NEUTRINO COMPLEMENTARITY,SOME RECENT RESULTS

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Complementarity

- (I) Complementarity and CP violation:The complex bimaximal model.
- (II) Quark lepton Complementariy and predictions for PMNS matrix.
- Based on works:

1. **Complementarity and neutrino oscillations, Chauhan, Pulido (Lisboa), Picariello (Milan), E.T-L, hepph/0605231.**
2. **CP Complementarity and Complex Bimaximality. E. T-L, hepph/0606nnn**

GENERAL: COMPLEMENTARITY.

- LEPTONS:$\theta_{sun} \equiv \theta_{12} \sim 32.3^0 \pm 2.4^0, \theta_{23} \sim 45^0, \theta_{13} \sim 3^0 <\sim 9^0$.
- QUARKS: $\theta_C(M_Z) \equiv \theta_{12} \sim 12.8 \pm 0.15^0, \quad \theta_{23} \sim 2.4^0, \quad \theta_{13} \sim 0.2^0, \; and \; \delta_{ckm} \sim 65^0$.

- CONCLUSION: The mechanisms that determine lepton (and quark?) masses are different for charged and neutral leptons. $\longrightarrow$ any hope of a link?.

- Complementarity,Quark-lepton complementarity (QLC): Minakata 2004

$$(C1) \quad \theta_{sun} + \theta_C = \theta_{12}^{PMNS} + \theta_{12}^{CKM} = 45.1^0 \pm 2.4^0 \simeq \frac{\pi}{4}; \qquad \theta_{23}^{PMNS} \pm \theta_{23}^{CKM} \simeq 45^o .$$

and

$$(C2) \quad \theta_{13}^{PMNS} \sim \theta_{13}^{CKM}, \quad or \quad \theta_{13}^{PMNS} \sim \theta_{23}^{CKM}$$

- Accidental?, GUT?, Q-L symmetry?,flavor dynamics?, nothing related to θ_C?. Maybe (C1), (C2) of different origin?.

$\longrightarrow$

$$U_{PMNS} = U_{CKM}^{\dagger} \, U_{bimax}$$

.

TYPES OF COMPLEMENTARITY

- MORE IN GENERAL: Effective description of the (small) deviation from maximal mixing$\longrightarrow$ parametrize U_{PMNS} with λ, and $U(\lambda) \sim I$.

$$\boldsymbol{U_{PMNS} = U^\dagger(\lambda)\ U_{bimax}}$$

DIFFERENT CASES:

- **QUARK-LEPTON COMPLEMENTARITY (QLC):** $\longrightarrow$ $\boldsymbol{\lambda \sim \sin\theta_C;\ U(\lambda) \sim U_{CKM}}$ smallness of $U_{e3} \sim$ small θ_C. $\longrightarrow$ GUT theories + flavour symmetries

- **CP COMPLEMENTARITY:** $\boldsymbol{\lambda \sim J, a_{CP};\quad U(\lambda) \sim U(J),}$ $\longrightarrow$ smallness of $U_{e3} \sim$ small a_{CP}.

$\longrightarrow$ USE COMPLEX BIMAXIMAL MATRICES Torrente-Lujan 1997 $\longrightarrow$ LR models with SCPV

- **MIXED POSSIBILITY:** $\boldsymbol{U_{PMNS} = U^\dagger_{CKM}(\theta_C)U^\dagger_{CP}(J)U_{bimax}}$ $\longrightarrow$ U_{CP} explains smallness θ_{13} (C2), $\longrightarrow$ U_{CKM} explains (C1).

- Why we want U_{bimax} instead of, for example, U_{trimax}?$\longrightarrow$ theoretical prejudices.

GENERAL: MECHANISM TOOLBOX& COMPLEMENTARITY

- Natural mechanisms to explain (unusual) neutrino flavour properties:
- 1) **SEE-SAW**$\longrightarrow$ very small masses.
- 2) **Froggatt-Nielsen mechanism**: abelian flavor symmetry breaking$\longrightarrow$ hierarchy of masses$\sim$ small mixing angles.

$$\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}$$

$\longrightarrow$ in reasonable agreement with CKM matrix and θ_C.

$\longrightarrow$ fine tuning is needed if want to produce large mixing angles and use see-saw mechanism.

- 3) **Non-abelian flavour SB**: continuous or discrete.$\longrightarrow$ a robust mechanism, it admits only vanishing or maximal mixing angles (at least in simple $SO(3)_F$, $SU(2)_F$.) Barbieri-Hall-Kane 99,Raidal 04

- Realistic models might need all:

$\longrightarrow$ Abelian: generates light charged fermions,

$\longrightarrow$ Non-Abelian: generates maximal mixing

$\longrightarrow$ See-Saw: generates very small neutrino masses

$$U_{PMNS} = U^{\dagger}_{CKM}(\theta_C)\ U_{bimax}$$

NEUT=Froggat. X NONABELIAN F

GUTS+Flavour$\longrightarrow$ QLC

- GUTS: additional relations. **SU(5)**: $Y_d \sim Y_l^T \longrightarrow U_d \sim V_l^\star$ **SO(10)**: $Y_u \sim Y_D \longrightarrow U_u \sim U_D$

$$\begin{aligned} U_{PMNS} &= U_l^\dagger U_D V_M \\ &= U_l^\dagger U_d U_{CKM} V_M \quad (SU(5)) \\ &= V_d^T U_d U_{CKM} V_M \quad (SO(10)) \end{aligned}$$

