Electroweak Constraints on Effective Theories

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Outline

- **TeV scale new physics and electroweak precision tests**
- **Effective theory analysis of electroweak data**
- **Applications to little Higgs models**
- **Conclusion**

The standard model and electroweak precision tests

- The standard model of electroweak physics is very successful experimentally.
- Electroweak precision tests (EWPTs) have been performed and have confirmed the SM.
  - Good precision: 1% level or better.
  - Experiments include
    * Atomic parity violation experiments;
    * Deep inelastic scattering: neutrino-nucleon, neutrino-electron scattering;
    * $e^+e^- \to \bar{f}f$, $e^+e^- \to W^+W^-$ scattering;
    * W boson mass;

      ...

TeV scale new physics

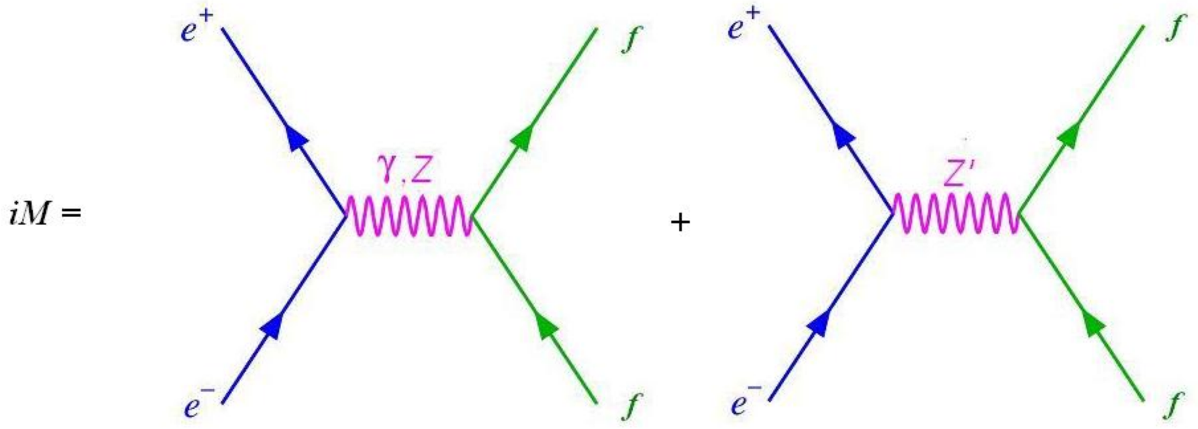
- The hierarchy problem.
- Fine-tuning to the Higgs-mass-square less than 10% $\Longrightarrow$ TeV scale extension of the SM.

SUSY, technicolor, extra-dimensions, little Higgs, ...

- Predict heavy particles $\longrightarrow$ awaiting direct probe at Tevatron, LHC, (ILC), ...
- Indirect information from EWPTs.

Electroweak constraints on new physics

- An example: Z' gauge boson.
  - Affect $e^+e^- \to \bar{f}f$ scattering:



  - No significant deviation from the SM prediction.
    $\Longrightarrow$ Constraints on Z' mass or Z'-fermion couplings.
- Electroweak constraints reduce the number of models or parameter space, and allow us to focus on more promising ones.

Model independent analysis

- Global analysis—using all relevant data.
- Model-independent method—avoid repeating the calculations.
  - Example: oblique parameters describe the corrections to gauge boson propagators. $S, T, U, \ldots$
    $\longrightarrow$ Not enough, non-oblique corrections are common. (e.x. Z').
    $\longrightarrow$ Model-dependent corrections to observables calculated from time to time in the literature.
  - Effective theory approach

The effective theory approach

- Below the cutoff Λ, after integrating out the heavy particles:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \sum_i c_i O_i + \frac{1}{\Lambda^4}(\dots) + \dots$$

- Only the SM fields appear in the operators O_i.
- The operators O_i conserve the SM gauge symmetry. $\Longrightarrow$ The number of O_i is finite to a given order in Λ.
- The coefficients c_i record the effects of the heavy particles. If taken arbitrary $\longrightarrow$ model-independent analysis.

