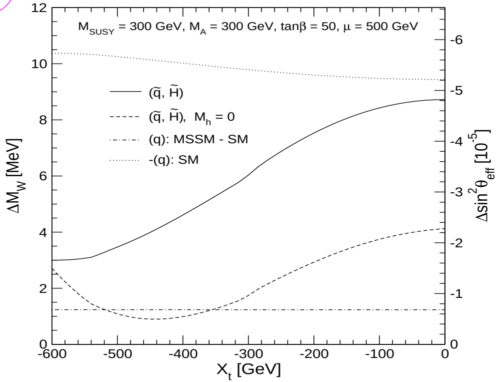
new results

$\Delta\rho$ at 2-loop order

New result on Yukawa-enhanced corrections of $\mathcal{O}(\alpha_{t,b}^2)$



[Haestier,Heinemeyer,DS,Weiglein'05]



induced shift in M_W as function of stop mixing: up to $\Delta M_W \approx 8$ MeV,

SUSY loops can be as large as SM quark loops

New result for M_W in the MSSM

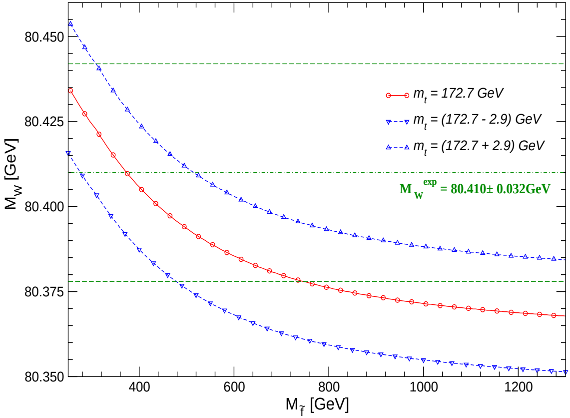
[Heinemeyer,Hollik,DS,Weber,Weiglein '06]

- Complete 1-loop calculation of Δr with complex parameters
- Incorporation of 2-loop calculation of $\Delta\rho$
  [Haestier,Heinemeyer,DS,Weiglein'05]
- Incorporation of higher-order SM contributions

$$\Delta r^{\mathrm{MSSM}} = \Delta r^{\mathrm{SM}} + \Delta r^{\mathrm{MSSM-SM}}_{1-\mathrm{loop}} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{MSSM-SM}}_{2-\mathrm{loop}}$$

Best available prediction of M_W in the MSSM $\rightarrow$ implemented in computer program

Compare MSSM prediction for M_W with experimental result

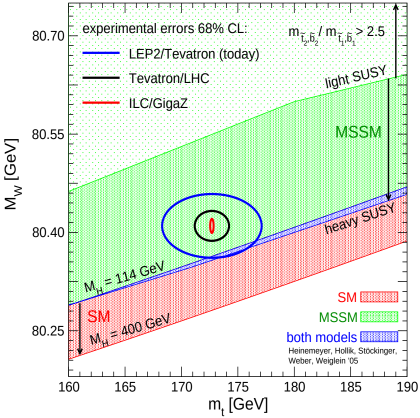


[Heinemeyer,Hollik,DS,Weber,Weiglein '06]

$\tan\beta = 10$
$A_{t,b} = 2M_{\tilde f}$
$\mu = 300$ GeV
$m_{\tilde g} = 300$ GeV
$M_A = 300$ GeV
$M_2 = 300$ GeV

Preference for light SUSY scale $M_{\tilde f} = M_{\tilde t,\tilde b}$

Compare MSSM prediction for M_W with experimental result



[Heinemeyer,Hollik,DS,Weber,Weiglein '06]

