A Solution for Little Hierarchy Problem and $b \rightarrow s \gamma$

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Little Hierarchy Problem

- SM Higgs $\geq 114.4$ GeV \quad \text{LEP}
- If lighter MSSM Higgs $\geq 114.4$ GeV
  \quad \rightarrow \quad \text{Large stop mass } \geq 500$ GeV
  \quad \rightarrow \quad \text{Requires a tuning to obtain EW scale}

Little Hierarchy Problem

- But the Higgs bound at LEP is for SM, not for MSSM.
- We must examine the LEP results in MSSM.
Higgs Search at LEP

- The SM Higgs mass $> 114.4 \text{ GeV (95\% CL)}$

\[ |D_\mu H|^2 \rightarrow g^2 ZZHH \rightarrow g^2 \langle H \rangle ZZH = g_{ZZH} ZZH \]
LEP experiment for MSSM Higgs

- MSSM has two Higgs doublets. One of two Higgs have vanishing VEV.
- For simplicity, we take $\langle H_u \rangle >> \langle H_d \rangle$
  $$H_u \rightarrow g_{ZZH_u} \sim g_{ZZH} \quad \text{SM like Higgs}$$
  $$H_d \rightarrow g_{ZZH_d} << g_{ZZH} \quad \text{cannot be seen in LEP}$$
- If $m_{H_u} > m_{H_d}$, lighter Higgs becomes $H_d$
  Lighter Higgs $H_d$ cannot be seen.

Is it possible to realize the above situation in a natural way?
Mass matrix of CP even Higgs (tree)

- When $\langle H_u \rangle >> \langle H_d \rangle$ (i.e. $\tan \beta >> 1$)

$$m_{h,H}^2 = \begin{pmatrix}
H_d & H_u \\
H_u & \end{pmatrix} \begin{pmatrix}
m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_A^2 + m_Z^2) \sin \beta \cos \beta \\
-(m_A^2 + m_Z^2) \sin \beta \cos \beta & m_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta
\end{pmatrix} \sim \begin{pmatrix}
m_A^2 & 0 \\
0 & m_Z^2
\end{pmatrix}$$

$m_A$ CP odd Higgs mass

- It is easy to satisfy $m_{Hu} > m_{Hd}$, if we take $m_A^2 < m_Z^2$
  $\Rightarrow$ All the Higgs boson mass scales are EW scale!

- Is it possible to satisfy $m_{Hu} > 114.4 \text{GeV}$ naturally?
  $\rightarrow$ loop corrections and diagonalization of actual mass matrix must be taken into account
Experimental aspect

Results of LEP II

- 115 GeV ~1.7 σ excess may be explained by the heavier (the SM like) CP-even MSSM Higgs boson
- 98 GeV ~2.3 σ excess may be explained by the lighter CP-even MSSM Higgs with small coupling
Previous works

- Both excesses can be explained in the MSSM, if SUSY breaking parameters have not mSUGRA type boundaries.

\[ 90\text{GeV} \leq m_A \leq 175\text{GeV}, \quad 110\text{GeV} \leq m_{H^\pm} \leq 200\text{GeV} \]

- They didn't take care about the fine-tuning problem enough.

\[ 100\text{GeV} \leq m_{\tilde{t}}, m_{H_u}, m_{H_d}, \mu, A_t \leq 2\text{TeV} \]

  e.g., large \( \mu \) requires fine-tuning twice, because

\[ m_1^2 = m_{H_d}^2 + \mu^2, \quad m_2^2 = m_{H_u}^2 + \mu^2 \approx O((100\text{GeV})^2) \]

- It is not obvious whether it is possible within natural SUSY breaking parameters.

\[ m_{\tilde{t}}, m_{H_u}, m_{H_d}, \mu, A_t \approx O((100\text{GeV})) \]
Set up for numerical calculation

- Natural SUSY breaking parameters
  \[ m_\tilde{t}, m_{H_u}, m_{H_d}, \mu, A_t \approx O((100\text{GeV})) \]

- GUT relation for gaugino masses

- LEP bound \( m_{\tilde{\chi}^0} \geq 46\text{GeV}, \ m_{\tilde{\chi}^\pm} \geq 94\text{GeV} \)

- Consistent with LEP Higgs search
  \[ \xi = g_{ZZh} / g_{ZZH_{\text{SM}}} \leq 0.50 \]
  \[ 90\text{GeV} \leq m_h (\leq 117\text{GeV}) \]

- Signal(98GeV)
  \[ \xi = g_{ZZh} / g_{ZZH_{\text{SM}}} \leq 0.50 \]
  \[ 95\text{GeV} \leq m_h \leq 101\text{GeV} \]
Results of numerical analyses

$90 \leq \xi \leq 5.00 \, \text{GeV}$

$5.00 \, \text{GeV} \leq \xi \leq 101 \, \text{GeV}$

$\tan \beta$

$m_{Q_3}=350\,\text{GeV}, \, m_{U_3}=300\,\text{GeV}, \, \mu =200\,\text{GeV}, \, A=325\,\text{GeV}$

$HH$ : heavy Higgs, $CH$ : Charged Higgs

1. $0 \leq \xi \leq 0.5$
2. $90\,\text{GeV} \leq m_h$

$0 \leq \xi \leq 0.50$
$95\,\text{GeV} \leq m_h \leq 101\,\text{GeV}$
1st summary and questions

- Light CP-even Higgs boson with small $g_{Zh}$ coupling can be consistent with the LEP data in the MSSM with natural SUSY breaking parameters. (2 excesses can be signals.)
- Mass scales of the MSSM Higgs bosons are EW scale.
  
  \[ m_h \sim 98\text{GeV}, \quad m_H \sim (115-120)\text{GeV} \]
  \[ m_A \sim 100\text{GeV}, \quad m_{H^\pm} \sim 130\text{GeV} \]

- Is such a light charged Higgs boson consistent with $\text{Br}(b\rightarrow s\gamma)$? [next topic]
  
  cf. $m_{H^\pm} \geq 350\text{GeV}$ in the type II 2HDM

- What is the essential point for obtaining lighter stop?
  