Reduce the number of operators

- Given the current experimental precision, enough to focus on dimension-6 operators, since higher order operators are suppressed by more powers of Λ^2.
- Keep only independent operators
- Keep operators that are relevant to TeV scales: imposing symmetries on the operators:
  - CP
  - Baryon and lepton number conservation
  - Flavor universality, $U(3)^5$ symmetry for q, l, u, d, e.
- Remove operators not tightly constrained by EWPTs. e.x.: operators that contain only gluons and quarks.

The 21 relevant operators

- Operators modifying gauge boson propagators:

$$O_{WB} = (h^\dagger \sigma^a h) W^a_{\mu\nu} B^{\mu\nu}, \quad O_h = |h^\dagger D_\mu h|^2;$$

  These two operators correspond respectively to the S and T parameters.

- Four-fermion operators:

$$O^s_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l), \quad O^t_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l),$$
$$O^s_{lq} = (\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q), \quad O^t_{lq} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q),$$
$$O_{le} = (\bar{l}\gamma^\mu l)(\bar{e}\gamma_\mu e), \quad O_{qe} = (\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e),$$
$$O_{lu} = (\bar{l}\gamma^\mu l)(\bar{u}\gamma_\mu u), \quad O_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d),$$
$$O_{ee} = \frac{1}{2}(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e), \quad O_{eu} = (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u), \quad O_{ed} = (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d);$$

The 21 operators

- Operators modifying gauge-fermion couplings:

$$
\begin{aligned}
&O^s_{hl} = i(h^\dagger D^\mu h)(\bar{l}\gamma_\mu l) + \text{h.c.}, \quad O^t_{hl} = i(h^\dagger \sigma^a D^\mu h)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.},\\
&O^s_{hq} = i(h^\dagger D^\mu h)(\bar{q}\gamma_\mu q) + \text{h.c.}, \quad O^t_{hq} = i(h^\dagger \sigma^a D^\mu h)(\bar{q}\gamma_\mu \sigma^a q) + \text{h.c.},\\
&O_{hu} = i(h^\dagger D^\mu h)(\bar{u}\gamma_\mu u) + \text{h.c.}, \quad O_{hd} = i(h^\dagger D^\mu h)(\bar{d}\gamma_\mu d) + \text{h.c.},\\
&O_{he} = i(h^\dagger D^\mu h)(\bar{e}\gamma_\mu e) + \text{h.c.}\,;
\end{aligned}
$$

- Operator modifying the triple-gauge couplings:

$$
O_W = \epsilon^{abc}\, W^{a\nu}_{\mu} W^{b\lambda}_{\nu} W^{c\mu}_{\lambda}.
$$

The calculation

- $$\mathcal{L} = \mathcal{L}_{SM} + a_{WB}O_{WB} + a_h O_h + \ldots a_W O_W$$
$$a_i \text{ dimension } (-2) \quad \sim \frac{1}{\Lambda^2}$$

.

- Calculate to linear order in a_i the corrections to the observables—only consider the interference between the SM and the new physics contribution:

Tree level calculation, amplitude $\mathcal{M}_{NP}$ linear in a_i

$$X_{th}(a_i) \sim |\mathcal{M}|^2 = |\mathcal{M}_{SM} + \mathcal{M}_{NP}|^2 = |\mathcal{M}_{SM}|^2 + 2Re(\mathcal{M}_{SM}\mathcal{M}^*_{NP})$$

$X_{th}(a_i)$: theoretical prediction for an observable X, linear in a_i.

$$X_{th}(a_i) = X_{SM} + \sum_i a_i \Delta X_i.$$

The observables

	Standard Notation	Measurement
Atomic parity violation	$Q_W(Cs)$ $Q_W(Tl)$	Weak charge in Cs Weak charge in Tl
DIS	g_L^2, g_R^2 R^ν κ $g_V^{\nu e}, g_A^{\nu e}$	ν_μ-nucleon scattering from NuTeV ν_μ-nucleon scattering from CDHS and CHARM ν_μ-nucleon scattering from CCFR ν-e scattering from CHARM II
Z-pole	Γ_Z σ_h^0 $R_f^0(f = e, \mu, \tau, b, c)$ $A_{FB}^{0,f}(f = e, \mu, \tau, b, c)$ $\sin^2\theta_{eff}^{lept}(Q_{FB})$ $A_f(f = e, \mu, \tau, b, c)$	Total Z width e^+e^- hadronic cross section at Z pole Ratios of decay rates Forward-backward asymmetries Hadronic charge asymmetry Polarized asymmetries
Fermion pair production at LEP2	$\sigma_f(f = q, \mu, \tau)$ $A_{FB}^f(f = \mu, \tau)$ $d\sigma_e/d\cos\theta$	Total cross sections for $e^+e^- \to f\overline{f}$ Forward-backward asymmetries for $e^+e^- \to f\overline{f}$ Differential cross section for $e^+e^- \to e^+e^-$
W pair	$d\sigma_W/d\cos\theta$	Differential cross section for $e^+e^- \to W^+W^-$
	M_W	W mass