Scan of all SUSY parameters

Preference for MSSM over SM

Outline

$(g-2)_\mu$: Status

- BNL experiment: finalized, very stable development
- SM-theory: several errors fixed, e^+e^- data preferred

$$a_\mu(\mathrm{exp}-\mathrm{SM}) = (23.9 \pm 9.9) \times 10^{-10}$$

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$$a_\mu(\mathrm{exp}-\mathrm{SM}) = (23.9 \pm 9.9) \times 10^{-10}$$

Historical comparison: 1978, CERN experiment:

$$a_\mu(\mathrm{CERN}-\mathrm{SM, without\ had}) = (720 \pm 85) \times 10^{-10}$$
$$a_\mu(\mathrm{SM,\ had}) = (667 \pm 81) \times 10^{-10}$$

Established had contributions, agreement with SM

$(g-2)_\mu$: Status

- BNL experiment: finalized, very stable development
- SM-theory: several errors fixed, e^+e^- data preferred

$$a_\mu(\mathrm{exp}-\mathrm{SM}) = (23.9 \pm 9.9) \times 10^{-10}$$

Now: compare with weak contributions only:

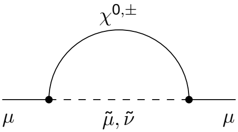
$$a_\mu(\mathrm{BNL}-\mathrm{SM}, \mathrm{without\ weak}) = (39.3 \pm 9.9) \times 10^{-10}$$
$$a_\mu(\mathrm{SM},\ \mathrm{weak}) = (15.4 \pm 2.2) \times 10^{-10}$$

existence of weak contributions, but disagreement with SM! $\rightarrow$ SUSY?

Status of SUSY prediction

1-Loop

$\propto \tan\beta$



[Fayet '80],...

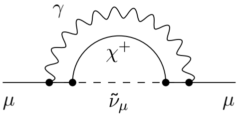
[Kosower et al '83],[Yuan et al '84],...

[Lopez et al '94],[Moroi '96]

complete

2-Loop logs

$\propto \log \frac{M_{\rm SUSY}}{m_\mu}$

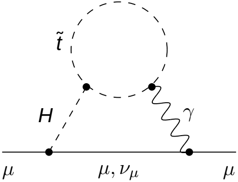


[Degrassi,Giudice '98]

complete

2-Loop non-log

$\propto \tan\beta\, \mu\, m_t$



[Chen,Geng'01][Arhib,Baek '02]

[Heinemeyer,DS,Weiglein '03]

[Heinemeyer,DS,Weiglein '04]

half

(no internal $\tilde{\mu}, \tilde{\nu}$)

SUSY prediction

Implementation of 1-Loop and leading 2-Loop straightforward:

1-Loop:

$$a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2}\sum_{i,m}\left\{-\frac{m_\mu}{12m^2_{\tilde\mu m}}(|n^L_{im}|^2+|n^R_{im}|^2)F_1^N(x_{im})+\frac{m_{\chi_i^0}}{3m^2_{\tilde\mu m}}\mathrm{Re}[n^L_{im}n^R_{im}]F_2^N(x_{im})\right\},$$

$$a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2}\sum_{k}\left\{\frac{m_\mu}{12m^2_{\tilde\nu_\mu}}(|c^L_k|^2+|c^R_k|^2)F_1^C(x_k)+\frac{2m_{\chi_k^\pm}}{3m^2_{\tilde\nu_\mu}}\mathrm{Re}[c^L_k c^R_k]F_2^C(x_k)\right\},$$

$$n^L_{im} = \frac{1}{\sqrt{2}}(g_1N_{i1}+g_2N_{i2})U^{\tilde\mu}_{m1}{}^* - y_\mu N_{i3}U^{\tilde\mu}_{m2}{}^*,$$
$$n^R_{im} = \sqrt{2}g_1N_{i1}U^{\tilde\mu}_{m2}+y_\mu N_{i3}U^{\tilde\mu}_{m1},$$
$$c^L_k = -g_2V_{k1},$$
$$c^R_k = y_\mu U_{k2}.\ F_1^N(x) = \frac{2}{(1-x)^4}\left[1-6x+3x^2+2x^3-6x^2\log x\right],$$
$$F_2^N(x) = \frac{3}{(1-x)^3}\left[1-x^2+2x\log x\right],$$
$$F_1^C(x) = \frac{2}{(1-x)^4}\left[2+3x-6x^2+x^3+6x\log x\right],$$
$$F_2^C(x) = \frac{3}{(1-x)^3}\left[-3+4x-x^2-2\log x\right],$$

