A Peculiar Dynamically Warped Theory Space

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- C. D. Carone, J. Erlich and B. Glover, "Dynamically warped theory space and collective supersymmetry breaking," JHEP **0510**, 042 (2005) [arXiv:hep-ph/0509002].

Deconstruction:

4D theory that appears 5D in the IR

Initial Motivation

Can a 4D moose model dynamically generate a profile of link vevs that approximates deconstructed AdS space?

Phenomenologically relevance

Example: bulk fields whose first few modes fake the spectrum of RS models at the LHC.

Given a linear U(1)^n moose, link charges $(q, -q)$, the effect of the metric

$$ds^2 = e^{-f(y)} dx^2 + dy^2$$

is encoded in the vev profile

$$v_i = \frac{e^{-f_i/2}}{qa} \qquad f_j = 2kaj$$

where $R = na$ and k is the AdS curvature.

Models of Interest

$$V = V_b(\phi_1) + \sum_{i=1}^{n-1} V_i(\phi_i, \phi_{i+1}),$$

where V_b represents special boundary effects.

V_b's job is to trigger a profile.

Example: $\mathrm{U}(1)^n$ gauge theory, gauge couplings $g_i = g$ and $n-1$ link fields $\phi_i \sim (+1,-1)$. Take

$$V_b = (a_1 - m^2)^2 \ ,$$

$$V_i = (\lambda\, a_i - a_{i+1})^2 \ .$$

where $a_i = \phi_i^\dagger \phi_i$, for $i = 1 \ldots n-1$, and $a_n = 0$.

Minimization conditions:

$$(1+\lambda^2)\, a_\ell - \lambda\,(a_{\ell+1} + a_{\ell-1}) = 0 \quad \text{(bulk)}$$

$$(1+\lambda^2)\, a_1 - m^2 - \lambda\, a_2 = 0 \quad \text{(boundary)}$$

$$-\lambda\, a_{n-2} + (1+\lambda^2)\, a_{n-1} = 0 \quad \text{(boundary)}$$

These have the general solution

$$a_j = \frac{m^2}{\lambda}\left(\frac{1}{1-\lambda^{2n}}\,\lambda^j + \frac{1}{1-\lambda^{-2n}}\,\lambda^{-j}\right) \ ,$$

or for large n and $\lambda < 1$

$$a_j \approx m^2\, \lambda^{j-1}$$

One can check that this is a minimum of the potential.

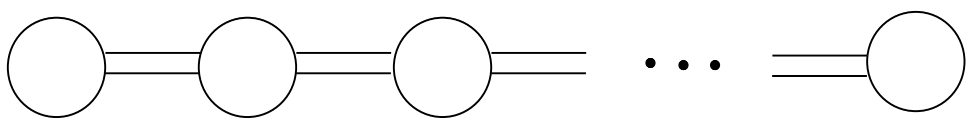
SUSY models are more interesting.

A Deconstructed SUSY U(1) Theory

- 4D $\mathcal{N}$=1 SUSY U(1)^n moose theory
- chiral multiplets ϕ_i with charges

$$(q_i, q_{i+1}) = (+1, -1)$$

- conjugate superfields $\overline{\phi}_i$ to cancel anomalies.



Scalar Potential:

$$V_D = \sum_{i=1}^{n} D_i^2,$$

$$D_i = g\left(|\phi_i|^2 - |\phi_{i-1}|^2 - |\overline{\phi}_i|^2 + |\overline{\phi}_{i-1}|^2 + \xi_i\right).$$

where we define $\phi_0 = \phi_n = 0$.

Stationary points:

$$\langle\phi_i\rangle\left(\langle D_i\rangle - \left\langle D_{i+1}\right\rangle\right) = 0 \ .$$

Vacua generically have equal D-terms,

$$\langle D_i\rangle = \frac{\sum_j g\xi_j}{n} \equiv D$$

Scalar VEVs v_i and $\overline{v}_i$ satisfy

$$(|v_{i+2}|^2 - |\overline{v}_{i+2}|^2) - 2(|v_{i+1}|^2 - |\overline{v}_{i+1}|^2) +$$

$$(|v_i|^2 - |\overline{v}_i|^2) = (\xi_{i+1} - \xi_{i+2})$$

This is a discretized form of

$$\frac{\partial^2 |\phi(y)|^2}{\partial y^2} = -\frac{\xi'(y)}{a},$$

where a is a lattice spacing (to be defined).

