Mirage unification at TeV scale and natural electroweak symmetry breaking in minimal supersymmetry

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Kiwoon Choi, Kwang-Sik Jeong and KO. JHEP 0509:039,
Kiwoon Choi, Kwang-Sik Jeong, Tatsuo Kobayashi and KO. Phys Lett B 633 (2006)

I. Introduction

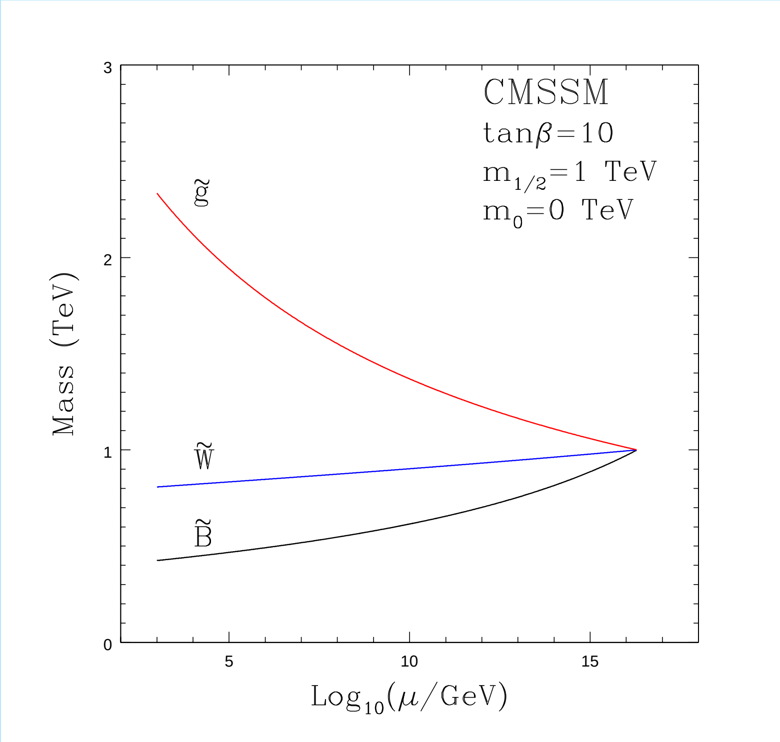
- Supersymmetry (SUSY) has been considered to be the leading candidate for physics beyond the SM not only as a solution of the hierarchy problem but also for many attractive features of the minimal model like gauge coupling unification and natural candidate for cold dark matter.

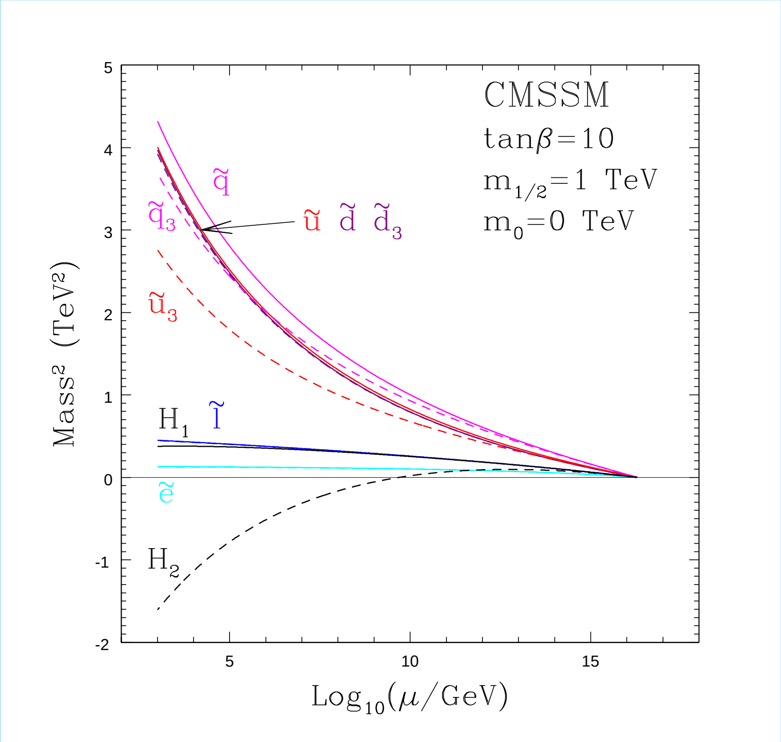
- However lower bound for m_{h^0} measured at LEPII favors heavy $\tilde{t}$ and this requires uncomfortable fine-tuning ($\lesssim$ a few %) in electroweak symmetry breaking of the MSSM (SUSY fine-tuning problem), leading to proliferation of alternatives and extensions.

- In this talk we come back to the minimal model again and propose a new scenario which solves the SUSY fine-tuning problem without any modification of the MSSM at low energy based on a SUSY breaking model inspired from KKLT flux string compactification.

II. Supersymmetric Fine-tuning Problem

Radiative electroweak symmetry breaking





L.E.Ibanez and G.G.Ross, K.Inoue, A.Kakuto, H.Komatsu and S.Takeshita, J.R.Ellis, D.V.Nanopoulos and K.Tamvakis, L.Alvarez-Gaume, J.Polchinski and M.B.Wise

Tuning in the radiative electroweak symmetry breaking

Radiative correction to $m_{H_2}^2$ is order of $m_{\tilde{t}}^2$

$$\Delta m_{H_2}^2 \sim -\frac{3}{4\pi^2} y_t^2 m_{\tilde{t}}^2 \ln\left(\frac{\Lambda}{m_{\tilde{t}}}\right) \approx -2m_{\tilde{t}}^2$$

$m_Z^2/2$ is given by the difference between $|m_{H_2}^2|$ and $|\mu|^2$.

$$\frac{m_Z^2}{2} = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - |\mu|^2 \approx -m_{H_2}^2 - |\mu|^2$$