- $U_{PMNS} = A U_{CKM} B.$ Diverse possibilities:

1)Flavor-Gauge SYMs: To include further assumptions, for example Y_d symmetric so that $U_d = V_d^\star$, or $U_d \sim U_l$ to obtain, in any case

$$U_{PMNS} = U_{CKM}^\dagger \; V_M$$

$\longrightarrow$ V_M contains two large angles (BIMAX?), they could require non-abelian flavour symmetries.

2) No assumptions as before.

$$U_{PMNS} = K \; U_{CKM}^\dagger \; V_M$$

$\longrightarrow$ V_M could contain only one large angle: a more easy alternative (Standard F-N mechanism).

CP COMPLEMENTARITY: Left Right Models with SCPV

- LR model with SCPV. EW VEVS Φ:$(\kappa e^{i\alpha_\kappa}, \kappa' e^{i\alpha_{\kappa'}})$: VEVS Δ_L,Δ_R triplets: $(v_R e^{i\alpha_R}, v_L e^{i\alpha_L})$:

SCPV$\longrightarrow$ κ, κ' : real,positive. Ys: Yukawa matrices, Hermitian, real.

- Mass matrices: Dirac, Majorana:

$$
\begin{aligned}
&\boldsymbol{M_u = \kappa Y_{qL} + \kappa' e^{-i\alpha_{\kappa'}} Y_{qR};} && \boldsymbol{M_{\nu,D} = \kappa Y_{lL} + \kappa' e^{-i\alpha_{\kappa'}} Y_{lR}} \\
&\boldsymbol{M_d = \kappa' e^{i\alpha_{\kappa'}} Y_{qL} + \kappa Y_{qR};} && \boldsymbol{M_l = \kappa' e^{i\alpha_{\kappa'}} Y_{lL} + \kappa Y_{lR}} \\
&\boldsymbol{M_{\nu,RR} = v_R Y_M; \, M_{\nu,LL} = v_L e^{i\alpha_L} Y_M}
\end{aligned} \tag{1}
$$

- $\kappa'/\kappa \simeq m_t/m_b >> 1$, $\mid \kappa \mid^2 + \mid \kappa' \mid^2 = v^2$, $v_L \, v_R = \beta \mid k \mid^2$,$\beta \sim 1$.
- CP(quark) related to CP(lepton) through $\alpha_{k'}$, $o(\kappa'/\kappa)$. $\alpha_{\kappa'} \sim 0$ (to avoid FCNC).
- SEE-SAW TYPE II: $\boldsymbol{M_\nu = M_\nu^{II} - M_\nu^{I}}$

$$
\begin{aligned}
M_\nu^I &= M_{\nu D}^t M_{\nu,RR} M_{\nu,D} \\
M_\nu^{II} &= M_{\nu,LL} = v_L e^{i\alpha_L} Y_M
\end{aligned}
$$

- $\kappa' << \kappa$$\longrightarrow$ $M_\nu^I \simeq \frac{k^2}{v_R} Y_{lL}^T Y_M^{-1} Y_{lL} = \frac{v_L}{\beta} Y_{lL}^T Y_M^{-1} Y_{lL}$. $\longrightarrow$ FULL SEE SAW (II+I),

$$
\boldsymbol{M_\nu = v_L e^{i\alpha_L} Y_M - \frac{v_L}{\beta} Y_{lL}^T Y_M^{-1} Y_{lL}}
$$

$\longrightarrow$ **CP Dirac and Majorana phases $\boldsymbol{\delta_D, \alpha_{1M}, \alpha_{2M}}$: depend on a single phase $\boldsymbol{\alpha_L}$.**

$\longrightarrow$ **GUT embedings,additional relations for Yuks**

CP COMPLEMENTARITY: MAXIMAL MATRICES.

- Unitary Mixing matrices depend: on 4 moduli + sgn(J) ⟶ BASICALLY 2 POSSIBILITIES:

- **SIMPLEST POSSIBILITY:** Threefold maximal lepton mixing,No free parameters: ⟶ all the elements have equal modulus. J=0. ⟶ $P_{ll} = P(\nu_l \to \nu_l) = P = 1/3$(SUN). Harrison 1995

- **NEXT TO SIMPLEST: QUASI MAXIMAL(COMPLEX BIMAXIMAL):** ⟶ maximal in a 2×2 subsector: One free parameter, $J \neq 0$ ⟶ connect Ps and CP asymmetry. Torrente-Lujan,1996 .

$$U_{PMNS} = \frac{1}{k} \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & \exp i\lambda_1 & \exp i\lambda_2 \\ \delta & \exp i\lambda_3 & \exp i\lambda_4 \end{pmatrix}$$

PARTICULAR CASES:

⟶ BIMAXIMAL ANSATZ:real matrix, ACP=0,No free parameters. Barger,Pakvasa 1998 .