Integrate

$$\frac{\partial |\phi(y)|^2}{\partial y} = \frac{-\xi(y) + D/g}{a}$$

$$D/g = \int_0^R dy\, \xi(y)/R$$

This can be related to the warp factor $f(y)$:

$$\frac{\partial e^{-f(y)}}{\partial y} = (-g^2 \xi(y) + gD)a$$

Examples:

- $\xi(y) = D/g = const$ yields flat space
- $\xi(y) = const[1 - R\,\delta(y)]$ yields linear profile

Let's make a desired warp factor by setting

$$\xi(y) = \tilde{\xi}(y) + D/g = \tilde{\xi}(y) + \int_0^R dy\, \xi(y)/R$$

where $\tilde{\xi}(y)$ determines the profile.

A solution exists if

$$\int_0^R dy\, \tilde{\xi}(y) = 0$$

Desired bulk warp factor requires opposite sign contribution to $\xi(y)$ at a boundary.

The KK Spectrum

Masses of vector and chiral multiplets originate from

$$\mathcal{L} \supset \int d^4\theta \sum_i \Phi_i^\dagger \exp\left[g(V_i - V_{i+1})\right] \Phi_i + \overline{\Phi}_i^\dagger \exp\left[g(-V_i + V_{i+1})\right] \overline{\Phi}_i,$$

Thus, $m^2_{\text{gauge}} =$

$$2g^2 \begin{pmatrix} v_1^2 & -v_1^2 & & & \\ -v_1^2 & v_1^2 + v_2^2 & -v_2^2 & & \\ & -v_2^2 & v_2^2 + v_3^2 & -v_3^2 & \\ & & \ddots & \ddots & \\ & & & -v_{n-1}^2 & v_{n-1}^2 \end{pmatrix}$$

VEVS contribute to the fermion spectrum as well:

$$\mathcal{L} \supset i\frac{g}{\sqrt{2}}\sum_i \left[\lambda_i(v_i\psi_i - v_{i-1}\psi_{i-1})\right] + \text{h.c.}$$
$$= i\frac{g}{\sqrt{2}}(\lambda_i \mid \psi_i) \left(\begin{array}{c|c} & \Theta \\ \hline \Theta^\dagger & \end{array}\right)_{ij} \left(\frac{\lambda_j}{\psi_j}\right) + \text{h.c.}$$

where the $n \times (n-1)$ dimensional matrix Θ is,

$$\Theta = \begin{pmatrix} v_1 & & & \\ -v_1 & v_2 & & \\ & \ddots & \ddots & \\ & & -v_{n-2} & v_{n-1} \\ & & & -v_{n-1} \end{pmatrix} .$$

Hence,

$$M^2_{\text{fermions}} = 2g^2 \begin{pmatrix} \Theta\Theta^\dagger & \\ & \Theta^\dagger\Theta \end{pmatrix}$$

Interestingly,

$$2g^2\Theta\Theta^\dagger \equiv m^2_{\text{gauge}}$$

i.e., half the fermion spectrum coincides exactly with the gauge spectrum.

Scalar Potential

Let $d_i \equiv \langle D_i \rangle = v_i^2 - v_{i-1} + \xi_i$, and $\phi_i = v_i + \varphi_i$.

$$V_D = g^2 \sum_i \Big[(\varphi_{i+1} + v_{i+1})(\varphi_{i+1}^\dagger + v_{i+1}) - (\varphi_i + v_i)(\varphi_i^\dagger + v_i) - |\overline{\phi}_{i+1}|^2 + |\overline{\phi}_i|^2 + \xi_{i+1} \Big]^2$$

This contains the quadratic terms

$$2g^2 \sum_i (d_i - d_{i+1}) \left(|\varphi_i|^2 + v_i (\varphi_i^\dagger + \varphi_i) - |\overline{\phi}_i|^2 \right)$$

$$+\frac{1}{2}(\varphi_i^\dagger \mid \varphi_i)\, g^2 \left(\begin{array}{c|c} \Theta^\dagger\Theta & \Theta^\dagger\Theta \\ \hline \Theta^\dagger\Theta & \Theta^\dagger\Theta \end{array} \right) \left(\begin{array}{c} \varphi_i \\ \hline \varphi_i^\dagger \end{array} \right)$$

- Imaginary modes $(\varphi_i - \varphi_i^\dagger)/\sqrt{2}$ have vanishing masses (goldstones, U$(1)^n \rightarrow U(1)$).
- Real modes $(\varphi_i + \varphi_i^\dagger)/\sqrt{2}$ have mass matrix

$$M^2_{scalars} = 2g^2 \Theta^\dagger \Theta$$

same as the remaining massive fermion modes.

