What can we learn from a Higher-Dimensional Decaying Black Hole?

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Outline :

1. Extra Dimensions and Black Holes
2. Evaporation of Black Holes - Hawking Radiation
   - Schwarzschild Black Holes
   - Kerr Black Holes
   - Schwarzschild-de Sitter Black Holes
3. Conclusions

In collaboration with :

C. Harris (Cambridge), E. Winstanley (Sheffield),

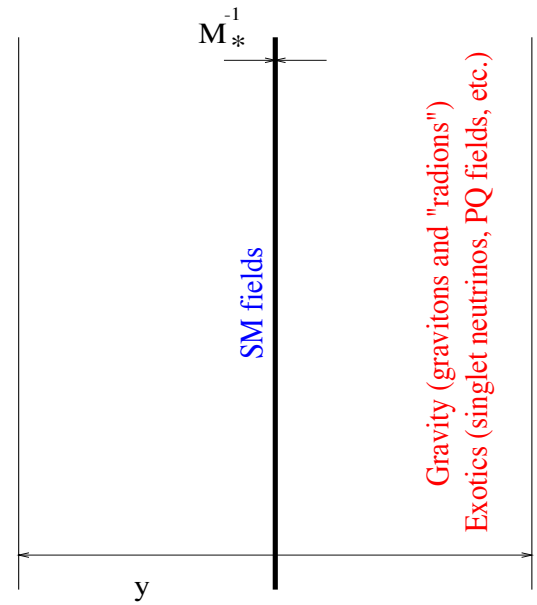
G. Duffy & M. Casals (Dublin),

J. Grain & A. Barrau (Grenoble)

Introduction : Extra Dimensions

- Large Extra Dimensions (1998) :

  - A 4D **Brane** with all the SM fields and scale for gravity $M_P = 10^{19}$ GeV
  - A $(4+n)$D Extra Space (**Bulk**) with gravitons and scale for gravity M_*
  - Then, we obtain:



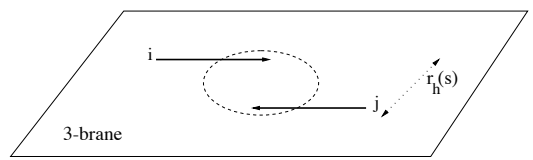
$$M_P^2 \simeq \mathcal{R}^n \, M_*^{2+n}$$

If $\mathcal{R} \leq 1$ mm, then $M_* \geq 1$ TeV $\Rightarrow$ resolution of the hierarchy problem (Arkani-Hamed, Dimopoulos & Dvali; Antoniadis & ADD)

Introduction : Black Hole Creation

• During gravitational collapse on the brane, we expect the formation of a black hole, that will extend off the brane

▷ Microscopic Black Holes may be created during the scattering of high energy particles (Banks & Fischler; Giddings & Thomas; Dimopoulos & Landsberg)



For every center-of-mass-energy $\sqrt{s}$, there is a Schwarzschild radius r_H – if $b < r_H$, a black hole will be created (Thorne)

▷ For a black hole to be classical, it must have:

• in $4+n$ Dimensions: $M_{BH} > M_* \geq 1$ TeV

Introduction : Black Hole Creation

- Stages of the life of the produced black hole: A highly asymmetric, rotating object that goes through the following: (Giddings & Thomas)

  - Balding phase: shedding of all quantum numbers apart from (M, Q, A) – mainly invisible energy
  - Spin-down phase: Loss of angular momentum – Hawking radiation – visible energy
  - Schwarzschild phase: Loss of mass – Hawking radiation – visible energy
  - Planck phase: when $M_{BH} \sim M_*$ – a few energetic quanta, or a stable "quantum" remnant?