$m_{\tilde{t}} \sim m_{H_2} \approx \mu > 500GeV$ means $<$ 2% fine-tuning in the measure,

$$\Delta_{\mu^2}^{-1} \equiv \left(\frac{\partial \ln(m_Z^2)}{\partial \ln(\mu^2)}\right)^{-1} \approx \frac{-m_{H_2}^2 - |\mu|^2}{|\mu|^2} \approx \frac{m_Z^2}{2|\mu|^2}$$

Radiative correction in the lightest Higgs boson mass

Theoretical upper bound for m_{h_0} is given by m_Z at tree-level. However, radiative correction from y_t can raise the bound,

H.E.Haber and R.Hempfling, Y.Okada, M.Yamaguchi and T.Yanagida, J.R.Ellis, G.Ridolfi and F.Zwirner

$$m_{h^0}^2 < m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{X_t^2}{12 m_{\tilde{t}}^2}\right)\right]$$

where $X_t = A_t - \mu\cot\beta$. (Taken from M.Carena and H.Haber Prog.Part.Nucl.Phys.50:63-152,2003)

For instance, the current SM bound is translated into,

$$m_{h^0} > 114.4 \text{ GeV} \rightarrow m_{\tilde{t}} \gtrsim 500 \text{ GeV} \qquad (X_t^2 << m_{\tilde{t}}^2)$$

Here we call this tension between the tuning in determination of m_Z and m_{h^0} lower bound as supersymmetric fine-tuning problem.

- Enhance quartic coupling in the Higgs potential and raise tree-level Higgs mass $\leftrightarrow$ perturbative unification.
  - A.Brignole, J.A.Casas, J.R.Espinosa and I.Navarro, J.A.Casas, J.R.Espinosa and I.Hidalgo
  - P. Batra, A.Delgado, D.E.Kaplan and T.M.P. Tait
  - R. Harnik, G.D.Kribs, D.T.Larson and H.Murayama
  - S. Chang, C. Kilic and R. Mahbubani
  - A. Birkedal, Z. Chacko and Y. Nomura
  - K.S.Babu, I. Gogoladze and C. Kolda
  - .....

- Realize little hierarchy between stop and Higgs soft breaking masses $\leftrightarrow$ large radiative correction.
  - T. Kobayashi and H. Terao, T.Kobayashi, H.Nakano and H.Terao
  - A. Birkedal, Z. Chacko and M.K. Gaillard
  - Z.Chacko, Y.Nomura and D.Tucker-Smith
  - Z. Berezhiani, P.H. Chankowski, A. Falkowski and S. Pokorski
  - T. Roy and M. Schmaltz
  - C. Csaki, G. Marandella, Y. Shirman and A. Strumia
  - A. Falkowski, S. Pokorski and M. Schmaltz
  - S. Chang, L.J. Hall and N. Weiner
  - .....

Sorry for incomplete references

Complicated new fields and thresholds at low energy are inevitable?

Complicated new fields and thresholds at low energy are inevitable?

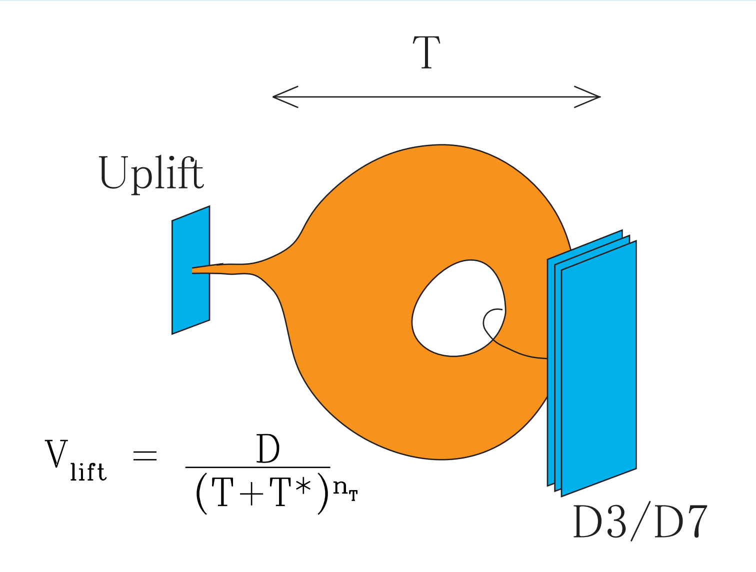
$\rightarrow$ No, we can have an explicit model at least effective supergravity level which realizes the little hierarchy within the MSSM and virtually no threshold up to the unification scale.

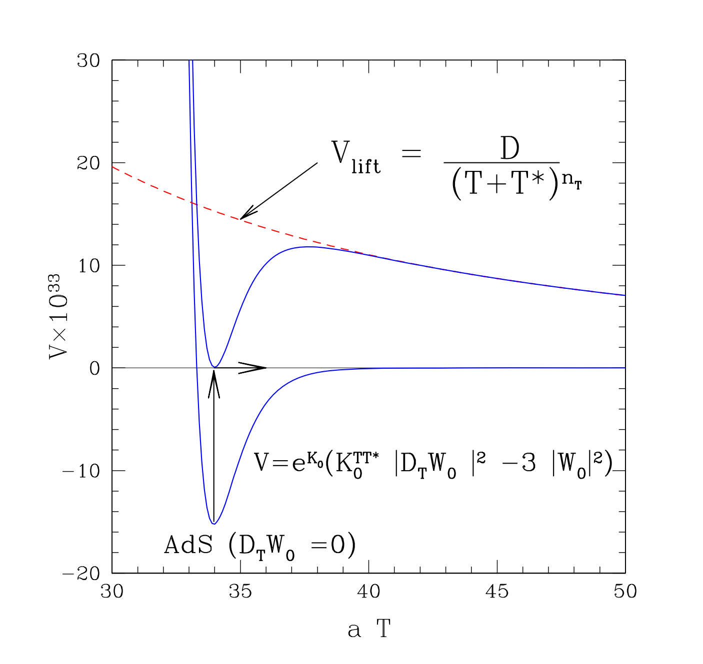
"Mirage unification" in the mixed modulus-anomaly mediation (mirage mediation)

III. Mixed Modulus-Anomaly Mediation in KKLT model

Compactified string theory predicts moduli fields (S, T, Z^α) in 4D.
KKLT stabilized all of them with tunable positive cosmological constant.
S, Z^α: flux, $K_0 = -3\ln(T+T^*)$, $W = w_0 - A\exp(-aT)$

Type IIB orientifold





S.Kachru, R.Kallosh, A.Linde and S.P.Trivedi (2003)

Mixed modulus-anomaly mediation

SUSY breaking by uplifting potential is mediated to visible fields on D3/D7 branes via modulus F-term $F^T/(T+T^*)$, which is hierarchically smaller than $m_{3/2}$ ($\approx m_{3/2}/4\pi^2$) $\rightarrow$ anomaly mediation is same order!