⟶ BIMAXIMAL WITH CP MAXIMAL: Georgi,Glashow 1998 .

- **OTHER:** Perturbations of the real bimaximal. Rodejohann 2005-2006,Ferrandis, Pakvasa (2002,2005). TRIBIMAXIMAL. Mohapatra 1998

CP COMPLEMENTARITY:A CONCRETE TOY MODEL.

- Working examples (α, β real):

$$(I)U_{MNS}(\lambda) = \frac{1}{ik}\begin{pmatrix} \beta & -\beta & \alpha \\ \exp(-i\lambda) & \exp i\lambda & \beta \\ \exp i\lambda & \exp(-i\lambda) & -\beta \end{pmatrix}; \quad (II)U_{MNS}(\lambda) = \frac{1}{ik}\begin{pmatrix} \alpha & -\beta & \beta \\ -\beta & \exp i\lambda & \exp(-i\lambda) \\ \beta & \exp(-i\lambda) & \exp(i\lambda) \end{pmatrix}$$

- TO SHOW: $U_{PMNS}(\lambda) = U^{\dagger}(\lambda)U_{bimax}$.
- CASE I: $\theta_{12} = \theta_{23} = \pi/4$. BUT $\theta_{13} \neq 0$. CASE II: $\theta_{23} = \pi/4$.
- By unitarity: All related to λ. $\alpha = 2\sin(\lambda), \beta^2 = 2\cos(2\lambda), k^2 = 2(1+\cos(2\lambda))$
- $Det\ U(\lambda) = sign \cos\lambda (= 1$, if $\mid \lambda \mid < \pi/2)$.
- $\lambda = 0$: BIMAXIMAL, $J = 0$, $(4s = m_3 + m_2 - 2m_1, 2d = m_2 - m_3)$

$$u = \begin{pmatrix} -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{i}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{i}{\sqrt{2}} \end{pmatrix}; \quad M = uM_D u^{\dagger} = \begin{pmatrix} s & s & d \\ s & s & d \\ d & d & s \end{pmatrix} \tag{2}$$

- Aproximate Complex Bimaximal maximal (unitary to 2nd order):

$$U(\lambda)_{MNSP} = \begin{pmatrix} -i/\sqrt{2} & i/\sqrt{2} & i\lambda \\ 1/2 - i\lambda/2 & 1/2 + i\lambda/2 & -i/\sqrt{2} \\ 1/2 + i\lambda/2 & 1/2 - i\lambda/2 & i/\sqrt{2} \end{pmatrix} + O(\lambda^2)$$

COMPLEMENTARITY: Factorization

- SHOW COMPLEMENTARITY: $U_{PMNS}(\lambda) = U^\dagger(\lambda) U_{bimax}$

⟶ A non trivial property, non shared by approximate bimax perturbations.

- $U(\lambda)\ U^\dagger(\lambda) = 1$. And $U(0) = U_{bimax}$. Let us define $W(\lambda) = U_{bimax} U^\dagger_{PMNS}(\lambda)$.⟶ $W(0) = I$.

⟶ Stone theorem, apply it to W:

$$\frac{dW}{d\lambda} |_{\lambda=0} = U_{bimax} \frac{dU^\dagger}{d\lambda} |_{\lambda=0} \equiv iA; \quad W(\lambda) = \exp(iA\lambda)$$

$$U_{PMNS}(\lambda) = \exp(-iA\lambda) U_{bimax}$$

- Computing,We have

$$U_{PMNS}(\lambda) \;=\; \exp\left(-i\frac{\lambda}{\sqrt{2}}(-J_{12}+J_{13})\right) \times \begin{pmatrix} -i/\sqrt{2} & i/\sqrt{2} & 0 \\ 1/2 & 1/2 & -i/\sqrt{2} \\ 1/2 & 1/2 & i/\sqrt{2} \end{pmatrix}$$

where

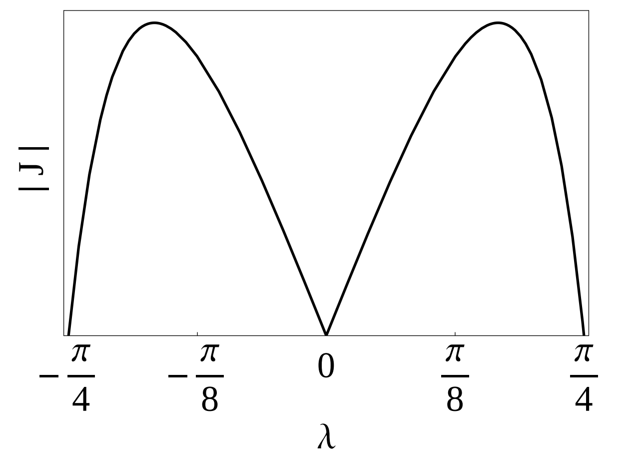
$$J_{12} = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad J_{13} = i\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

⟶ VERY SIMPLE rotation around the BIMAX(J=0): $(iJ_- = J_{12} - J_{13})$, ⟶ symmetry Hint?.

