FERMION MASSES AND SO(10)

BORUT BAJC

(LJUBLJANA, SLOVENIA)

SUSY 06
IRVINE

WORK DONE WITH

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WE WILL CONSIDER

THE YUKAWA STRUCTURE IN SO(10)

I.E.

HOW ARE M_U, M_D, M_E, M_N

FERMION MASSES & MIXINGS

CONSTRAINED BY SO(10)

WE WILL ASSUME:

(1) ONLY SO(10)

NO FLAVOUR SYMMETRY

NO SINGLETS

(2) RENORMALIZABILITY

- NEED IT FOR PREDICTIONS
- NOT SO CRAZY:

$$W = \frac{c}{M_{Pl}} QQQL \qquad (16_F^4)$$

$c \lesssim 10^{-7}$ PHENOMENOLOGICAL CONSTRAINT

THE MOST GENERAL YUKAWA

$$\mathcal{L}_Y = 16_F^T \left(10_H Y_{10} + 120_H Y_{120} + 126_H Y_{126} \right) 16_F$$

(Y_{10}, Y_{120}, Y_{126}:) 3 X 3 MATRICES

$$Y_{10,126}^T = + Y_{10,126}$$

$$Y_{120}^T = - Y_{120}$$

FROM SO(10)

THE MOST GENERAL CASE
NOT RESTRICTIVE:
TRY TO MODEL SIMPLE,
MINIMAL SUBCASES

126:
- (1) BREAKS RANK OF SO(10)
- (2) GIVES MASS TO $\nu_{R,L}$ (MAJORANA)
- (3) CORRECTS LIGHT FERMION MASSES

MATTER

$$3 * 16_F$$

$$\hookrightarrow \underset{SM}{15} + \underset{\nu_R}{1}$$

HIGGSES – YUKAWA SECTOR

$$16_F * 16_F = 10_H + 126_H + 120_H$$

$$m_{\nu_L} \sim \frac{M_W^2}{m_{\nu_R}} \quad \text{(SEESAW)}$$

m_{ν_L} ↑ SMALL, m_{ν_R} ↑ LARGE

ν_R = SM SINGLET

⇓

(LARGE) M_{ν_R} = SM SINGLET $\subset \boxed{\overline{126}_H}$!

IN SUSY LARGE $\langle \overline{126}_H \rangle \sim M_{\nu_R}$

MUST BE CANCELED IN

D-TERMS BY 126_H

$(2,1,4) + (1,2,\bar{4})$

$16_F \, Y_{126} \, \overline{126}_H \, 16_F$ (YUKAWA)

3X3 MATRIX IN GENERATION SPACE

$(1,3,10)$ — MAJORANA MASS TO ν_R

$+$

$(2,2,15)$ — DIRAC MASS TO CHARGED FERMIONS

$+$

$(3,1,\overline{10})$ — MAJORANA MASS TO ν_L

$+$

...

⇗ PATI-SALAM DECOMPOSITION

$SU(2)_L \times SU(2)_R \times SU(4)_C$

$\subset SO(10)$

ANOTHER REASON TO USE 126_H :

$\langle (1,3,10) \rangle \neq 0$ HAS $B-L = 2$

$\Downarrow$

R - PARITY UNBROKEN AT HIGH ENERGY

MOHAPATRA, 86
FONT, IBAÑEZ, QUEVEDO, 89
MARTIN, 92

$$R = (-1)^{3(B-L)}$$

IT REMAINS UNBROKEN ALSO AT LOW ENERGY

AULAKH, BENAKLI, SENJANOVIĆ, 97
AULAKH, MELFO, SENJANOVIĆ, 98
AULAKH, MELFO, RAŠIN, SENJANOVIĆ, 99

LSP DARK MATTER CANDIDATE

$\overline{126}_H$:

$\langle (2,2,15) \rangle \Longrightarrow M_E = -3 M_D$

GOOD FOR 2^{ND} GEN.

$m_\mu \approx -3 m_s$

BAD FOR $1^{ST}, 3^{RD}$ GEN.

NEED FOR $\boxed{10_H}$:

$(2,2,1) + \ldots$

$\langle (2,2,1) \rangle \Longrightarrow M_E = M_D$

GOOD FOR 3^{RD} GEN.