- $\bar{\phi}$ scalars (and fermions) remain massless.

In spite of the n FI terms, the spectrum is supersymmetric!

How can this be?

Consider a different theory: 4D $\mathcal{N}=1$ SUSY U(1) with no matter, plus an FI term

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\, \xi V + \int d^2\theta\, \frac{1}{32} W^\alpha W_\alpha$$

$$m_{gauge} = m_{fermion} = 0$$

Spectrum is supersymmetric (ignoring gravity), but there is a cosmological constant:

$$V_D = \xi^2$$

Our case: precisely the same!

Recall: $\sum_j \xi_j = 0$ implies vanishing D-terms.

So define:

$$\tilde{\xi}_i \equiv \xi_i - \frac{\sum_i \xi_i}{n} \ ,$$

$$\widetilde{D}_i \equiv g\left(|\phi_i|^2 - |\phi_{i-1}|^2 - |\overline{\phi}_i|^2 + |\overline{\phi}_{i-1}|^2 + \tilde{\xi}_i\right)$$

Then our potential turns out to be:

$$V_D = \sum_i \widetilde{D}_i^2 + \frac{(\sum_i \xi_i)^2}{n} \ .$$

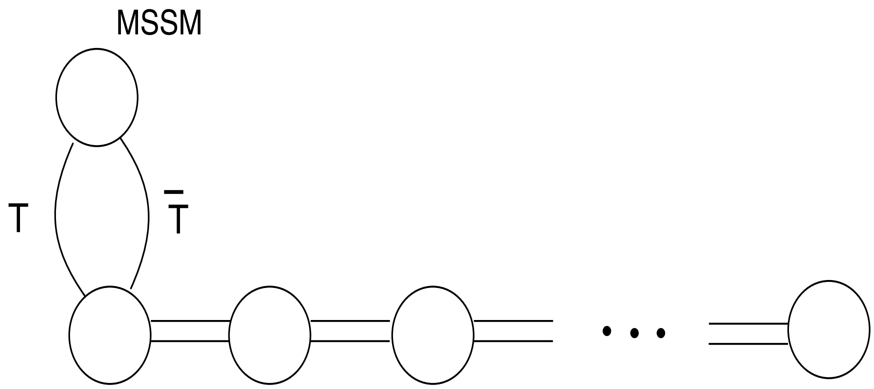
Note: the Goldstino $\lambda = \sum_i \lambda_i / \sqrt{n}$ transforms homogeneously under a shifted SUSY:

$$\widetilde{\delta}\lambda = i\theta \frac{\sum_i \widetilde{D}_i}{\sqrt{n}} + \sigma^{\mu\nu}\theta \frac{\sum_i F^i_{\mu\nu}}{\sqrt{n}}$$

$$\widetilde{\delta}\widetilde{D}_i = \delta D_i = \overline{\theta}\,\overline{\sigma}^{\mu}\partial_\mu \lambda_i - \theta\sigma^{\mu}\partial_\mu \overline{\lambda}_i$$

The Return of SUSY breaking

Everything changes if we couple the theory to the world:



Assume random FI terms of size ξ, so that $\sum_i \xi_i \sim \sqrt{n}\,\xi$, and $g^2\xi \sim M_{Pl}^2$. Then

$$M^2_{\mathrm{SUSY}} = \langle D_1 \rangle = \frac{g^2 \sum_i \xi_i}{n} \sim \frac{g^2 \xi}{n^{1/2}} \sim \frac{M_{Pl}^2}{n^{1/2}}$$

$$M^2_{\mathrm{grav}} = \left(\sum_i D_i^2 \right)^{1/2} \sim g^2 \xi \sim M_{Pl}^2$$

$$m_{3/2} = M^2_{\mathrm{grav}} / M_{Pl} \sim M_{Pl}$$

- SUSY scale is suppressed by number of lattice sites: collective SUSY breaking
- Scalar masses of T, $\overline{T}$ split by $\pm 2\,\langle D_1 \rangle$.

Conclusions

- It is possible to construct SUSY theories that dynamically generate a warp factor.

- The theory discussed here employs Fayet-Iliopoulos terms at each site, and a special FI term at a boundary.

- The theory possesses a SUSY spectrum in the absence of a coupling to the MSSM.

- Including a messenger sector, SUSY is collectively broken: the SUSY scale is suppressed by the number of lattice sites.

- The model can be distinguished from other D-term breaking scenarios: SUSY-breaking scale can be much lower than the gravitino mass.