CP asymmetry: relate λ with CP

- J and Acp:

$$J = Im[u_{11}u_{22}u^\star_{21}u^\star_{12}] = \frac{1}{2}\mid\frac{\alpha}{k}\mid^2\mid\frac{\beta}{k}\mid^2 = \frac{1}{16}\frac{\sin(4\lambda)}{\cos^4(\lambda)} \sim \frac{\lambda}{4}$$



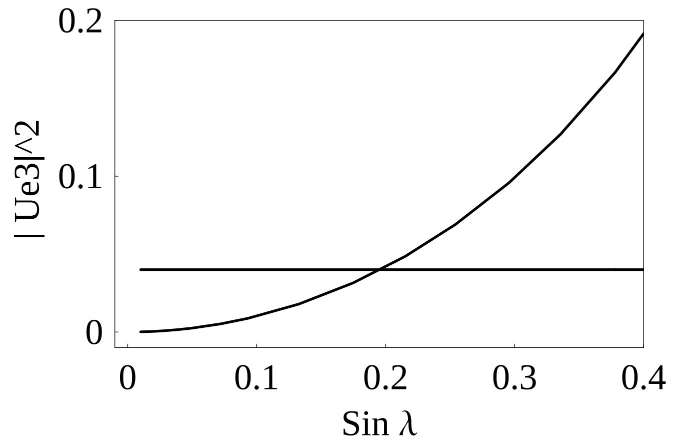
$$\mid a_{CP}\mid \equiv 2\frac{\mid \Im(u_{11}u_{22}u^*_{12}u^*_{21})\mid}{\mid u_{11}u_{22}\mid^2 + \mid u_{12}u_{21}\mid^2} = \sin 2\lambda \sim 2\lambda.$$

- $\mid U_{e3}\mid^2 = \mid\frac{\beta}{k}\mid^2$, $\mid U_{e2}\mid^2 = \mid\frac{\alpha}{k}\mid^2$, → MODEL PREDICTION

$$\mid U_{e3}\mid^2\mid U_{e2}\mid^2 = 2J$$

- ATM,CHOOZ → $\mid U_{e3}\mid^2 < 0.04(95\%CL)$

$$\mid U_{e3}\mid^2 = \mid\frac{\alpha}{k}\mid^2 = \frac{2\sin^2\lambda}{1+\cos(2\lambda)} \sim \lambda^2$$



$$\lambda \sim < \pi/16, \quad \sin\lambda \sim< 0.2,$$
$$acp = \sin 2\lambda \sim < 0.38,$$
$$J \sim< 5 \times 10^{-2}$$

CP COMPLEMENTARITY: EXP EVIDENCE

- FIT TO DATA?,at least as BIMAX,$\longrightarrow$ one free parm$\longrightarrow$ better fits
- ANALYSIS including kamLAND $\chi^2 = \chi^2_{glob,KL}$;

$\longrightarrow$ Take $\boldsymbol{\Delta m^2_{23} \sim 3 \times 10^{-3}\ eV^2}$.

$\longrightarrow$ KL global (766 t/yr) $\boldsymbol{R^{exp} = 0.66 \pm 0.05}$.

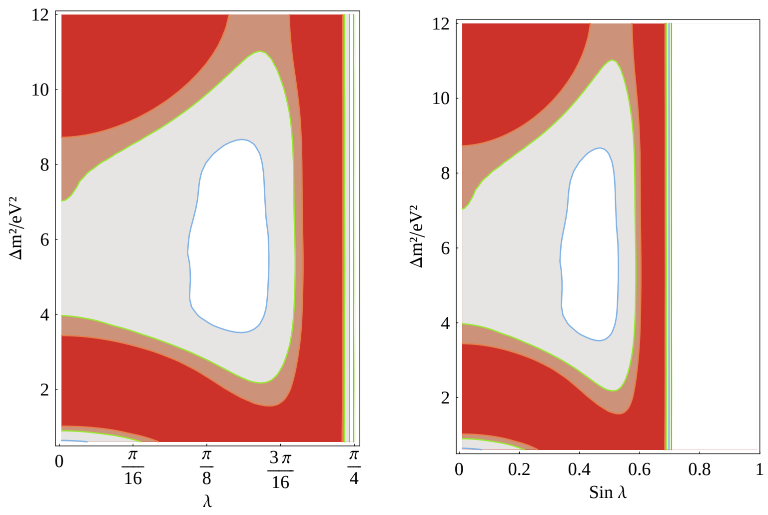
- RESULTS: Minima $\chi^2_{min} \sim$ better than BIMAX

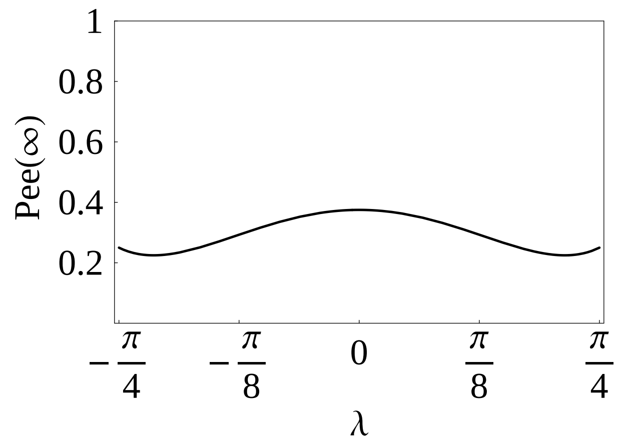
$$\boldsymbol{\lambda_{min} = 0.17}$$
$$\boldsymbol{\Delta m^2_{12} = 10 \times 10^{-5}\ eV^2}$$

