Extracting Flavor from Quiver Gauge Theories

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Based on collaboration with Yaron Antebi and Yossi Nir, [Phys. Rev. D73, 075009].
Motivation

- The current information on the (dimensionless parameters of the) neutrino sector:

\[
\begin{align*}
\tan^2 \theta_{23} &\sim 0.53-1.89 \\
\tan^2 \theta_{12} &\sim 0.31-0.61 \\
|U_{e3}| &\leq 0.17 \\
\sqrt{\Delta m^2_{21}/|\Delta m^2_{32}|} &\sim 0.17-0.20
\end{align*}
\]

- In contrast to the quarks and charged leptons, the parameters above are not particularly small nor hierarchical.

- Thus it could be that the neutrino sector is anarchical. [Hall, Murayama, Weiner (2000)]

- This can be tested by measuring the mass scales, \(m_i\) and the mixing angle \(\theta_{13}\).
So why are neutrinos so special? Why are they flavor blind?

- It is possible to generate such anarchy through the FN mechanism (approximate symmetry).

- But the FN mechanism has limitations: there are many other FN models which do not predict anarchy.

Can we motivate one model over others? Is there a framework which predicts anarchy in the neutrino sector?

- In this talk we'll consider a string motivated class of models: quiver gauge theories.

- Certain class of (heterotic) string motivated FN models were investigated in the past and found to be more constraining.

[Refs: Ibanez, Ross (1994)]
[Binetruy, Ramond (1995)]
[Elwood, Irges, Ramond (1997),(1998)]
[Binetruy, Lavignac, Ramond (1996)]
[Dreiner, Murayama, Thormeier (2005)]
To Make a Long Story Short:

The FN mechanism within quiver gauge theories is more predictive.

Under mild assumptions, anarchy is predicted.

A bottom-up approach in which phenomenology constrains a certain class of string theories.
**The FN Mechanism**

- Provides an explanation to the flavor puzzle. [Froggatt, Nielsen (1979)] [Leurer, Nir, Seiberg (1993)]

- $U(1)_{FN}$ horizontal symmetry which is spontaneously broken by a VEV of a scalar field $S$, $<S>/M_V = \varepsilon \ll 1$.

- $M_V$ is the scale at which the breaking is communicated to the visible sector through massive fields in vector-like representations of the gauge group.

- The charges of the various fields dictate the parametric suppression in $\varepsilon$. 

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For the FN mechanism, one chooses the small parameter, \( \varepsilon \), and a set of charges. These dictate the parametric suppression of the various operators.

Thus the mechanism is limited:
- The value of the small parameter is arbitrary.
- The charges of the various fields are not predicted.
- No information on the \( O(1) \) parameters is given.

To demonstrate, consider for example the SU(5) models:

\[
\begin{align*}
\mathbb{P} (10_i) &= (2,1,0) \\
\mathbb{P} (\overline{5}_i) &= (0,0,0) \\
M_u &= \langle H_u \rangle \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} \\
M_d &= \langle H_d \rangle \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ 1 & 1 & 1 \end{pmatrix} \\
M_v &= \frac{\langle H_u \rangle^2}{M} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \\
\varepsilon &= 0.05 \\
\end{align*}
\]

\[
\begin{align*}
\mathbb{P} (H_u) &= 0 \\
\mathbb{P} (H_d) &= 0 \\
\mathbb{P} (S) &= -1 \\
\text{Different Neutrino Structure} \\
\end{align*}
\]

\[
\begin{align*}
\mathbb{P} (10_i) &= (4,2,0) \\
\mathbb{P} (\overline{5}_i) &= (1,1,0) \\
M_u &= \langle H_u \rangle \begin{pmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1 \end{pmatrix} \\
M_d &= \langle H_d \rangle \begin{pmatrix} \varepsilon^5 & \varepsilon^5 & \varepsilon^4 \\ \varepsilon^3 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon & \varepsilon & 1 \end{pmatrix} \\
M_v &= \frac{\langle H_u \rangle^2}{M} \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon^2 & \varepsilon \\ \varepsilon & \varepsilon & 1 \end{pmatrix} \\
\varepsilon &= 0.23 \\
\end{align*}
\]
Quiver Gauge Theories