Table 1: Relevant measurements

- Compare with experiments and calculate the χ^2 distribution in terms of a_i:

$$\chi^2(a_i) = \sum_X \frac{(X_{th}(a_i) - X_{exp})^2}{\sigma_X^2} = \chi^2_{SM} + a_i \hat{v}_i + a_i M_{ij} a_j.$$

M_{ij}, v_i: our results

M_{ij} : 21 by 21 symmetric matrix; v_i : 21-vector

- Diagonal elements M_{ii}, tell us how well individual operators are constrained. $\Lambda \sim M_{ii}^{\frac{1}{4}} = 1.3 \sim 17$ TeV.

- Constraints on individual operators not useful, because corrections are correlated.

- Constraints can be obtained from the χ^2 for arbitrary linear combinations of the operators. $\rightarrow$ Constrain generic models.

Numerical results

	a_{WB}	a_h	a^s_{ll}	a^t_{ll}	a^s_{lq}	a^t_{lq}	a_{le}	a_{qe}	a_{lu}	a_{ld}	a_{ee}	a_{eu}	a_{ed}	a^s_{hl}	a^t_{hl}	a^s_{hq}	a^t_{hq}	a_{hu}	a_{hd}	a_{he}	a_W
a_{WB}	9.1e4																				
a_h	2.4e4	7.9e3																			
a^s_{ll}	−78.	−51.	5.8e2																		
a^t_{ll}	−3.9e4	−1.2e4	6.7e2	2.2e4																	
a^s_{lq}	−1.4e3	−1.6e2	0.	1.5e2	2.7e3																
a^t_{lq}	−5.5e2	−1.4e2	0.	5.9e2	4.6e2	2.9e3															
a_{le}	−56.	−9.7	2.8e2	3.0e2	0.	0.	1.3e3														
a_{qe}	1.3e3	72.	0.	−1.4e2	−2.7e3	−7.4e2	0.	2.8e3													
a_{lu}	−4.0e2	3.8	0.	−1.1e2	1.2e3	−2.5e2	0.	−1.2e3	7.1e2												
a_{ld}	−6.9e2	−6.9	0.	66.	1.4e3	3.3e2	0.	−1.4e3	5.8e2	7.8e2											
a_{ee}	−59.	−42.	5.3e2	6.1e2	0.	0.	2.6e2	0.	0.	0.	4.8e2										
a_{eu}	7.8e2	1.1e2	0.	−2.1e2	−1.3e3	−9.1e2	0.	1.4e3	−4.8e2	−7.3e2	0.	8.4e2									
a_{ed}	4.2e2	−83.	0.	1.7e2	−1.3e3	5.5e2	0.	1.3e3	−7.3e2	−6.8e2	0.	4.7e2	8.8e2								
a^s_{hl}	−1.7e4	−4.1e3	1.5e2	9.7e3	−5.9e2	8.3e2	17.	3.7e2	−3.9e2	−1.6e2	1.3e2	66.	3.8e2	5.5e4							
a^t_{hl}	5.9e4	1.7e4	−43.	−3.0e4	−7.1e2	−6.6e2	−31.	6.6e2	−82.	−3.4e2	−32.	4.9e2	47.	1.5e4	6.3e4						
a^s_{hq}	−1.9e3	−1.4e3	0.	2.7e3	−2.6e3	−72.	0.	2.6e3	−1.2e3	−1.4e3	0.	1.2e3	1.4e3	−6.6e3	−8.7e3	6.0e3					
a^t_{hq}	−9.3e3	−4.5e3	0.	8.7e3	−49.	3.5e2	0.	56.	−1.4e2	−36.	0.	−64.	1.8e2	−2.4e4	−3.1e4	7.7e3	2.6e4				
a_{hu}	−6.1e2	−6.6e2	0.	1.2e3	−1.2e3	−4.	0.	1.2e3	−5.1e2	−6.9e2	0.	5.7e2	6.7e2	−3.7e3	−4.4e3	2.2e3	4.1e3	1.4e3			
a_{hd}	1.2e3	4.3e2	0.	−8.1e2	−1.4e3	−1.3e2	0.	1.4e3	−6.9e2	−7.2e2	0.	6.7e2	7.3e2	3.3e3	3.6e3	4.2e2	−2.9e3	1.6e2	1.1e3		
a_{he}	−2.8e4	−4.6e3	−1.1e2	9.0e3	4.6e2	−1.6e2	23.	−4.5e2	2.5e2	2.4e2	−96.	−1.7e2	−3.0e2	−2.5e4	−3.2e4	4.5e3	1.7e4	2.3e3	−2.1e3	3.2e4	
a_W	7.7	4.5	0.	−4.2	0.	0.	0.	0.	0.	0.	0.	0.	0.	6.3	−1.7	0.	0.8	0.	0.	1.4	2.6