SUSY prediction

Implementation of 1-Loop and leading 2-Loop straightforward:

2-Loop: $$a_\mu^{\rm logs} = -\frac{4\alpha}{\pi}\,\log\frac{M_{\rm SUSY}}{m_\mu}\,a_\mu^{\rm 1-Loop}$$

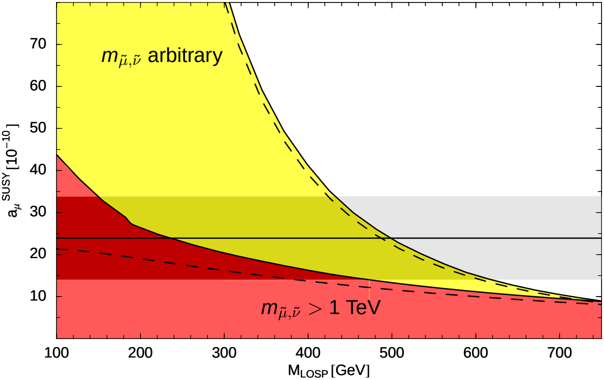
$$a_\mu^{(\chi\gamma H)} = \frac{\alpha^2 m_\mu^2}{8\pi^2 M_W^2 s_W^2}\sum_{k=1,2}\left[{\rm Re}[\lambda_\mu^{A^0}\lambda_{\chi_k^+}^{A^0}]\,f_{PS}(m_{\chi_k^+}^2/M_{A^0}^2) + \sum_{S=h^0,H^0}{\rm Re}[\lambda_\mu^S\lambda_{\chi_k^+}^S]\,f_S(m_{\chi_k^+}^2/M_S^2)\right],$$
$$a_\mu^{(\tilde f\gamma H)} = \frac{\alpha^2 m_\mu^2}{8\pi^2 M_W^2 s_W^2}\sum_{\tilde f=\tilde t,\tilde b,\tilde\tau}\sum_{i=1,2}\left[\sum_{S=h^0,H^0}(N_c Q^2)_{\tilde f}\,{\rm Re}[\lambda_\mu^S\lambda_{\tilde f_i}^S]\,f_{\tilde f}(m_{\tilde f_i}^2/M_S^2)\right].$$

$$\lambda_\mu^{\{h^0,H^0,A^0\}} = \left\{-\frac{s_\alpha}{c_\beta},\frac{c_\alpha}{c_\beta},t_\beta\right\},$$
$$\lambda_{\chi_k^+}^{\{h^0,H^0,A^0\}} = \frac{\sqrt{2}M_W}{m_{\chi_k^+}}\left(U_{k1}V_{k2}\{c_\alpha,s_\alpha,-c_\beta\} + U_{k2}V_{k1}\{-s_\alpha,c_\alpha,-s_\beta\}\right).$$
$$\lambda_{\tilde t_i}^{\{h^0,H^0\}} = \frac{2m_t}{m_{\tilde t_i}^2 s_\beta}\left(+\mu^*\{s_\alpha,-c_\alpha\} + A_t\{c_\alpha,s_\alpha\}\right)(U_{i1}^{\tilde t})^*\,U_{i2}^{\tilde t},$$
$$\lambda_{\tilde b_i}^{\{h^0,H^0\}} = \frac{2m_b}{m_{\tilde b_i}^2 c_\beta}\left(-\mu^*\{c_\alpha,s_\alpha\} + A_b\{-s_\alpha,c_\alpha\}\right)(U_{i1}^{\tilde b})^*\,U_{i2}^{\tilde b},$$
$$\lambda_{\tilde\tau_i}^{\{h^0,H^0\}} = \frac{2m_\tau}{m_{\tilde\tau_i}^2 c_\beta}\left(-\mu^*\{c_\alpha,s_\alpha\} + A_\tau\{-s_\alpha,c_\alpha\}\right)(U_{i1}^{\tilde\tau})^*\,U_{i2}^{\tilde\tau}.$$