K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski and S. Pokorski (2004)
K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski (2005)

Relative significance α is calculable and controlled by the power of modulus in the uplifting potential [$\overline{D3}$ uplifting (KKLT) predicts $\alpha \approx 1 (n_T = 2)$].

$$\alpha \equiv \frac{m_{3/2}}{\langle aT \rangle} \frac{1}{M_0} \approx \frac{2}{n_T} \quad \left(\langle aT \rangle \approx \ln\left(\frac{M_P}{m_{3/2}}\right) \approx 4\pi^2 \right), \qquad M_0 \equiv \frac{F^T}{T+T^*}$$

Visible fields on D3/D7 brane ($W = \lambda_{ijk} Q_i Q_j Q_k$),

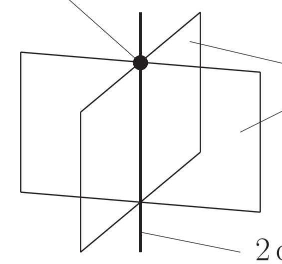
$$\mathcal{L}_{soft} = -\frac{1}{2} M_a \lambda^a \lambda^a - m_i^2 |\tilde{Q}_i|^2 - \left(\frac{1}{6} A_{ijk} y_{ijk} \tilde{Q}_i \tilde{Q}_j \tilde{Q}_k + \text{h.c.} \right)$$

Moduli mediation:

Gauge k-fn. & Kähler on D3/D7:

$$f_a = T^{l_a},$$
$$\mathcal{K}_{\rm eff} = K_0 + Z_i Q_i^* Q_i,$$
$$Z_i = 1/(T+T^*)^{n_i}$$

n_i: Modular weight analogue



$$M_a = F^T \partial_T \ln(Re(f_a)) = l_a M_0, \qquad M_0 \equiv F^T/(T+T^*)$$
$$A_{ijk} = -F^T \partial_T \ln\left(\frac{\lambda_{ijk}}{e^{-K_0} Z_i Z_j Z_k}\right) = (3 - n_i - n_j - n_k) M_0,$$
$$m_i^2 = \frac{2}{3} V_0 - F^T F^{T*} \partial_T \partial_T^* \ln\left(e^{-K_0/3} Z_i\right) = (1 - n_i)|M_0|^2.$$

Anomaly-Mediation: Randall and Sundrum (1998), G.F.Giudice, M.A.Luty, H. Murayama and R.Rattazzi (1998)

$$
\begin{aligned}
M_a &= \frac{\beta_a}{g_a} m_{3/2} \\
A_{ijk} &= -\frac{1}{16\pi^2}\left(\gamma_i + \gamma_j + \gamma_k\right) m_{3/2} \\
m^2 &= -\frac{1}{32\pi^2}\frac{d\gamma_i}{d\ln\mu} m_{3/2} \\
&\quad + \frac{1}{8\pi^2}\left\{T\left(\frac{\partial\gamma_i}{\partial T} M_0 m_{3/2} + \mathrm{H.c.}\right)\right\}
\end{aligned}
$$

where $\frac{\gamma_i}{8\pi^2} = \frac{d\ln Z_i}{d\ln\mu}$.

β_a, $\gamma_i/(8\pi^2) \rightarrow$ 1-loop suppressed, but always exists if $m_{3/2} \neq 0$
Interference term in m_i^2 via modulus dependence of γ_i.

K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski and S. Pokorski (2004)

IV. Mirage Unification and Little SUSY Hierarchy at TeV

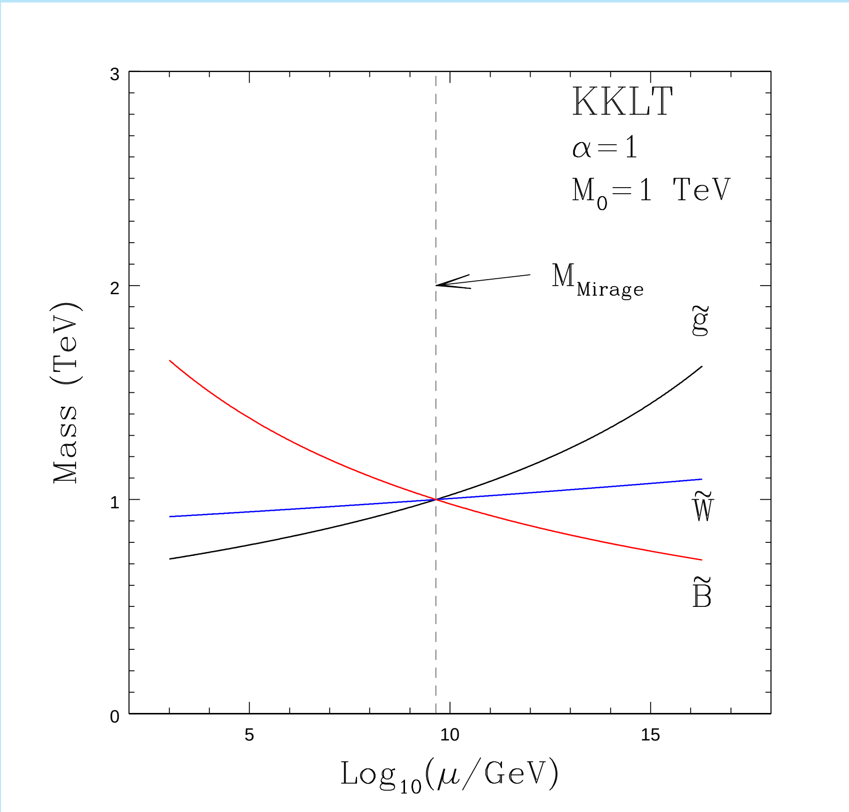
Correlation of R.G. running of modulus mediation with anomaly mediation.