Table 2: The elements of the matrix $\mathcal{M}$. Since it is a symmetric matrix we do not list the redundant elements. The matrix is equal to the numbers listed above times $10^{12}(\mathrm{GeV})^4$. We abbreviate the powers 10^n as en to save space.

$$\begin{aligned} \hat{v}_i \;&=\; \{1.5, 10^2, -23., 49., 76., -1.1, 10^2, -2.4, 10^2, 29., 1.4, 10^2, -36., -68., 44., \\ &\quad 1.0, 10^2, 15., -6.4, 10^2, -88., 1.0, 10^2, 1.7, 10^2, 71., 63., 1.8, 10^2, 1.0\} \end{aligned}$$

the S and T fit

- $$S = \frac{4scv^2 a_{WB}}{\alpha}, \quad T = -\frac{v^2}{2\alpha} a_h.$$
- Setting all a_i, but a_{WB} and a_h, to zero.

$$\begin{aligned} \chi^2 &= \chi_0^2 + (a_{WB} \quad a_h) \begin{pmatrix} 9.1\,10^{16} & 2.4\,10^{16} \\ 2.4\,10^{16} & 7.9\,10^{15} \end{pmatrix} \begin{pmatrix} a_{WB} \\ a_h \end{pmatrix} + 1.5\,10^8 a_{WB} - 2.3\,10^7 a_h \\ &= \chi_0^2 + (S \quad T) \begin{pmatrix} 5.4\,10^2 & -4.8\,10^2 \\ -4.8\,10^2 & 5.3\,10^2 \end{pmatrix} \begin{pmatrix} S \\ T \end{pmatrix} + 12.\,S + 5.9\,T. \end{aligned}$$

