ν tri-bi-maximal mixing from a non-Abelian discrete family symmetry

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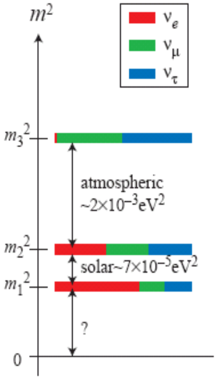
Outline

Tri-bi-maximal hypothesis

Harrison-Perkins-Scott

$$V_{PMNS} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



Objectives

- Straightforward to embed into GUT / Strings
- Obtain observed charged fermion (quark and lepton) data
- Obtain near tri-bi-maximal ν / lepton mixing

Why is $Z_3 \ltimes Z_3' \equiv \Delta(27)$ interesting?

- Non-Abelian, has "triplets" and "anti-triplets"
- Small discrete subgroup of $SU(3)_f$ (hep-ph/0507176)

$Z_3 \ltimes Z_3'$ invariants

Transformation properties

Field	Z_3	Z_3'
A_1	A_1	A_3
A_2	αA_2	A_1
A_3	$(\alpha)^2 A_3$	A_2

$$\alpha^3 \equiv 1$$

- Allowed: $SU(3)_f$ invariants
- Not allowed by Z_3: $A_1 B_1 + A_2 B_2 + A_3 B_3$

Sequential domination

Seesaw formula

$$m_\nu = \left(M^D_\nu\right) \left(M_{N_R}\right)^{-1} \left(M^D_\nu\right)^T$$

(Light) Sequential Domination

$$M_{N_R} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

- With $M_1 < M_2 \ll M_3$

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Dirac structure

M_1 then M_2... Then M_3

$$m_\nu = \begin{pmatrix} b_1 & c_1 & . \\ b_2 & c_2 & . \\ b_3 & c_3 & . \end{pmatrix} \begin{pmatrix} M_1^{-1} & 0 & 0 \\ 0 & M_2^{-1} & 0 \\ 0 & 0 & M_3^{-1} \end{pmatrix} \begin{pmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ . & . & . \end{pmatrix}$$

Wanted ν Dirac matrix

$$M_\nu^D = \begin{pmatrix} b_1 & c_1 & . \\ b_2 & c_2 & . \\ b_3 & c_3 & . \end{pmatrix} \propto \begin{pmatrix} 0 & x & . \\ y & x & . \\ -y & x & . \end{pmatrix}$$

Effective superpotential terms for Dirac masses

Yukawa leading order terms

- Charged fermion Dirac masses are similar to m_ν^D

$$P_Y \sim \frac{1}{M^2}(\phi_{23}^i \nu_i)(\phi_{123}^j \nu_j^c)H + \frac{1}{M^2}(\phi_{123}^i \nu_i)(\phi_{23}^j \nu_j^c)H + \frac{1}{M^2}(\phi_3^i \nu_i)(\phi_3^j \nu_j^c)H$$

Desired vevs

$$\langle \phi_3 \rangle \propto (0,0,1)$$

$$\langle \phi_{23} \rangle \propto (0,1,-1)$$

$$\langle \phi_{123} \rangle \propto (1,1,1)$$

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Table of fields

Field	$U(1)$	Z_2
ν	0	1
ν^c	0	1
θ	0	-1
H	0	1
H_{45}	2	1
ϕ_3	0	1
ϕ_{23}	-1	1
ϕ_{123}	1	1
$\bar{\phi}_3$	-1	-1
$\bar{\phi}'_3$	3	-1

Effective superpotential terms for Dirac masses

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Effective superpotential terms for Majorana masses

Majorana leading order terms

$$P_M \sim \frac{1}{M^5}(\phi_{123}\nu^c)(\phi_{123}\nu^c)(\theta\bar{\phi}_3)(\theta\bar{\phi}_3)$$

$$+\frac{1}{M^5}(\phi_{23}\nu^c)(\phi_{23}\nu^c)(\theta\bar{\phi}_3)(\theta\bar{\phi}'_3)$$

$$+\frac{1}{M}(\theta\nu^c)(\theta\nu^c)$$

More vevs

$$\langle\theta\rangle \propto (0,0,1)$$

$$\langle\bar{\phi}_3^T\rangle \propto (0,0,1)$$

$$\langle\bar{\phi}_3'^T\rangle \propto (0,0,1)$$

Alignment by soft terms

Quartic terms and minimization conditions

- Symmetry is discrete, breaks the continuum of vacuum states
- $V \sim -m^2(\varphi^i \varphi_i^*)$
  $\pm m_{3/2}^2 (\varphi^i \varphi_i^* \varphi^i \varphi_i^*)$
- For $\varphi = \phi_{123}$, positive coefficient yelds $\langle \phi_{123} \rangle \propto (1, 1, 1)$
- For $\varphi = \phi_3$, negative coefficient yelds $\langle \phi_3 \rangle \propto (0, 0, 1)$

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Relative alignment

Aligning ϕ_{23}

- $m^2_{3/2}(\phi_{23}\phi^*_{123})(\phi_{123}\phi^*_{23})$ with positive coupling
- $m^2_{3/2}(\phi_{23}\phi^*_{3})(\phi_{3}\phi^*_{23})$ with negative coupling
- Meaning $\langle\phi_{23}\rangle \propto (0, 1, -1)$

The predictions

Mixing angles values predicted

- $s_{12}^2 \approx \frac{1}{3}{}^{+0.052}_{-0.048}$
- $s_{23}^2 \approx \frac{1}{2}{}^{+0.061}_{-0.058}$
- $s_{13}^2 \approx 0.0028$

Mixing angles values measured experimentally

- $s_{12}^2 = 0.30 \pm 0.08$
- $s_{23}^2 = 0.50 \pm 0.18$
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Summary

- The model is viable and unifiable.
- Seesaw mechanism and Misalignment of vevs play key roles.
- Tri-bi-maximal mixing directly related to Z_3s of discrete group.

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