$$f_{PS}(z) = \frac{2z}{y}\left[{\rm Li}_2\left(1-\frac{1-y}{2z}\right) - {\rm Li}_2\left(1-\frac{1+y}{2z}\right)\right]$$
$$f_S(z) = (2z-1)f_{PS}(z) - 2z(2+\log z),$$
$$f_{\tilde f}(z) = \frac{z}{2}\left[2+\log z - f_{PS}(z)\right].$$

Numerical result

$\tan\beta = 50$, all parameters < 3 TeV



- typically: $13 \times 10^{-10}\ \tan\beta \times \left(\frac{100\,\mathrm{GeV}}{M_{\rm SUSY}}\right)^2\ \mathrm{sign}(\mu)$

SUSY contributions in the observed range for low $M_{\rm SUSY}$!

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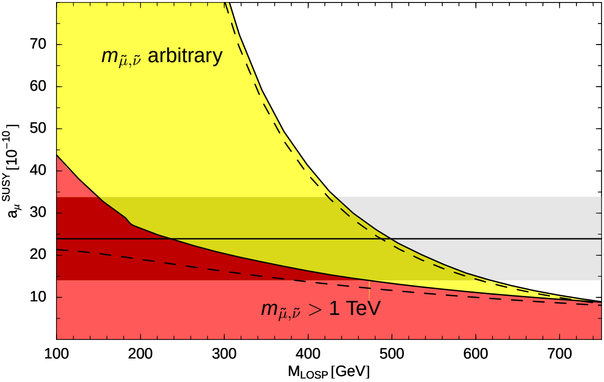

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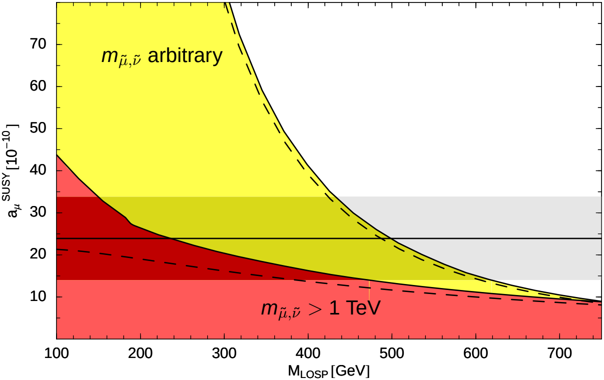

- typically: $13 \times 10^{-10}\ \tan\beta \times \left(\frac{100\,\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2\ \mathrm{sign}(\mu)$
- possible enhancement for large μ
- suppression for large $m_{\tilde{\mu},\tilde{\nu}} \leftrightarrow$ 2-Loop important

SUSY contributions in the observed range for low M_{SUSY}!

Conclusions

Two precision observables: M_W and $(g-2)_\mu$

- current measurements sensitive to 2-Loop SM and SUSY effects
- 2-Loop SUSY contributions:
  - M_W: Computer code for M_W^{MSSM}
  - $(g-2)_\mu$: 1-,2-Loop contributions easy to implement

[Haestier,Heinemeyer, DS, Weiglein '05]
[Heinemeyer, DS, Weiglein '03,'04]
[Heinemeyer,Hollik, DS, Weber,Weiglein '06]

- SUSY with low mass scale $\sim 200\ldots 600$ GeV fits very well
- SM cannot be excluded at the moment

Future, more precise measurements very important and promising!