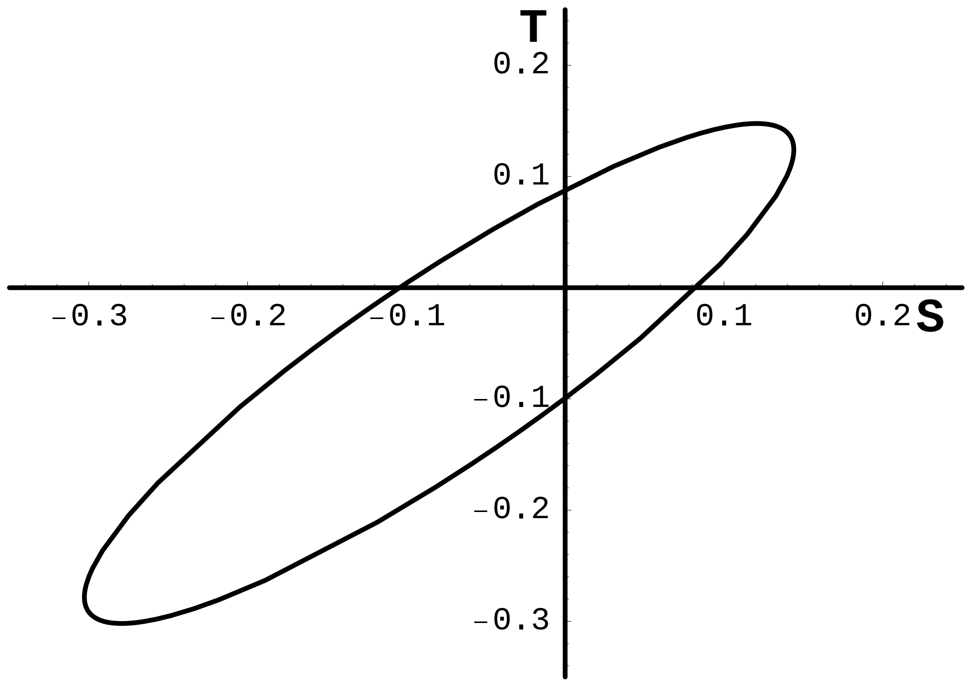
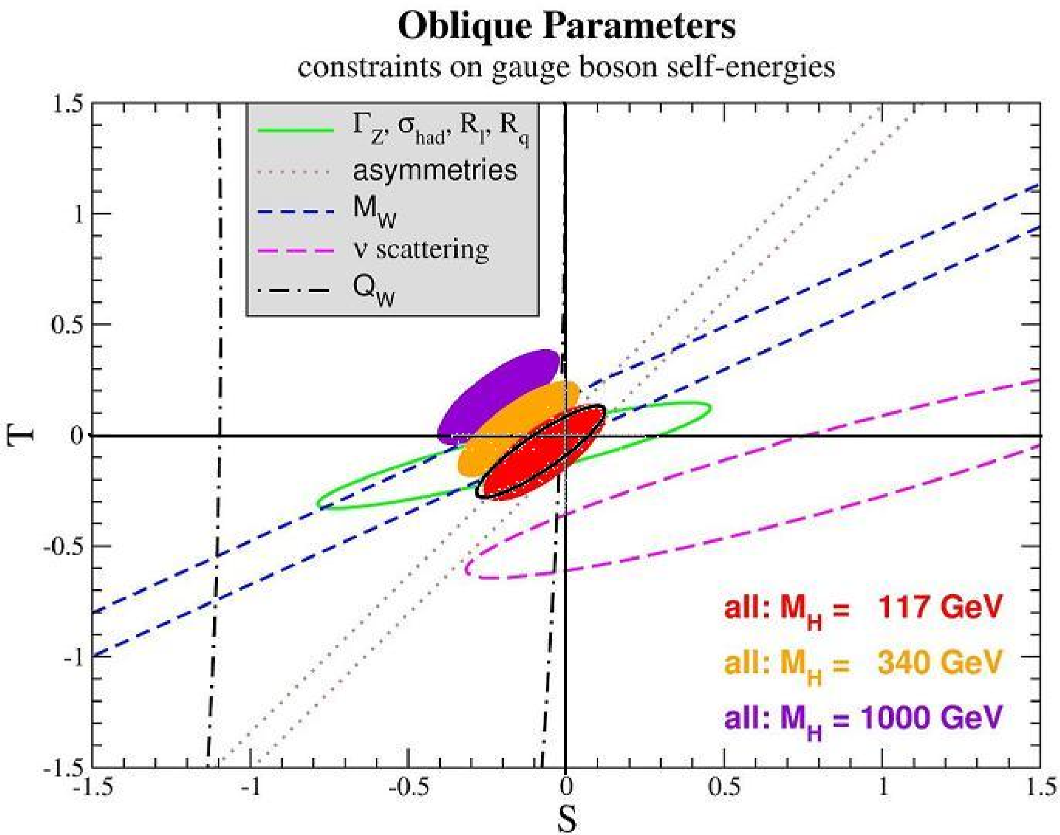


Figure 1: Allowed region for S and T at 90% confidence level.



From Erler & Langacker, Review of Particle Physics.

Applications

- General procedure
  - Integrating out the heavy particles.
  - Obtain operator coefficients a_i as functions of the parameters in the model.
  $$a_i = a_i(m, g, \ldots)$$
  - Substitute the coefficients in the χ^2 distribution.
  - Calculate bounds, draw plots, ...

Little Higgs models

TeV scale gauge bosons, fermions, scalars are present.

The littlest Higgs model

- Gauge group enlarged: $SU(2)_1 \times SU(2)_2 \times U(1)_1 \times U(1)_2 \longrightarrow SU(2)_W \times U(1)_Y$.

