FERMION MASSES AND SO(10)

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(LJUBLJANA, SLOVENIA)

SUSY 06
IRVINE

WORK DONE WITH

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WE WILL CONSIDER

THE YUKAWA STRUCTURE IN SO(10)

I.E.

HOW ARE $M_u, M_d, M_e, M_n$

FERMION MASSES & MIXINGS

CONSTRAINED BY SO(10)
WE WILL ASSUME:

1. ONLY SO(10)
   - NO FLAVOUR SYMMETRY
   - NO SINGLETS

2. RENORMALIZABILITY
   - NEED IT FOR PREDICTIONS
   - NOT SO CRAZY:

   \[ W = \frac{c}{M_{pe}} \text{QQQL} \quad (16 \gamma) \]

   \[ c \leq 10^{-7} \]

   PHENOMENOLOGICAL CONSTRAINT
**THE MOST GENERAL YUKAWA**

\[ L_Y = 16_F^T (10_H Y_{10} + 120_H Y_{120} + 126_H Y_{126}) 16_F \]

**3 X 3 MATRICES**

**THE MOST GENERAL CASE NOT RESTRICTIVE:**

**TRY TO MODEL SIMPLE, MINIMAL SUBCASES**

\[ Y_{10,126}^T = + Y_{10,126} \]

\[ Y_{120}^T = - Y_{120} \]

**FROM SO(10)**

\[ 126 \leq (1) \text{ BREAKS RANK OF SO}(10) \]

\[ (2) \text{ GIVES MASS TO } \nu_{R,L} \text{ (MAJORANA) } \]

\[ (3) \text{ CORRECTS LIGHT FERMION MASSES} \]
MATTER

\[ 3 \times 16_F \rightarrow 15 + 1 \text{ SM } \nu_R \]

HIGGSES - YUKAWA SECTOR

\[ 16_F \times 16_F = 10_H + 126_H + 120_H \]

\[ M_{\nu_L} \sim \frac{M_w^2}{M_{\nu_R}} \quad \text{(SEESAW)} \]

\[ \uparrow \quad \text{SHALL} \quad \text{UP} \quad \text{LARGE} \]

\[ \nu_R = \text{SM SINGLET} \]

\[ (\text{LARGE}) \quad M_{\nu_R} = \text{SM SINGLET} \subset \overline{126}_H \]
In SUSY large $\langle 126_H \rangle \sim M_{\nu R}$

must be canceled in d-terms by

\[ 126_H \]

\[ (2,1,4) + (1,2,5) \]

\[ 16_F \rightarrow 126 \rightarrow 126_H \rightarrow 16_F \]

\( (\text{Yukawa}) \)

3x3 matrix

in generation space

\( \rightarrow \)

Pati-Salam decomposition

\( \rightarrow \)

\( SU(2)_L \times SU(2)_R \times SU(4)_C \)

\( \rightarrow \)

\( SO(10) \)

Majorana mass to $\nu_R$

Dirac mass to charged fermions

Majorana mass to $\nu_L$
\[ \langle (1,3,10) \rangle \neq 0 \quad \text{has} \quad B-L = 2 \]

\[ \downarrow \]

\text{R-parity unbroken at high energy}

\[ 3(B-L) \]

\[ R = (-1) \]

It remains unbroken also at low energy.

Mohapatra, 86
Font, Ibañez, Quevedo, 89
Martin, 92

Aulakh, Benakli, Senjanović, 97
Aulakh, Helfo, Senjanović, 98
Aulakh, Helfo, Rašin, Senjanović, 99

LSP dark matter candidate
\[ \langle (2,2,15) \rangle \implies M_E = -3 M_D \]

Good for 2nd gen.

\[ \mu \approx -3 \mu_s \]

Bad for 3rd gen.

\[ \langle (2,2,1) \rangle \implies (2,2,1) + \ldots \]

\[ \langle (2,2,1) \rangle \implies M_E = M_D \]

Good for 3rd gen.

\[ u_{\mu} \approx u_{\mu_s} \]

Bad for 1st, 2nd gen.

Combination of $10_H$ and $126_H$ could be realistic.

- Lazarides, Shafi, Wetterich, 81
- Babu, Mohapatra, 92
\[ M_D = \nu_{10}^d Y_{10} + \nu_{126}^d Y_{126} \]
\[ M_U = \nu_{10}^u Y_{10} + \nu_{126}^u Y_{126} \]
\[ M_E = \nu_{10}^d Y_{10} - 3 \nu_{126}^d Y_{126} \]
\[ M_{\nu D} = \nu_{10}^u Y_{10} - 3 \nu_{126}^u Y_{126} \]

**ONLY** \(10_H\) : \( M_D = M_E \)

**ONLY** \(\overline{126}_H\) : \(-3 M_D = M_E\)

DUE TO \(\langle (2,2,15) \rangle \propto \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}\)

**NEUTRINO:**
\[ M_{\nu_R} = \nu_R Y_{126} \]
\[ M_{\nu_L} = \nu_L Y_{126} \]
\[ \nu_R = \langle (1,3,10) \rangle \]
\[ \nu_L = \langle (3,1,10) \rangle \]

\[ M_N = M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu D} + M_{\nu_L} \]

\( \overset{\text{TYPE I}}{\text{SEESAW}} \)
\( \overset{\text{TYPE II}}{\text{SEESAW}} \)
FIRST ATTEMPTS USED ONLY TYPE I

BABU, MOHAPATRA, 92
ODA, TAKASUGI, TANAKA, YOSHIMURA, 98
MATSUDA, KOIDE, FUKUYAMA, NISHIURA, O1
FUKUYAMA, OKADA, 02
NOT VERY SUCCESSFUL