$\longrightarrow$ CP "MEASUREMENT"

$$\boldsymbol{J \sim \lambda/4 = 0.042}$$

- Needed: Compare with solar data $\longrightarrow$ complicated, Matter effects, BUT qualitatively one expects good agreement, $Pee(Sun) \sim 0.3 - 0.5$ for $L/E \to \infty$.





QUARK-LEPTON COMPLEMENTARITY:, νs and SM $\longrightarrow \theta_{13}^{PMNS} = 9^{+1}_{-2}\, deg$.

Chauhan,Picariello, Pulido, E. T-L 2006, hepph/0605123

- ASSUME $U_{PMNS} = U_{CKM}^{\dagger} V_M$.$\longrightarrow$ **CORRELATION** $V_M = U_{CKM}\ U_{PMNS}$.
- V_M: $\Omega = \mathrm{diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3})$

$$V_M = U_{CKM}\ \Omega\ U_{PMNS}$$

- $V_M \equiv U_{23}(\theta)\Phi U_{13}(\theta)\Phi^{\dagger} U_{12}(\theta);\ U_{PMNS} = U_{23}\ \Phi\ U_{13}\ \Phi^{\dagger}\ U_{12}\ \Phi_m$.

QUESTIONS:

$\longrightarrow$ BOTTOM-UP:which V_M is allowed by data? is this $V_M)_{13} = 0$?, is this BIMAX, TRIMAX?

$\longrightarrow$ TOP-DOWN:assuming V_M bimax, or $V_M)_{13} = 0$?, any prediction for the less known θ_{13}?

Controversy?, Related to accidental complementarity or not. BIMAX$\longrightarrow$ 2 large angles$\longrightarrow$ non abelian flavour symmetries at work?.

BOTTOM-UP:Which V_M from DATA?: VM bimaximal?

- $V_{M,13}$ CAN VANISH, without special fine tuning (QLC): $\boldsymbol{V_M = U_{CKM}\ \Omega\ U_{PMNS}}$

$$\sin^2\theta_{13}^{V_M} = \left|\left(1-\frac{\lambda^2}{2}\right)e^{i(\omega_1-\omega_2-\phi)}\sin\theta_{13}^{PMNS} + \lambda\sin\theta_{23}^{PMNS}\cos\theta_{13}^{PMNS} + O(\lambda^3)\right|^2$$

$$\left|\frac{\sin\theta_{23}}{\tan\theta_{13}}\right| \sim \frac{1}{\lambda}, \text{ or, for } \theta_{13} \text{ small, } \theta_{23} \text{ maximal, } \quad \theta_{13}^{PMNS} \sim \frac{\lambda}{\sqrt{2}}$$

.

- USING CENTRAL VALUES:

1) CKM WOLFENSTEIN parameters $\lambda = 0.2237$, $\eta = 0.317$, $\rho = 0.225$, $|V_{cb}| \approx A\lambda^2 = 0.041$,

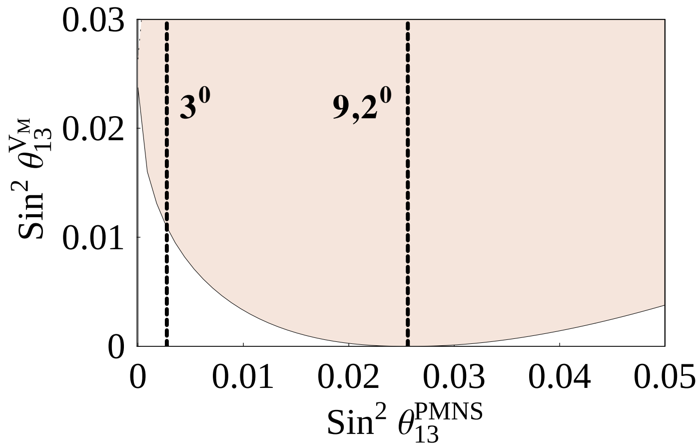
2) PMNS mixing angles $\theta_{12}^{PMNS} = 34°$, $\theta_{23}^{PMNS} = 45°$, $\theta_{13}^{PMNS} = FIXED(3°)$.

3) Ω phases: MC flat distributions in the interval $[0, 2\pi]$.

- range of values: $\boldsymbol{(V_M)_{13} = 0.10 - 0.22}$

$\longrightarrow$ 2) $(V_M)_{1,3} \neq 0$, not bi or tribimaximal. $\longrightarrow$ Already obtained.

BUT TOO NAIVE. OTHER CENTRAL VALUES?



BOTTOM-UP: A DETAILED STUDY.