Introduction: Black Hole Evaporation

- Hawking Radiation: What is it?
  - creation of a virtual pair of particles just outside the horizon
  - the antiparticle falls into the BH whose mass decreases
  - the particle escapes to infinity where it gets observed

The Radiation Spectrum: A nearly black-body spectrum with emission rate

$$\frac{d^2 E}{dt\, d\omega} = \frac{|\mathcal{A}_n^{(s)}(\omega)|^2\ \omega}{\exp\left(\omega/T_{BH}\right) \mp 1}\ \frac{1}{(2\pi)}$$

where $|\mathcal{A}_n^{(s)}(\omega)|^2$ is the absorption probability (greybody factor)

Introduction: Black Hole Evaporation

... defined in terms of the flux as

$$|\mathcal{A}^{(s)}_{n,\ell}(\omega)|^2 = \frac{\mathcal{F}_{\text{horizon}}}{\mathcal{F}_{\text{infinity}}}$$

▷ Particles in the Bulk : Gravitons, Scalars

▷ Particles on the Brane : (zero-mode) Scalars, Fermions, Gauge Bosons, (zero-mode) Gravitons

Here, we will focus on the emission of Standard Model particles on the brane – and for this, we will need to find the induced-on-the-brane $4D$ background in which these particles live during a given phase

Schwarzschild Black Holes

- The Background: Described by the spherically-symmetric $(4+n)$-dimensional Tangherlini line-element

$$ds^2 = -\left[1-\left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1-\left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2 d\Omega^2_{2+n}$$

where $d\Omega^2_{2+n}$ the line-element of a $(2+n)$-D unit sphere, and

$$r_H = \frac{1}{M_*}\left(\frac{M_{\rm B\,H}}{M_*}\right)^{\frac{1}{n+1}}\left(\frac{8\Gamma(\frac{n+3}{2})}{(n+2)\sqrt{\pi}^{(n+1)}}\right)^{1/(n+1)} \quad \text{and} \quad T_{\rm H} = \frac{n+1}{4\pi r_H}$$

For the induced-on-the-brane background, we set: $\theta_n = \frac{\pi}{2}$. Then

$$ds^2_4 = -\left[1-\left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1-\left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2\, d\Omega^2_2$$

Schwarzschild Black Holes

- The Equation of Motion: To find the Absorption Probability for a spin-s field, we must solve the following equation

(P.K.; Ida, Oda & Park)

$$\Delta^s \frac{d}{dr}\left[\Delta^{1-s}\frac{dP_s}{dr}\right] + \left[\frac{\omega^2 r^2}{h} + 2i\omega s r - \frac{is\omega r^2 h'}{h} - \Lambda_{sj}\right] P_s(r) = 0$$

where

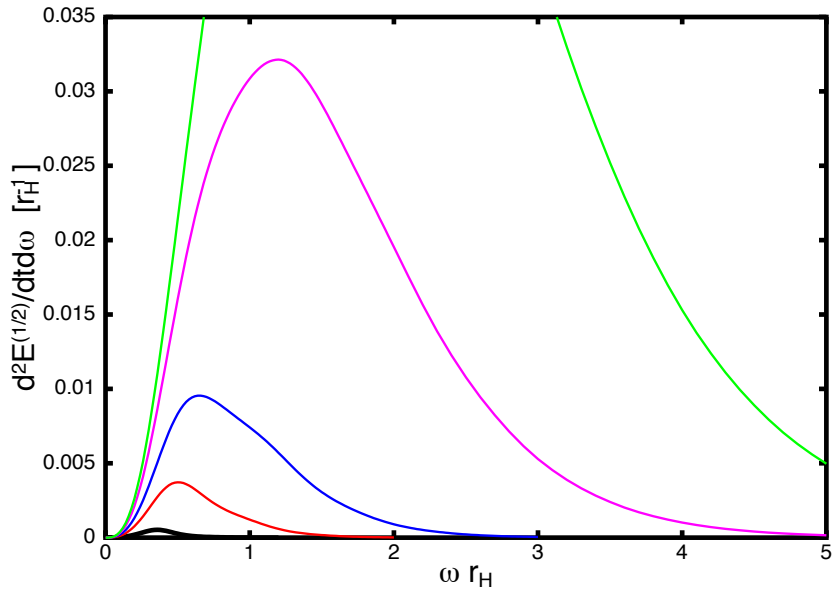
$$\Psi_s = e^{-i\omega t}\, e^{im\varphi}\, \Delta^{-s}\, P_s(r)\, S^m_{s,j}(\theta)$$

$$\Delta = r^2\, h = r^2\left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right], \qquad \Lambda_{sj} = j(j+1) - s(s-1)$$