IF ASSUME ONLY TYPE II

\[ M_N \times M_{\nu_2} \times M_D = M_E \]
- 2\textsuperscript{ND} + 3\textsuperscript{RD} GENERATIONS \( (m_2 \ll m_3) \)
- ASSUME SMALL MIXING ANGLES IN CHARGED SECTOR

\[ M_N \propto \left( \begin{array}{c}
\varepsilon \\
\varepsilon \\
\varepsilon \\
\varepsilon
\end{array} \right)
\begin{pmatrix}
\omega_b \\
\omega_{b-\omega_b}
\end{pmatrix} \]

IN TYPE II SEESAW LARGE ATMOSPHERIC ANGLE CONNECTED WITH B-TAU UNIFICATION

BAJC, SENJANOVIC, VISSANI, 01, 02
SEVERAL ANALYSIS PERFORMED

GOM, MOHAPATRA, NG, 03, 03
DUTTA, MHIURA, MOHAPATRA, 04, 04
BASIC, SENJANOVIC, VISSANI, 04
GOM, MOHAPATRA, NASRI, 04
BERTOLINI, FRIGERIO, MALINSKY, 04

RESULT:

THE YUKAWA SECTOR

\[ 16_F \left( 10_H Y_{10} + 126_H Y_{126} \right) 16_F \]

IS REALISTIC BOTH IN TYPE I AND TYPE II SEE-SAW

BERTOLINI, MALINSKY, 05
BABU, HACESANU, 05
WHAT ABOUT THE HIGGS SECTOR?

\[ 10_H + 126_H + \overline{126}_H \]

- Not enough to break \( \text{SO}(10) \rightarrow \text{SM} \)

- Not enough to fine-tune the doublet-triplet splitting

\[ W_H = M_{10} \overline{10}_H^2 + M_{126} \overline{126}_H . 126_H \]

\[ (\bar{D}^{10}, \bar{D}_{126}, \bar{D}_{126}) \begin{pmatrix} M_{10} & 0 & 0 \\ 0 & 0 & M_{126} \end{pmatrix} \begin{pmatrix} \bar{D}^{10} \\ \bar{D}_{126} \\ \bar{D}_{126} \end{pmatrix} \]

The doublets \( D, \bar{D} \):
To get

\[ \det M = 0 \] (MSSM Higgs!)

One needs to mix

\[ 10_H \] and \[ 126_H \] (\[ \overline{126}_H \])

\[ \downarrow \text{1 index} \]
\[ \downarrow \text{5 indices} \]

Connected by

\[ 2^{10}_H \] \[ \rightarrow \text{4 indices} \]

\( (1,1,1) \)
\( (1,1,15) \)
\( (1,3,15) \)
\( (2,2,10) \)
\( (2,2,\overline{10}) \)

\[ \text{together with} \]
\[ 126_H + \overline{126}_H \]

\[ \text{break} \]
\[ \text{SO}(10) \rightarrow \text{SM} \]

Other doublets
ALL TOGETHER

$10_H + 126_H + \overline{126}_H + 210_H$

$W_{\text{Higgs}} = M_{210} \overline{210}_H^2 + \lambda 210_H$

$+ M_{126} \overline{126}_H \overline{126}_H + \eta \overline{126}_H 210_H 126_H$

$+ M_{10} 10_H^2 + 210_H 10_H (\alpha 126_H + \beta \overline{126}_H)$

$W_{\text{Yukawa}} = 16_F \left( Y_{10} 10_H + Y_{126} \overline{126}_H \right) 16_F$

$L_{\text{Gauge}} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \ldots$

TOTAL NUMBER OF MODEL PARAMETERS =

$26$ (AS IN HSSM!)

(+ SUSY BREAKING)

$\Rightarrow$ MINIMAL GUT

aulakh, balo, melo, senjanovic, vissani, 03
So far the fitting of Yukawa sector has been done without assuming Higgs sector.

Why should it matter?

\[ M_{\text{Fermion}} = \mathbf{U} \mathbf{Y} \]

The VEVs determined by the Higgs sector parameters.

\[ \rightarrow \text{Type II See-Saw only unlikely} \quad \text{Aulakh, 05} \]

\[ \rightarrow \text{Type I See-Saw only unlikely} \quad \text{Bajc, Hell, Semjanovic, Vissani, 05} \]
VERY RECENTLY:

THE MINIMAL SUCH \( \mathcal{E}(\nu) \) IS INCONSISTENT WITH DATA.

\[ \text{BEHIND, HAENEL, SCHWARZ, 06} \]

BUT:

\( \chi^2 \) ANALYSIS DONE AT \( M_{\nu_\tau} \)

(DIFFICULT TO ESTIMATE THE ERRORS)

BETTER TO DO IT AT \( M_2 \)

PRELIMINARY RESULT:

\( \chi^2 \) EVEN LARGER

F. NESTI

(TO APPEAR)
CONCLUSIONS

1. MINIMAL RENORMALIZABLE SO(10)

\[ 3 \times 16_F + 10_H + 126_H + \overline{126}_H + 210_H \text{ + gauge:} \]

only 26 parameters (+ SUSY)

2. YUKAWA SECTOR ABLE TO DESCRIBE FERMION MASSES + MIXINGS

3. HIGGS SECTOR CONSTRAINTS

=> POSSIBLE PROBLEMS?

4. PREDICTIONS (MODULO 3):

- \(|U_{e3}| > 0.1\), all CP phases
- NEUTRINO HIERARCHICAL
- LOW-ENERGY SUSY WITH EXACT R-PARITY (LSP!)

5. OTHER POSSIBILITIES:

\[ \rightarrow 10_H + 120_H \text{ (SPLIT SUSY)} \]
\[ \rightarrow 126_H + 120_H \text{ (NON SUSY)} \]