- DETAILED STUDY, USE: $V_M = U_{CKM}\ \Omega\ U_{PMNS}$

1) Updated $CKM, PMNS$ at 95%CL CKMfitter(Charles:2004jd)

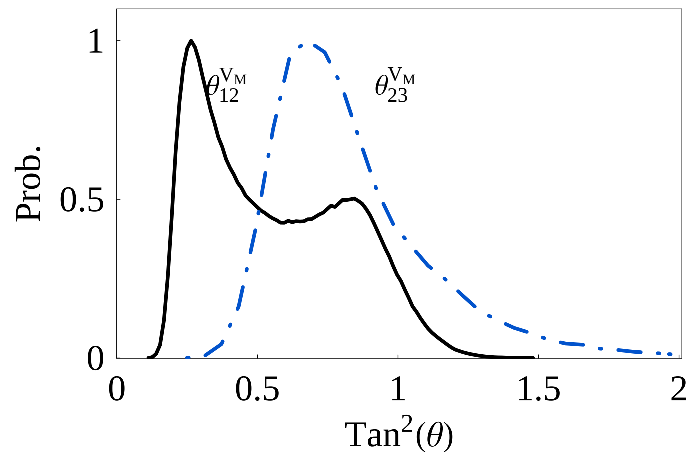
$$\lambda = 0.2265^{+0.0040}_{-0.0041}, A = 0.801^{+0.066}_{-0.041}, \overline{\eta} = 0.189^{+0.182}_{-0.114}, \overline{\rho} = 0.358^{+0.086}_{-0.085}$$

with

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\overline{\rho} + i\overline{\eta})}{\sqrt{1-\lambda^2}\left[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})\right]};$$

and

$$\sin^2\theta_{12}^{PMNS} = 0.44^{+0.41}_{-0.22}, \sin^2\theta_{23}^{PMNS} = 0.31^{+0.18}_{-0.15}, \sin^2\theta_{13}^{PMNS} = 0.009^{+0.02}_{-0.009}$$



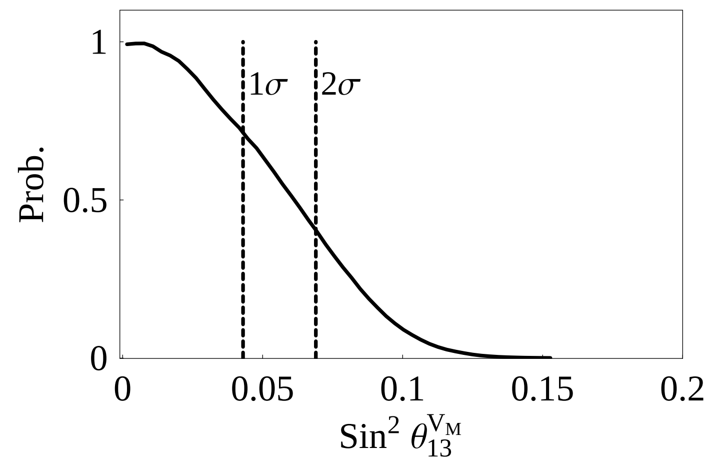
2) Two-sided Gaussian distributions CKM,PMNS parameters Unknown phases: vary in the interval $[0, 2\,\pi]$ (flat distribution).

- RESULTS:

$\longrightarrow$ $\tan^2\theta_{23}^{V_M} \sim [0.35, 1.4]$

$\longrightarrow$ $\tan^2\theta_{12}^{V_M} \sim 0.25 - 1.1.$ $(= 1.0$ BIMAX), $((= 0.5$ TRIBIMAX).

$\longrightarrow$ $\sin^2\theta_{13}^{V_M} = 0$ is preferred.



TOP-DOWN APPROACH: Predictions V_{PMNS}. θ_{13}^{PMNS}

- V_M BIMAXIMAL OR TRIBIMAXIMAL, $(V_M)_{13} \sim 0.$ $\longrightarrow$ PMNS $\longrightarrow$ θ_{13}^{PMNS}

- USE $\boldsymbol{U_{PMNS} = (U_{CKM}\ \Omega)^{-1}\ V_M}$

- CONDITIONS:

1)variation U_{CKM} two-sided Gaussians,

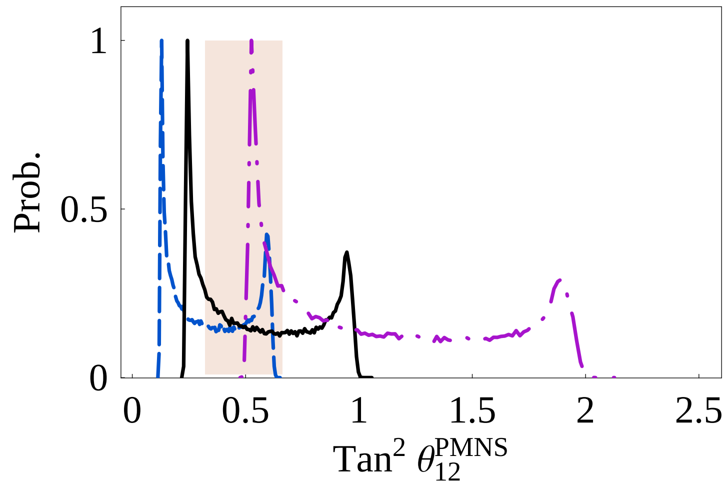
2) Ω phases: flat distributions in $[0, 2\pi]$.