Quiver gauge theories arise as low energy effective theories on D-branes placed on singular manifolds. An extended quiver diagram is a graph which efficiently describes the gauge theory for unoriented strings:

\[ N \quad \bullet = U(N), \text{SO}(N) \text{ or } \text{SP}(N) \]

\[ N \rightarrow M \]

\[ (N, \overline{M}) \]

\[ \text{Adj} \]

\[ (\overline{N}, \overline{M}) \]

\[ (N, M) \]

= Symmetric or Antisymmetric of \( N \times N \)
Consider as an example, $U(5) \times U(1)$ theory:

For each $U(N) = SU(N) \times U(1)$ factor, the charge under the $U(1)$ part of the gauge group is dictated by the representation of the non-abelian part.
The U(1)s pose problems to model building: For example in the $U(5) = SU(5) \times U(1)$

$$\Theta = H_d \cdot 10 \cdot \overline{5} + H_u \cdot 10 \cdot 10 + \frac{H_u \cdot H_u \cdot \overline{5} \cdot \overline{5}}{M}$$

the up-type Yukawa couplings cannot be turned on.

Such problems occur already in the SM and in GUT models which embed it.

There are three possible solutions:

1. Extend the particle content.
2. Spontaneously break the $U(1) \rightarrow Z_N$ by a composite field.
3. The anomalous $U(1)$ is broken by non-perturbative effects.

[Ibanez, Marchesano, Rabdan (2001)]
**Embedding the FN Mechanism**

- Within the quiver framework, the theory is more predictive.
- Horizontal symmetry through the U(1)s.
- Relation between Abelian and non-Abelian charges strongly restricts possible FN charges.
- In particular under any U(1), fields can be charged at most \( \pm 2 \).

\[ S_{\pm 2} \]

- The largest possible suppression for a \( U(1)_{\text{FN}} \) is therefore \( \epsilon^3 \).
SU(5) and Neutrino Anarchy

- It is not trivial to generate hierarchy in the up sector:
  All 10-plets have the same FN charge \( \Rightarrow \) No hierarchy!

- We must therefore consider extended gauge groups.

- This is generic in this framework: \( S \) breaks some of the non-abelian symmetry and therefore, \textit{FN requires an extended gauge group.}

- We therefore assume the simplest SU(5)\( \times \)SU(5) with a single FN field.
Motivation

The FN Mechanism

Quiver Gauge Theories

SU(5) and Anarchy

Summary

- S is a bifundamental, (5,5). After obtaining a VEV, S breaks the SU(5)×SU(5) → SU(5)_{diag}.

- S is charged (1,-1) under U(1)×U(1). Therefore U(1)_{FN} = U(1)_L - U(1)_R

- The 10-plets can have one of three possible charges: (2,0), (1,1), (0,2) under the U(1)×U(1). To explain the hierarchy in the up-sector each generation must have a different representation.

- The 5-plets can have one of two charges: (-1,0), (0,-1) under U(1)×U(1). Thus at least two generations have similar charges and therefore:
  - The neutrino sector must admit (at least) quasi-anarchy.
  - ms/m_b ≈ |V_{cb}|

- To have the correct hierarchy in the down sector one finds that all 5-plets must have the same charge!

In this framework anarchy in the neutrino sector is predicted.
A Simple SU(5) Quiver

- The above quiver correctly reproduce hierarchy in the quarks and charged leptons and predicts anarchy for the neutrinos.

- It is unique.
Summary

1) The FN mechanism within quiver gauge theories are much more restrictive:
   1) There is a relation between Abelian and non-Abelian charges.
   2) The FN framework requires product groups.

2) In the SU(5) scenario, anarchy in the neutrino sector is predicted.

3) The predictive power allows to experimentally constrain classes of string models.

4) A generic problem in quiver models: the U(1) factors do not allow all couplings.
   Solution: non-perturbative effects or extended particle content.
The above generates the correct hierarchy in quarks and charged leptons and predicts anarchy for the neutrinos.

- It is anomaly free.

- \( K \) break the \( U(1)_{L+R} \) spontaneously.