$$\text{Modulus}: \; M_a(\mu) = \frac{g_a^2(\mu)}{g_a^2(\Lambda)} M_0 = M_0 - \frac{\beta_a}{g_a} \ln\left(\frac{\Lambda}{\mu}\right)^2 M_0$$

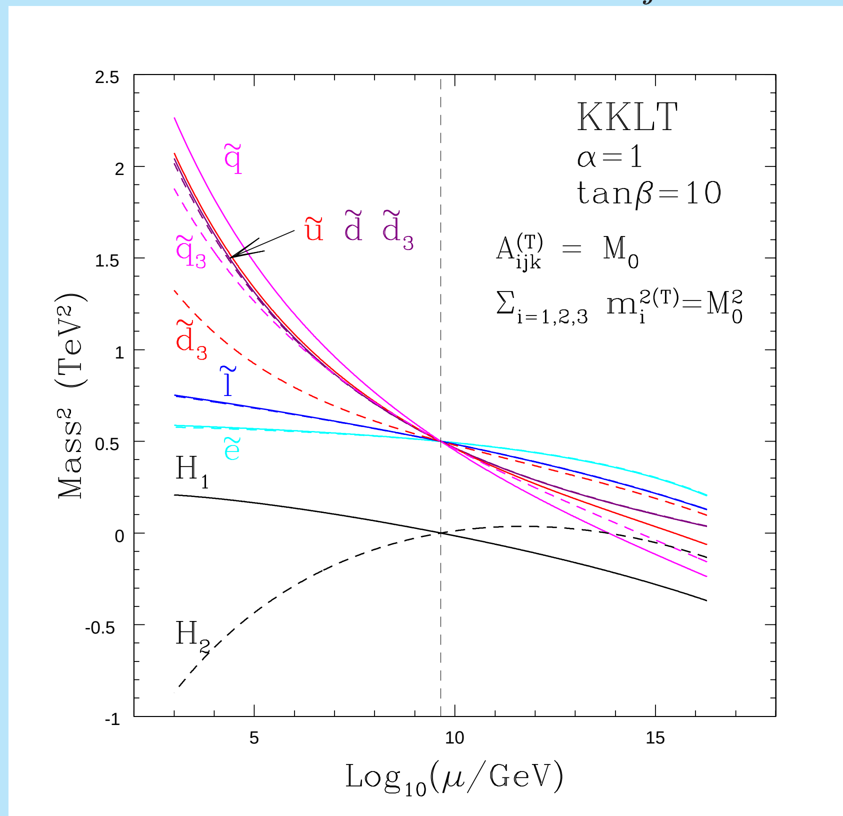
$$\text{Anomaly}: \; M_a(\mu) = \frac{\beta_a}{g_a} m_{3/2}$$

They cancel at $\mu = \Lambda \exp\left(-\frac{m_{3/2}}{2M_0}\right) = M_{Mirage} \approx \Lambda \left(\frac{m_{3/2}}{\Lambda}\right)^{\alpha/2}$.

$\overline{D3}$ uplifting (KKLT) predicts $M_{Mirage} \approx \sqrt{\Lambda m_{3/2}}$.



Yukawa coupling only for $n_i + n_j + n_k = 2$



Anomaly mediation effectively shifts the messenger scale.
(Mirage messenger scale : $M_{\rm Mirage}$) K. Choi, K-S. Jeong, KO. (2005)

Short proof (Similar for tri-linear coupling):

$$\frac{\partial}{\partial \ln\left(\frac{\mu}{\Lambda}\right)} \ln Z_i = \frac{1}{8\pi^2}\gamma_i = \frac{1}{8\pi^2}\left(2g_A^2 C_A(Q_i) - \frac{1}{2}\sum_{jk}\frac{|\lambda_{ijk}|^2}{e^{-K_0}Z_iZ_jZ_k}\right)$$

At $\mu = \Lambda$, $\quad \rightarrow g_A^2 = Re(T)^{-1}, \quad e^{-K_0}Z_iZ_jZ_k = (T+T^*)^{3-n_i-n_j-n_k}$

$$\gamma_i(Re(T)) = Re(T)^{-1}\gamma_i(1)$$

if $n_i + n_j + n_k = 2$

Radiative T dependence in $\ln Z_i$ is solved at all μ, as the compensator C,

$$\left(\frac{\mu}{\Lambda}\right) \rightarrow \left(\frac{\mu}{\Lambda\sqrt{CC^*}}\right)^{1/g_G^2 Re(T)}$$

F,D components vanish at M_{Mirage} (The mediation scale shifted from Λ).