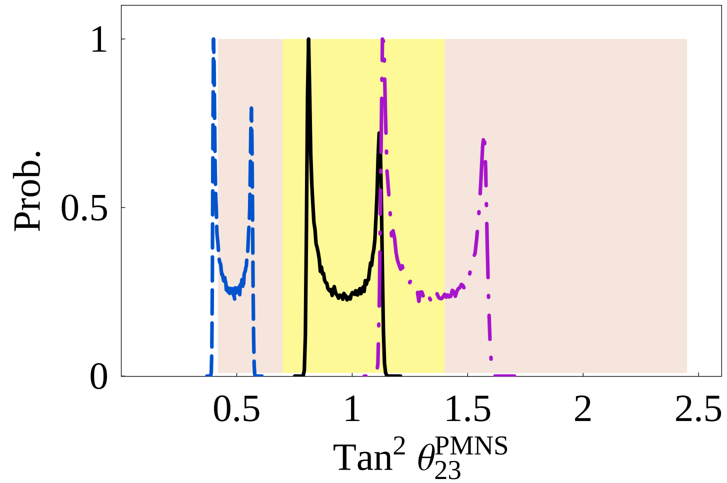
3) FIX V_M=BIMAX,TRIBIMAX.

$\longrightarrow$ **FIX $\tan^2\theta_{12}^{V_M} = 1.0$. (No dependence on $\tan^2\theta_{12}^{V_M}$)**

$\longrightarrow$ **FIX $\tan^2\theta_{23}^{V_M} \in \{0.5, 1.0, 1.4\}$.**

$\longrightarrow$ **FIX $\sin^2\theta_{13}^{V_M} = 0$.**





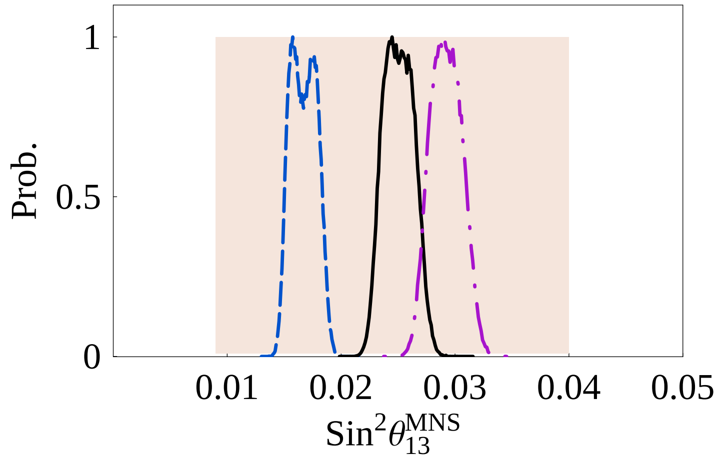
- **RESULTS:**

$\sin^2\theta_{13}^{PMNS}$ strongly peaked at $(7.3°, 8.9°, 9.8°)$.

FOR ALL the physical span of $\tan^2\theta_{23}^{V_M} \in \{0.5, 1.0, 1.4\}$.

$\longrightarrow$ **force θ_{13}^{PMNS} to be between 7° and 10°.**

$\longrightarrow$ VM BIMAX: $\sin^2\theta_{13}^{VM} = 0$, $\tan^2\theta_{23}^{VM} = 1$: $\boldsymbol{\theta_{13}^{PMN} \simeq 9 \pm 1°}$.

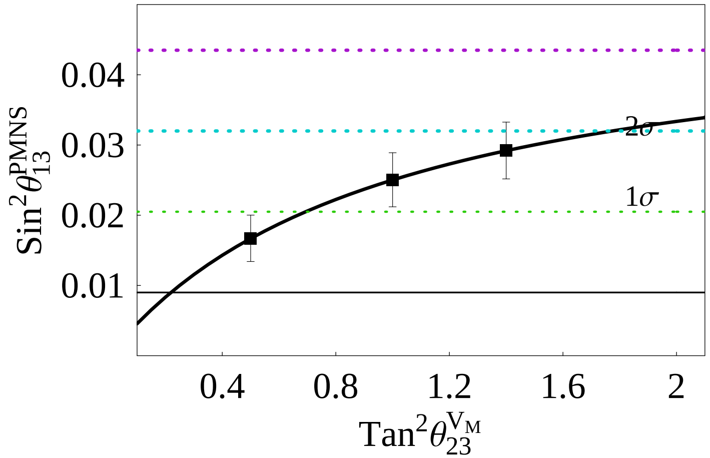


- **ANOTHER LOOK:**

$$(U_{PMNS})_{13} = e^{-i\omega_1}\Big[\left(1-\frac{\lambda^2}{2}\right)\sin\theta_{13}^{V_M} e^{-i\phi^{V_M}} - \lambda\sin\theta_{23}^{V_M}\cos\theta_{13}^{V_M} + A\lambda^3(-\rho + i\,\eta + 1)\cos\theta_{23}^{V_M}\cos\theta_{13}^{V_M} + O(\lambda^4)\Big],$$

so that, imposing $\sin^2\theta_{13}^{V_M} = 0$, $A \approx O(1)$.

$$\sin^2\theta_{13}^{PMNS} = \sin^2\theta_{23}^{V_M}\lambda^2 + O(\lambda^3),$$



Summary/Conclusions

- COMPLEMENTARITY: neutrino angles are nearly maximal, but not exactly maximal.