(4D : Teukolsky & Press; Starobinsky & Churilov; Unruh; Sanchez; Page; McGibbon & Webber)

Schwarzschild Black Holes

- The amount of energy emitted per unit time strongly depends on the number of transverse-to-the-brane spacelike dimensions

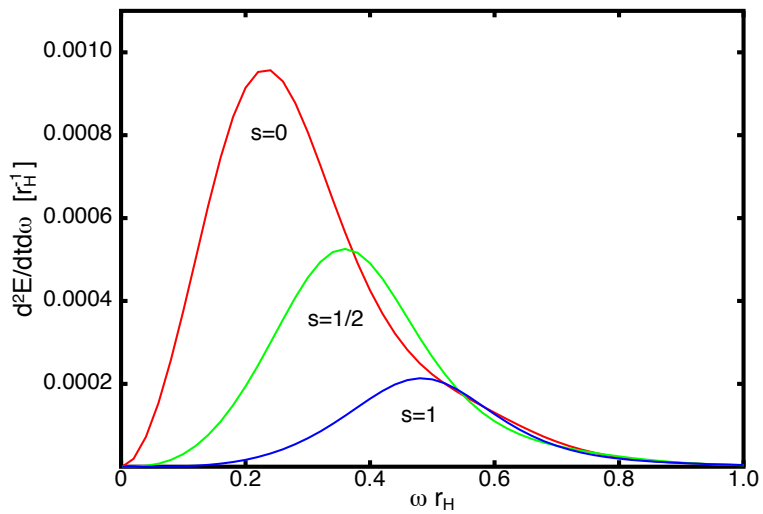


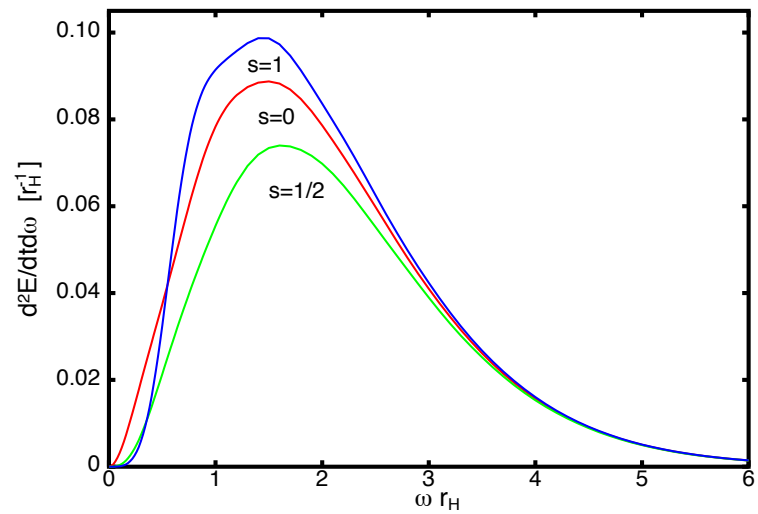
n	0	1	2	3	4	5	6	7
Scalars	1.0	8.94	36.0	99.8	222	429	749	1220
Fermions	1.0	14.2	59.5	162	352	664	1140	1830
G. Bosons	1.0	27.1	144	441	1020	2000	3530	5740

Schwarzschild Black Holes

- Relative Emission Rates : How do they change with n?

(Harris & P. K.)