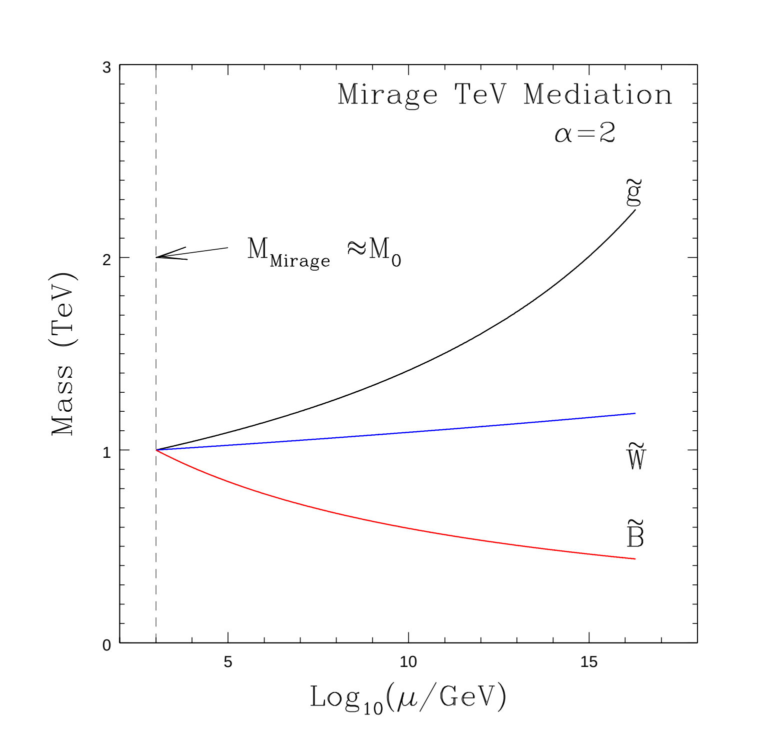
Although we do not know how to realize this in string inspired scenario, if we can obtain a sequestered uplifting with $n_T = 1$,

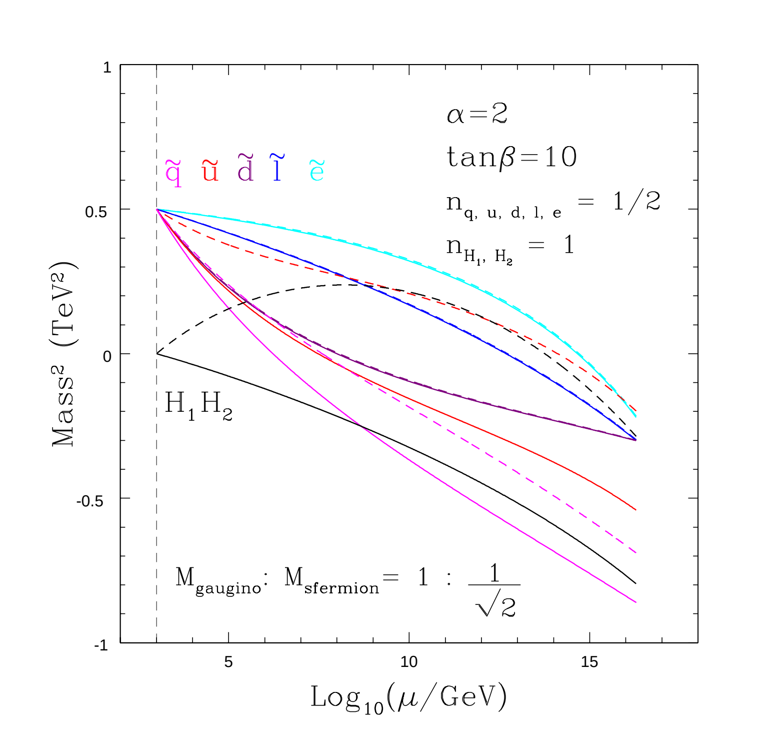
$$V_{\text{lift}} = \underbrace{\frac{D}{(T+T^*)^2}}_{\text{KKLT}} \to \frac{D}{(T+T^*)}$$

We can obtain mirage unification at TeV scale.

$$M_{\text{Mirage}} = \Lambda \left(\frac{M_0}{\Lambda} \right)^{\alpha/2} \approx M_0$$

We can realize the little hierarchy $m_{H_2} << m_{\tilde{t}}$ at $\approx M_0$.





Large logarithmic corrections are cancelled by anomaly mediation.

Bonus: Anomaly mediation often encounters a trouble in generating $\mathcal{L}_{\rm soft} = BH_1H_2$ but mirage unification at TeV provides a special exception.

$$\frac{1}{\Lambda^2}H_1H_2\langle W^AW^A\rangle \to \Delta\mathcal{W} = \tilde{A}\exp(-aT)H_1H_2$$

$$\mu = \frac{\tilde{A}e^{-aT}}{(T+T^*)^{(3-n_{H_1}-n_{H_2})/2}}$$

$$B = -m_{3/2}\left(1-\frac{2}{\alpha}\right) + (2-n_{H_1}-n_{H_2})M_0$$

$$\underbrace{+\frac{1}{16\pi^2}\left(\gamma_{H_1}+\gamma_{H_2}\right)m_{3/2}}_{\text{Cancel at } M_{\rm Mirage}}$$

Exact result $\alpha(n_T = 1)$ shows $B(M_{\rm Mirage}) = -(n_{H_1}+n_{H_2})M_0$.
$\leftarrow$ Similar order correction ΔB not under control ($\Delta\mathcal{K}$, $\Delta V_{\rm lift}$).

Requirement for natural electroweak symmetry breaking:

$$n_{q_3} + n_{u_3} + n_{H_2} = 2 \to \widetilde{m}_{q_3}^2 + \widetilde{m}_{u_3}^2 = M_0^2,\ \widetilde{m}_{H_2}^2 = 0$$

Two possibilities for m_{H_1} in the little hierarchy,

$$\begin{aligned} \text{Model (I)} &: \quad m_{H_1} \sim m_{H_2} \sim \mu \quad << m_{Q,U,D,L,E} \\ \text{Model (II)} &: \quad m_{H_2} \sim \mu << \quad m_{H_1} \sim m_{Q,U,D,L,E} \end{aligned}$$

$$B\mu = \frac{\tan\beta}{1+\tan^2\beta}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2) \to \tan\beta \sim \frac{m_{H_1}^2}{B\mu}$$

$$\text{Model (I)} : \tan\beta \sim M_0/(\sqrt{8\pi^2}B) \to B \lesssim M_0/\sqrt{8\pi^2}$$