  – heavy gauge bosons W', Z' of TeV scale mass;

  – gauge coupling g_1, g_2, g'_1, g'_2: $g = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$, $g' = \frac{g'_1 g'_2}{\sqrt{{g'}_1^2 + {g'}_2^2}}$;

  $$\text{Define: } g = g_1 s = g_2 c, \quad g' = g'_1 s' = g'_2 c'.$$

  – $Y = Y_1 + Y_2$.

  – $M_{W'} = \frac{gF}{2sc}$, $\quad M_{Z'} = \frac{g'F}{\sqrt{20}s'c'}$.

The heavy scalar and heavy fermion

- Triplet scalar ϕ with mass M_ϕ couples to Higgs:

$$-i\lambda(h^T\phi^\dagger h - h^\dagger \phi h^*)$$

- The heavy fermion mixes with the top quark, but does not affect EWPT at tree level.

The operators

$$
\begin{aligned}
a_h &= -\frac{5(c'^2 - s'^2)^2}{2F^2} + \frac{2\lambda^2}{M_\phi^4}, \\
a_{hq}^t &= a_{hl}^t = -\frac{(c^2 - s^2)c^2}{2F^2}, \\
a_{hf}^s &= \frac{5s'c'(c'^2 - s'^2)}{F^2}\left(Y_2^f \frac{s'}{c'} - Y_1^f \frac{c'}{s'}\right), \\
a_{lq}^t &= a_{ll}^t = -\frac{c^4}{F^2}, \\
a_{ff'}^s &= -\frac{20s'^2c'^2}{F^2}\left(Y_2^f \frac{s'}{c'} - Y_1^f \frac{c'}{s'}\right)\left(Y_2^{f'} \frac{s'}{c'} - Y_1^{f'} \frac{c'}{s'}\right).
\end{aligned}
$$

- To obtain bounds on physical mass:

$$
M_{W'} = \frac{gF}{2sc}; \quad M_{t'} \geq \sqrt{2}\lambda_t F, \text{ take } M_{t'} = \sqrt{2}F
$$

- To suppress the corrections? $Y_1^f = Y_2^f$, $s' = c'$.

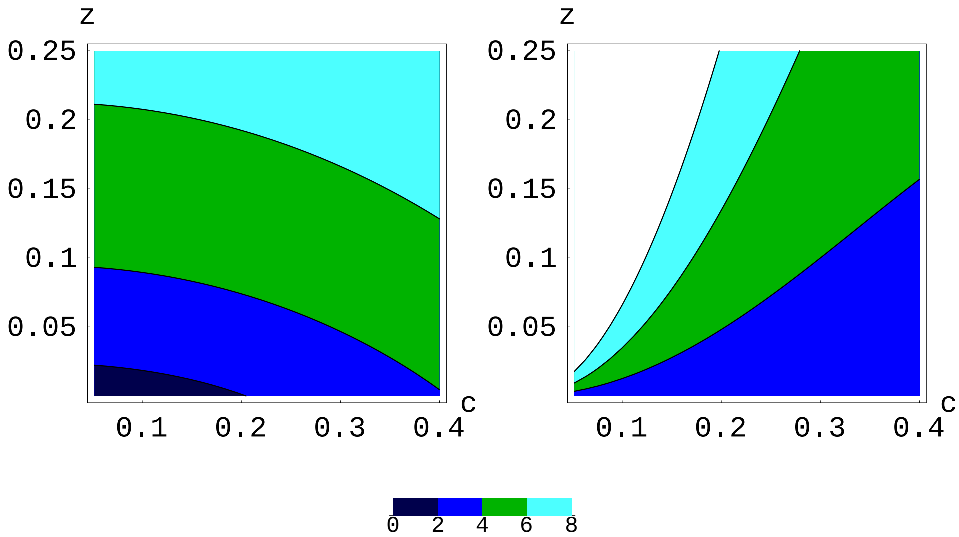


Figure 2: 95% CL lower bounds in TeV on $M_{t'}$ (left) and $M_{W'}$ (right) as functions of c and $z \equiv \lambda^2 F^2 / M_\phi^4$ for $Y_1^f = Y_2^f$ and $s' = c'$.

Conclusion

- **Electroweak precision tests can put constraints on TeV scale extensions of the SM.**
- **We have done a model-independent analysis on electroweak constraints, using the effective theory approach.**
- **Constraints on general TeV scale models can be easily obtained using our results.**