  $m_t \approx 300\text{GeV}$
Rich guy becomes richer

- The same radiative correction $\Delta(m_{\tilde{t}})$
  
  ① $m_{H_d}^2 \geq m_{H_u}^2 + \Delta(m_{\tilde{t}})$
  
  $114 \text{GeV} \leq m_h$

  ② $m_{H_d}^2 \leq m_{H_u}^2 + \Delta(m_{\tilde{t}})$

  $114 \text{GeV} \leq m_H$

- Off diagonal element $m_{12}^2 \propto \cot \beta$ increases
  (decreases) $m_H(m_h)$

- The usual case ① needs larger $\Delta(m_{\tilde{t}})$ and smaller $m_{12}^2$
  (larger $\tan \beta$) to satisfy the lower bound of SM like Higgs.

Large $\tan \beta$ is unfavored in case ②
Numerical calculation

\[ m_Q = 350\text{GeV}, \quad m_U = 300\text{GeV}, \]
\[ \mu = 200\text{GeV}, \quad A = 325\text{GeV} \]

\[ m_Q = 300\text{GeV}, \quad m_U = 250\text{GeV}, \]
\[ \mu = 300\text{GeV}, \quad A = 300\text{GeV} \]

\[ 0 \leq \xi \leq 0.50 \]
\[ 90\text{GeV} \leq m_h (\leq 117\text{GeV}) \]
$b \rightarrow s \gamma$ constraint

Is this scenario consistent with $b \rightarrow s \gamma$?

$m_{H^\pm} \approx 130\text{GeV} \iff m_{H^\pm} \geq 350\text{GeV}$
Branching ratio of $b \rightarrow s \gamma$

$$\text{Pre: }(3.60 \pm 0.30) \times 10^{-4}$$
$$\text{Exp: }(3.39 \pm 0.27) \times 10^{-4}$$

Fact: no $b \rightarrow s \gamma$ process if SUSY is exact [Ferrara and Remiddi ’74]

$m_W = m_{\tilde{W}}, m_{H^\pm} = m_{\tilde{H}^\pm}, m_t = m_{\tilde{t}}$

Naturalness $\Rightarrow$ The masses of every fields in the loops are the weak scale.

$m_W \sim m_{\tilde{W}}, m_{H^\pm} \sim m_{\tilde{H}^\pm}, m_t \sim m_{\tilde{t}}$

The cancellation between charged Higgs contribution and the chargino’s is expected.
Results (small coupling, naturalness)

- **Charged Higgs** \((m_{Q3}=350\text{GeV}, m_{U3}=300\text{GeV}, \mu_R=200\text{GeV}, \tan\beta=10)\)

- **Chargino**

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**SM** \(\Rightarrow\) Charged Higgs induced amplitude

\[
\begin{align*}
\text{Charged Higgs induced amplitude} & \quad \text{(Charged Higgs)} \\
\text{SM} & \quad \Rightarrow \\
\text{Chargino induced amplitude} & \quad \text{(Chargino)}
\end{align*}
\]
$B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}$

**Charged Higgs contribution in tree diagram.**

*There are no SUSY particle contributions at tree level.
  
i.e., no cancellations as in the case of $b \rightarrow s \gamma$!

$$Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau})_{SM+CH} = Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau})_{SM} \times r_H$$

$$r_H = (1 - m_B^2 \frac{\tan^2 \beta}{2})^2$$

$$m_{H^\pm} = 130 \text{GeV}$$

$*Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau})_{SM} = \frac{G_F m_B m_{\tau}^2 (1 - \frac{m_{\tau}^2}{m_B^2})^2 f_B V_{ub}^2}{8\pi} \tau_B$

$$= (1.59 \pm 0.40) \times 10^{-4}$$

$$Br(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau})_{Exp} = (1.06^{+0.34}_{-0.28} \text{(stat)} + 0.18_{-0.16} \text{(syst)}) \times 10^{-4}$$

**Belle hep-ex/0604018**

$$r_H \approx 0.67, m_{H^\pm} \approx 130 \text{GeV} \Rightarrow \tan \beta \approx 10.35$$
Summary

- Little hierarchy problem can be solved by lighter Higgs with smaller ZZ$^h$ coupling.
  Rich (poor) guy becomes richer (poorer).
- Every Higgs in MSSM have the weak scale masses.
  
  \[
  m_h \sim 98\text{GeV}, m_H \sim (115-120)\text{GeV} \\
  m_A \sim 100\text{GeV}, m_{H^\pm} \sim 130\text{GeV}
  \]
- Such small charged Higgs mass is consistent with Br(b$\rightarrow$s $\gamma$) because of cancellation.
  Naturalness requirement plays an important role in the cancellation.