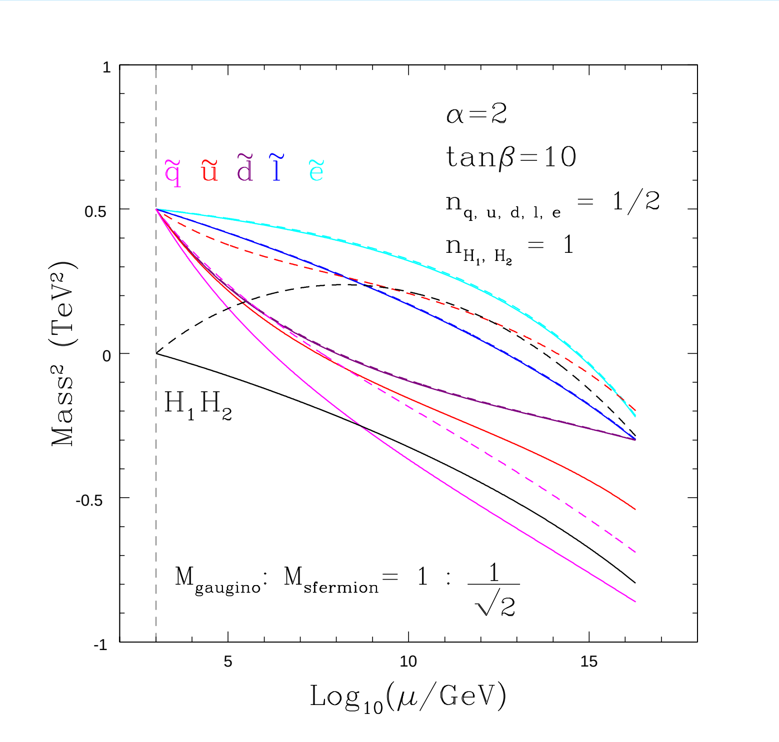
$$\text{Model (II)} : \tan\beta \sim \sqrt{8\pi^2}.$$

In case of Model (II) ($n_{H_1} \neq n_{H_2}$), we have tadpole contribution in RG running,

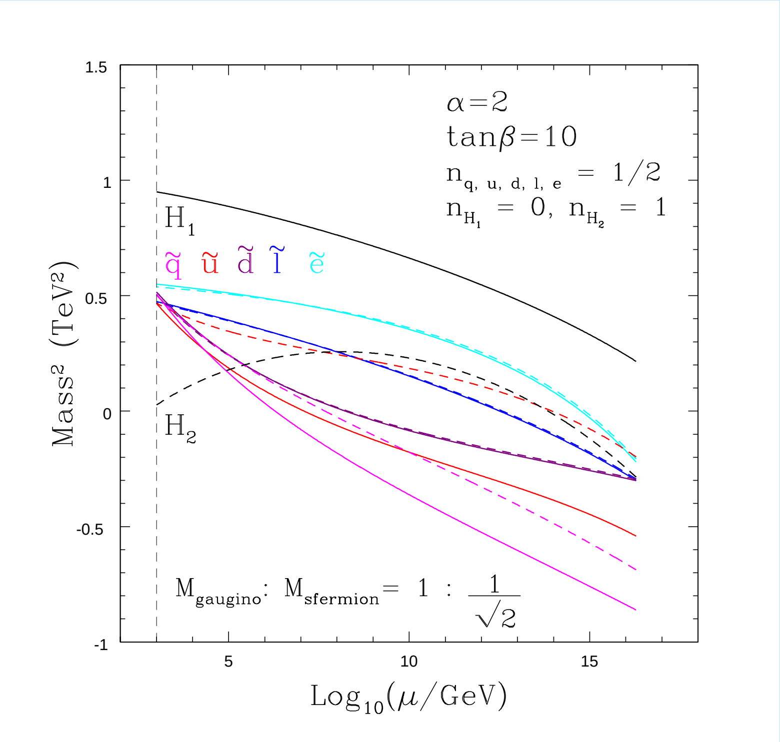
$$\delta m_{H_2}^2 = -\frac{1}{8\pi^2} Y_{H_2} \left(\sum_j Y_j (1 - n_j) \right) M_0^2 g_Y^2(\mu) \ln \left(\frac{\Lambda}{\mu} \right)$$

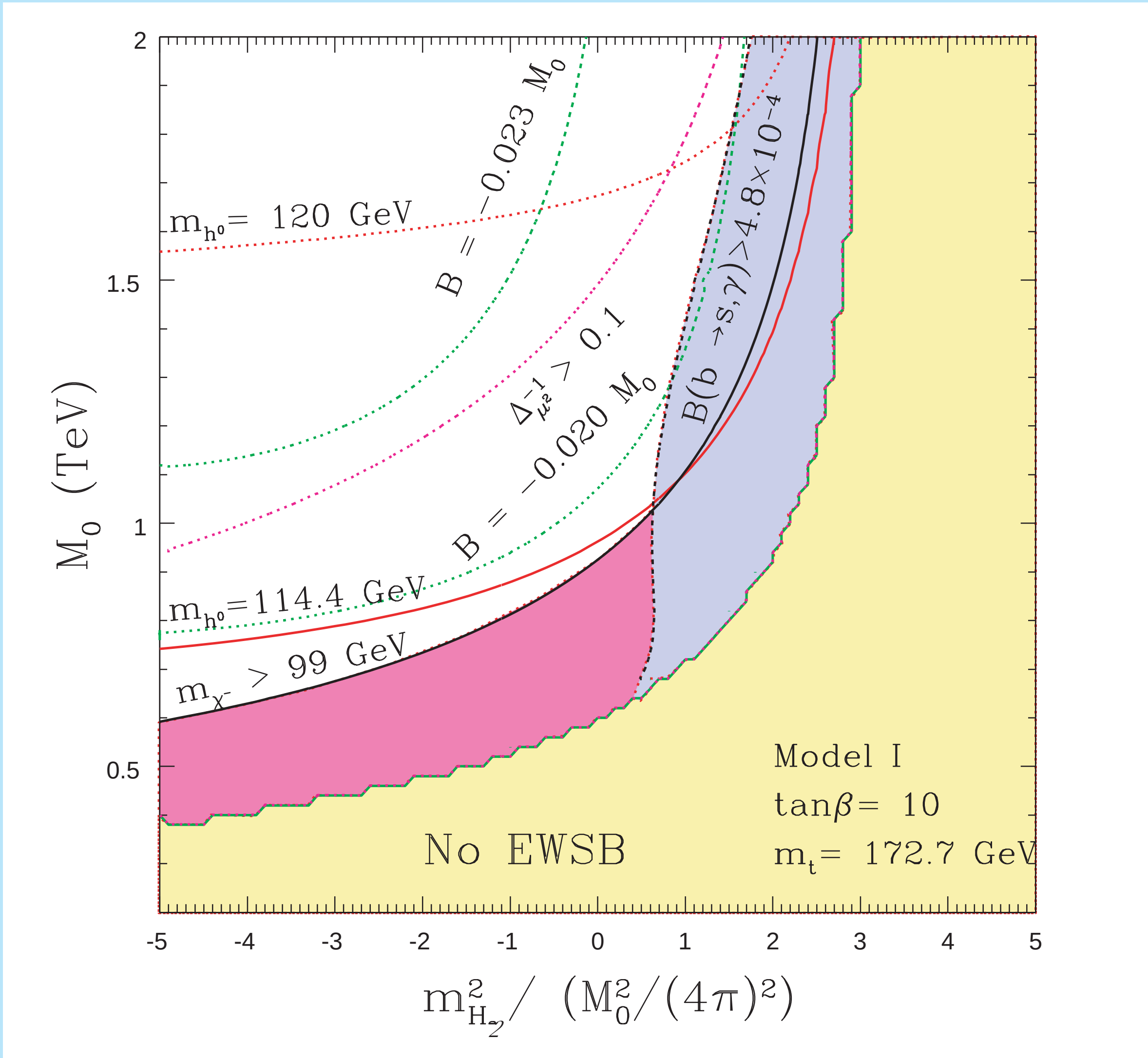
$\delta m_{H_2}^2 > 0$ for universal matter modular weights, $n_q = n_u = n_d = n_l = n_e$, however size is small enough ($\sim M_0^2/(4\pi)^2$) to be driven to negative by other corrections.