$m_\tau \approx m_b$

BAD FOR $1^{ST}, 2^{ND}$ GEN.

COMBINATION OF 10_H AND $\overline{126}_H$ COULD BE REALISTIC !

- LAZARIDES, SHAFI, WETTERICH, 81
- BABU, MOHAPATRA, 92

$$M_D = v_{10}^d Y_{10} + v_{126}^d Y_{126}$$

$$M_U = v_{10}^u Y_{10} + v_{126}^u Y_{126}$$

$$M_E = v_{10}^d Y_{10} - 3\, v_{126}^d Y_{126}$$

$$M_{\nu_D} = v_{10}^u Y_{10} - 3\, v_{126}^u Y_{126}$$

ONLY 10_H : $M_D = M_E$

ONLY $\overline{126}_H$: $-3 M_D = M_E$

-3 DUE TO $\langle (2,2,15) \rangle \propto \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$

NEUTRINO:

$$M_{\nu R} = v_R Y_{126} \qquad v_R = \langle (1,3,10) \rangle$$

$$M_{\nu L} = v_L Y_{126} \qquad v_L = \langle (3,1,\overline{10}) \rangle$$

$$M_N = \underbrace{M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}}_{\text{TYPE I SEESAW}} + \underbrace{M_{\nu_L}}_{\text{TYPE II SEESAW}}$$

FIRST ATTEMPTS USED ONLY TYPE I

BABU, MOHAPATRA, 92
ODA, TAKASUGI, TANAKA, YOSHIMURA, 98
MATSUDA, KOIDE, FUKUYAMA, NISHIURA, 01
FUKUYAMA, OKADA, 02 ← !

NOT VERY SUCCESSFUL

IF ASSUME ONLY TYPE II

$$M_N \propto M_{\nu_L} \propto M_D - M_E$$

- 2^{ND} + 3^{RD} GENERATIONS ($m_2 \ll m_3$)
- ASSUME SMALL MIXING ANGLES IN CHARGED SECTOR

$$M_N \propto \begin{pmatrix} \epsilon & \epsilon \\ \epsilon & m_b - m_\tau \end{pmatrix}$$

IN TYPE II SEESAW LARGE ATMOSPHERIC ANGLE CONNECTED WITH B-TAU UNIFICATION

BAJC, SENJANOVIĆ, VISSANI, 01, 02

SEVERAL ANALYSIS PERFORMED

GOH, MOHAPATRA, NG, 03, 03
DUTTA, MIMURA, MOHAPATRA, 04, 04
BAJC, SENJANOVIĆ, VISSANI, 04
GOH, MOHAPATRA, NASRI, 04
BERTOLINI, FRIGERIO, MALINSKY, 04

RESULT :

THE YUKAWA SECTOR

$$16_F \left(10_H Y_{10} + \overline{126}_H Y_{126} \right) 16_F$$

IS REALISTIC BOTH

IN TYPE I

AND TYPE II

SEE-SAW

BERTOLINI, MALINSKY, 05
BABU, MACESANU, 05

[redacted]

WHAT ABOUT THE HIGGS SECTOR?

$10_H + 126_H + \overline{126}_H$

- NOT ENOUGH TO BREAK

  $SO(10) \to SM$

- NOT ENOUGH TO FINE-TUNE THE DOUBLET – TRIPLET SPLITTING

$$W_H = M_{10}\, 10_H^2 + M_{126}\, \overline{126}_H \cdot 126_H$$

$10_H \rightsquigarrow (2,2,1)$, $\overline{126}_H \rightsquigarrow (2,2,15)$, $126_H \rightsquigarrow (2,2,15)$

THE DOUBLETS $D, \bar{D}$:

$$\left(\bar{D}_{10}, \bar{D}_{126}, \bar{D}_{\overline{126}}\right) \begin{pmatrix} M_{10} & 0 & 0 \\ 0 & 0 & M_{126} \\ 0 & M_{126} & 0 \end{pmatrix} \begin{pmatrix} D_{10} \\ D_{126} \\ D_{\overline{126}} \end{pmatrix}$$