DIFFERENT SCENARIOS:

- **CP COMPLEMENTARITY.** $U_{PMNS} = U^{\dagger}(J)\ V_M$

⟶ Complex Bimaximal matrices, one parameter: $\lambda \sim J, a_{CP}$, ⟶ smallness of $U_{e3} \sim$ small a_{CP}.

⟶ $U_{e3} \simeq J$.

⟶ Estimation of Acp from **A) CHOOZ** $J \sim< 5\ \times 10^{-2}$. **B) Kamland:** $J = 0.042$

⟶ any link to LR SCPV models?.

- **QUARK-LEPTON COMPLEMENTARITY (QLC)**⟶ smallness of $U_{e3} \sim$ small θ_C.

a) DATA(NEUTRINOS+CKM) favour $(V_M)_{1,3} \sim 0$. ⟶ V_M can be bimax or tribimax (Non abelian F symms?).

b) $(V_M)_{13} = 0 +$ CKM data $\theta_{13}^{PMNS} = 9^{+1}_{-2}\, deg.$

- CKM-PMNS correlation from GUT+Non Abelian Flavour symmetries.

- **A THIRD POSSIBILITY? CP +CKM complementarity?:** two small parameters ⟶ fits.

$$U_{PMNS} = U^{\dagger}_{CKM} U^{\dagger}_{CP}(J)\ V_M$$

SOME FORMULAS,SEE SAW

- QUARKS, LEPTONS: $M_u, M_d, M_l, M_{\nu,D}$: Dirac mass matrices for the up, down sectors. They are diagonalized by

$$\begin{aligned} M_u &= U_u M_u^D V_u^\dagger; \qquad M_l = U_l M_l^D V_l^\dagger \\ M_d &= U_d M_d^D V_d^\dagger; \qquad M_{\nu D} = U_{\nu,D} (M_{\nu,D})^D (V_{\nu,D})^T \end{aligned} \tag{3}$$

⟶ CKM and lepton (DIRAC) mixing $\boldsymbol{U_{CKM} = U_u^\dagger\ U_d}$. ⟶ $\boldsymbol{U_{PMNS} = U_l^\dagger\ U_{\nu,D}}$.

- MAJORANA NEUTRINOS: (SEE SAW TYPE I), the light neutrino mass matrix M_ν: complex, symmetric.

$$\boldsymbol{M_\nu \ = \ M_D\ (M_R)^{-1}\ M_D^T = U_\nu\ M_\nu^D\ U_\nu^T} \tag{4}$$

⟶ FULL PMNS: lepton mixing matrix $\boldsymbol{U_{PMNS} = U_l^\dagger\ U_\nu}$.

- DEFINE New matrix C: $\boldsymbol{M_\nu \equiv U_{\nu,D} C U_{\nu,D}^T}$ then

$$C \ \equiv \ (M_{\nu,D})^D (V_{\nu,D})^\dagger M_R^{-1} (V_{\nu,D})^\star (M_{\nu,D})^D.$$

⟶ The matrix V_M diagonalizes this matrix

$$\boldsymbol{C = V_M M_\nu^D V_M^T, \quad U_{PMNS} = U_l^\dagger\ U_{\nu,D} V_M.}$$

We can ask:

⟶ where the large angles reside?, none,one,two, in V_M?.

⟶ can we relate the hierarchy nature of M_R to the number of small or large angles in V_M?

⟶ if $M_R \propto M_R I$ ⟶ C diagonal ⟶ $V_M = I$. ⟶ V_M measures the degeneracy of M_R.

⟶ If signficant nonunitarity: until which degree V_M is non-unitary?.

QLC and renormalization

- QLC holds at low energies, but Q-L symmetry or unification is most probable and some high energy GUT scale. ⟶ are renorm effects small enough to keep this relation from high to low scales?.

- For θ_C: in SM or MSSM the effect is small.

in MSSM For $\tan\beta = 50$, $\sin\theta_C(M_Z) = 0.2225$, $\sin\theta_C(GUT) = 0.2224$.

- For νs: strong dependence on the mass hierarchy.

⟶ normal mass hierchy $m_1 < m_2 << m_3$: very small effect.

⟶ inverted mass hierarchy,degenerate or quasi degenerate spectrum: the effect can be large (depending on the y_τ yukawa). Specially in MSSM with large $\tan\beta$.

$$\frac{d\theta_{12}}{dt} \propto \frac{C y_\tau^2}{32\pi^2} = \tag{5}$$

$$= 310^{-7}\ (SM) \tag{6}$$

$$= 310^{-7}(1+\tan^2\beta)\ (MSSM) \tag{7}$$