Model I

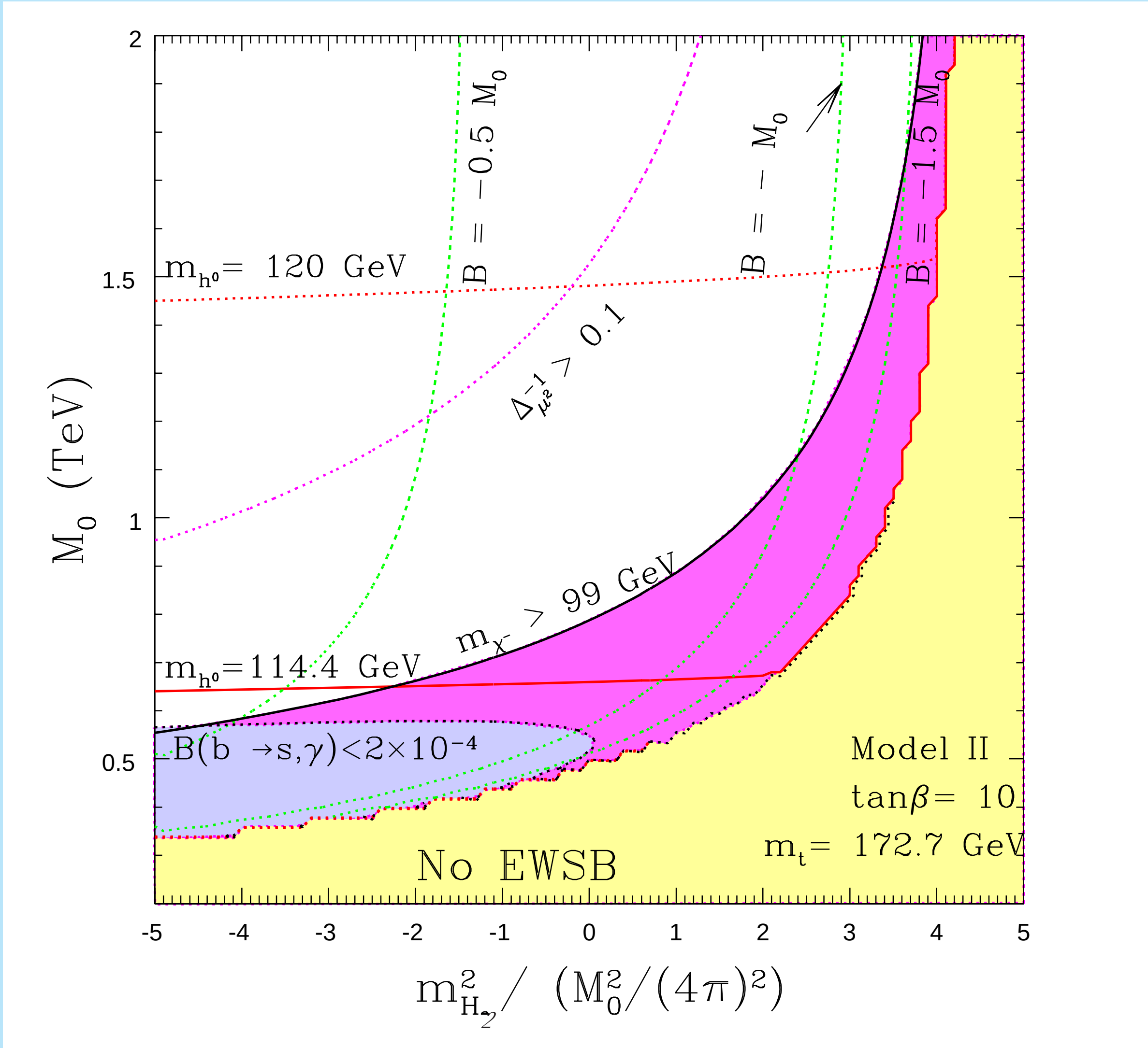


Model II

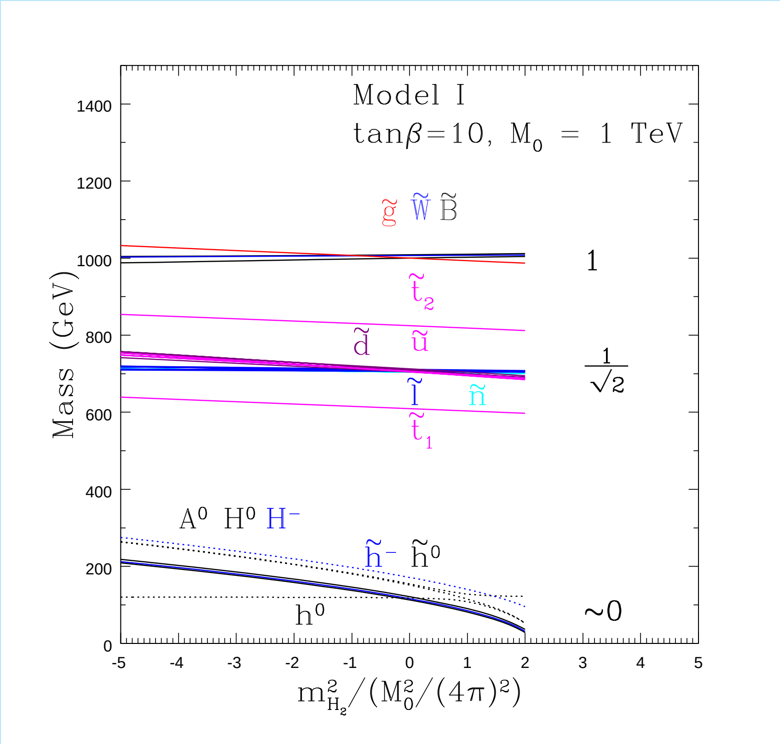


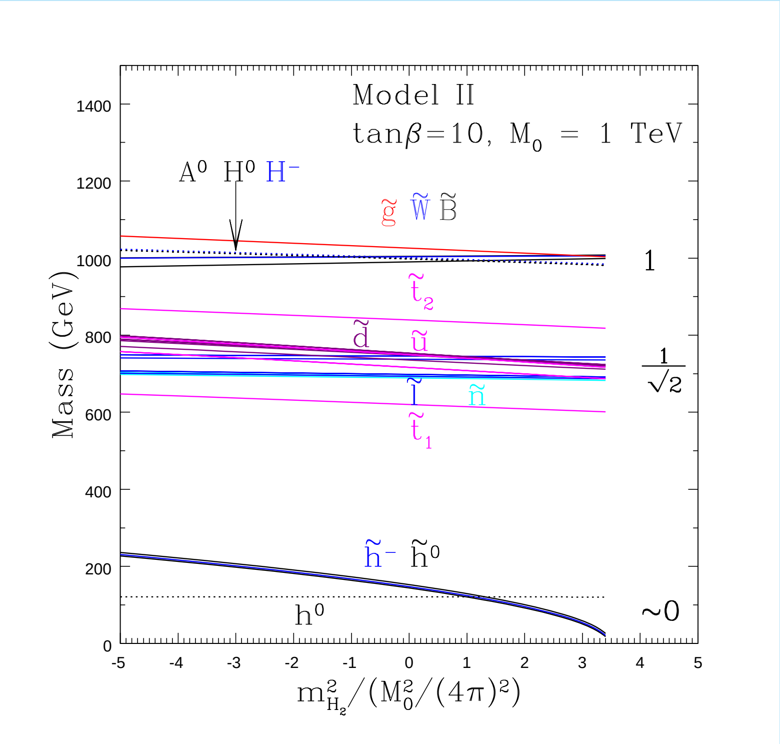


m_{h^0}= 120 GeV
B = −0.023 M_0
$\Delta^{-1}_{\mu^2}$ > 0.1
B = −0.020 M_0
B(b →s,γ)>4.8×10^{-4}
m_{h^0}=114.4 GeV
m_{χ^-} > 99 GeV
No EWSB
Model I
tanβ= 10
m_t= 172.7 GeV
M_0 (TeV)
$m^2_{H_2}$/ ($M_0^2/(4\pi)^2$)



m_{h^0}= 120 GeV
B = −0.5 M_0
B = − M_0
B = −1.5 M_0
$\Delta^{-1}_{\mu^2} > 0.1$
$m_{\chi^-} > 99$ GeV
m_{h^0}=114.4 GeV
B(b →s,γ)<2×10⁻⁴
Model II
tanβ= 10
m_t= 172.7 GeV
No EWSB
M_0 (TeV)
$m^2_{H_2}/\ (M_0^2/(4\pi)^2)$


Model I
tanβ=10, M0 = 1 TeV
g̃ W̃ B̃
1
t̃2
d̃ ũ
1/√2
l̃ ñ
t̃1
A0 H0 H−
h̃− h̃0
h0
~0
Mass (GeV)
m²H2/(M²0/(4π)²)


Model II
tanβ=10, M0 = 1 TeV
A0 H0 H−
g̃ W̃ B̃
1
t̃2
d̃ ũ
1/√2
l̃ ñ
t̃1
h̃− h̃0
h0
~0
Mass (GeV)
m²H2/(M²0/(4π)²)

VI. Conclusion

- Raising lower bound for m_{h^0} favors heavy $\tilde{t}$ and m_{H_2} in general which leads to fine-tuning in the electroweak symmetry breaking of the MSSM.

- We proposed a new scenario where the little hierarchy between Higgs and SUSY particles is realized by mirage unification in the mixed modulus-anomaly mediation without any modification of the MSSM.

- Tuning parameter $\Delta_{\mu^2}^{-1}$ can be naturally above 10% and m_{h^0} can comfortably accomodate the SM bound.

- The scenario favors light SUSY particles $\lesssim 1$ TeV and predicts distinctive relation among the gaugino and sfermion masses.

- Higgsinos are predicted around 100 $\sim$ 200 GeV and LSP is pure neutral higgsino.

Example in CMSSM