TO GET

$$\det M_{\text{Doublets}} = 0 \quad \left(\begin{matrix} \text{MSSM} \\ \text{HIGGS} \end{matrix} ! \right)$$

ONE NEEDS TO MIX

10_H AND 126_H $\left(\overline{126}_H \right)$

10_H ⇝ 1 INDEX

126_H, $\overline{126}_H$ ⇝ 5 INDICES

CONNECTED BY

$\boxed{210_H}$ ⇝ 4 INDICES

$$
\begin{aligned}
210_H = \; & (1,1,1) \\
+ \; & (1,1,15) \\
+ \; & (1,3,15)
\end{aligned}
$$

TOGETHER WITH $126_H + \overline{126}_H$ BREAK $SO(10) \to SM$

$$
\begin{aligned}
+ \; & (2,2,10) \\
+ \; & (2,2,\overline{10})
\end{aligned}
$$

OTHER DOUBLETS

$+ \ldots$

ALL TOGETHER

$$10_H + 126_H + \overline{126}_H + 210_H$$

CLARK, KUO, NAKAGAWA, 82
AULAKH, MOHAPATRA, 82
LEE, 94

$$W_{HIGGS} = m_{210}\, 210_H^2 + \lambda\, 210_H^3$$
$$+ m_{126}\, \overline{126}_H\, 126_H + \eta\, \overline{126}_H\, 210_H\, 126_H$$
$$+ m_{10}\, 10_H^2 + 210_H\, 10_H \left(\alpha\, 126_H + \bar{\alpha}\, \overline{126}_H \right)$$

$$W_{YUKAWA} = 16_F \left(Y_{10}\, 10_H + Y_{126}\, \overline{126}_H \right) 16_F$$

$$\mathcal{L}_{GAUGE} = -\frac{1}{4g^2} F^A_{\mu\nu} F^{A\mu\nu} + \ldots$$

TOTAL NUMBER OF MODEL PARAMETERS =

26 ⇝ (AS IN MSSM!)

(+ SUSY BREAKING)

⟹ MINIMAL GUT

AULAKH, BAJC, MELFO, SENJANOVIĆ, VISSANI, 03

SO FAR THE FITTING OF YUKAWA SECTOR HAS BEEN DONE WITHOUT ASSUMING HIGGS SECTOR

WHY SHOULD IT MATTER?

$$M_{FERMION} = v\, Y$$

v → HIGGS SECTOR

Y → YUKAWA SECTOR

THE VEVS DETERMINED BY THE HIGGS SECTOR PARAMETERS

→ TYPE II SEE-SAW ONLY UNLIKELY

AULAKH, 05

BAJC, MELFO, SENJANOVIĆ, VISSANI, 05

→ TYPE I SEE-SAW ONLY UNLIKELY

AULAKH, GARG, 05

VERY RECENTLY:

THE MINIMAL SUSY SO(10) IS INCONSISTENT WITH DATA

BERTOLINI, MALINSKY, SCHWETZ, 06

BUT:

χ^2 ANALYSIS DONE AT M_{GUT}

(DIFFICULT TO ESTIMATE THE ERRORS)

BETTER TO DO IT AT M_Z

PRELIMINARY RESULT:

χ^2 EVEN LARGER

F. NESTI

(TO APPEAR)

CONCLUSIONS

1. MINIMAL RENORMALIZABLE SO(10)

   $= 3 * 16_F + 10_H + 126_H + \overline{126}_H + 210_H$ + GAUGE:

   ONLY 26 PARAMETERS (+ ~~SUSY~~)

2. YUKAWA SECTOR ABLE TO DESCRIBE FERMION MASSES + MIXINGS

3. HIGGS SECTOR CONSTRAINTS

   $\Rightarrow$ POSSIBLE PROBLEMS ?

4. PREDICTIONS (MODULO ③):
   - $|U_{e3}| > 0.1$, ALL ~~CP~~ PHASES
   - NEUTRINO HIERARCHICAL
   - LOW-ENERGY SUSY WITH EXACT R-PARITY (LSP!)

5. OTHER POSSIBILITIES:

   $\rightarrow 10_H + 120_H$ (SPLIT SUSY)

   $\rightarrow 126_H + 120_H$ (NONSUSY)