$(n = 0)$ $\boxed{1 : 0.55 : 0.23}$ $(n = 6)$ $\boxed{1 : 0.84 : 1.06}$

- The type of the emitted radiation also depends strongly on n

Kerr Black Holes

- The line-element on the brane has the form of a rotating, neutral, n-dependent Myers-Perry solution

$$ds^2 = \left(1 - \frac{\mu}{\Sigma\, r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma\, r^{n-1}}\, dt\, d\varphi - \frac{\Sigma}{\Delta} dr^2 - \Sigma\, d\theta^2 - \left(r^2 + a^2 + \frac{a^2 \mu \sin^2\theta}{\Sigma\, r^{n-1}}\right) \sin^2\theta\, d\varphi^2,$$

where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}} \quad \text{and} \quad \Sigma = r^2 + a^2 \cos^2\theta$$

and the parameters μ and a are associated to the black hole mass and angular momentum as

$$M = \frac{(n+2) A_{n+2}}{16\pi G}\, \mu \qquad \text{and} \qquad J = \frac{2}{n+2}\, M a$$

Kerr Black Holes

- In this case, the "master" e.o.m. becomes (P.K.; Ida, Oda & Park)

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR_s}{dr}\right)+$$

$$\left[\frac{K^2-iKs\Delta'}{\Delta}+4is\omega r+s\left(\Delta''-2\right)-\Lambda_\ell^m\right]R_s=0$$

where $K=\left(r^2+a^2\right)\omega-am$.

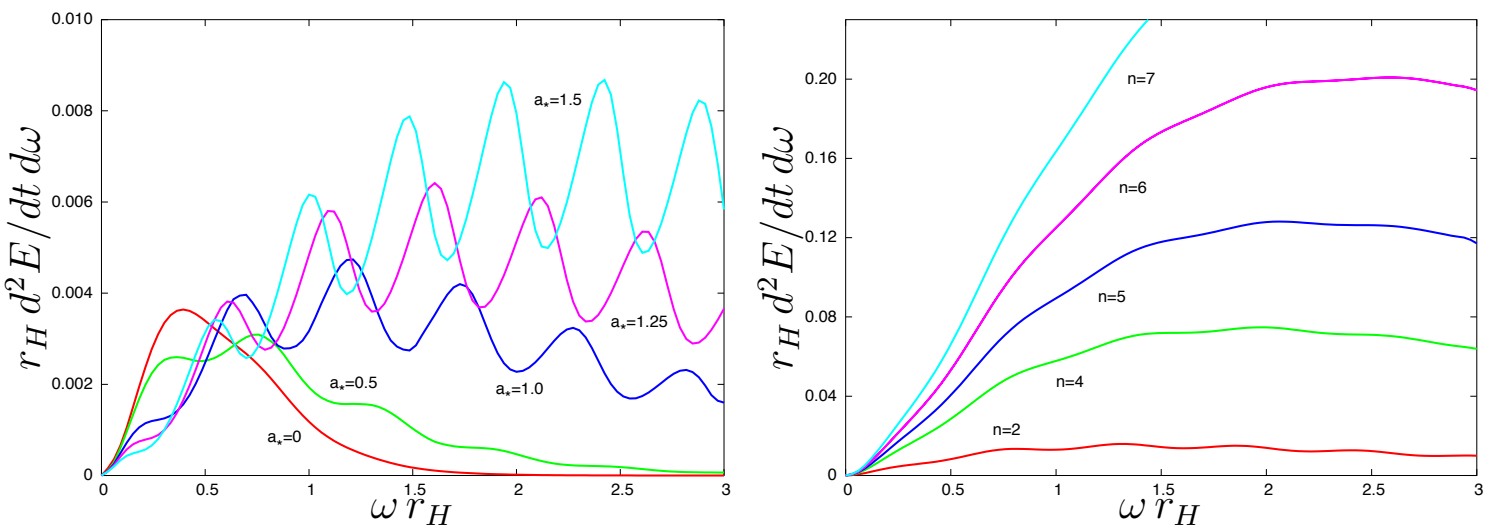
The Hawking temperature and rotation velocity of this brane black hole is

$$T_H=\frac{(n+1)\,r_H^2+(n-1)\,a^2}{4\pi(r_H^2+a^2)\,r_H}\,,\qquad \Omega=\frac{a}{(r_H^2+a^2)}$$

Kerr Black Holes

The differential energy emission rate is now given by:

$$\frac{d^2 E}{dt\, d\omega} = \sum_{\ell,m} |\mathcal{A}^m_{\ell,n}|^2 \frac{\omega}{\exp\left[(\omega - m\Omega)/T_H\right] - 1} \frac{1}{2\pi}$$

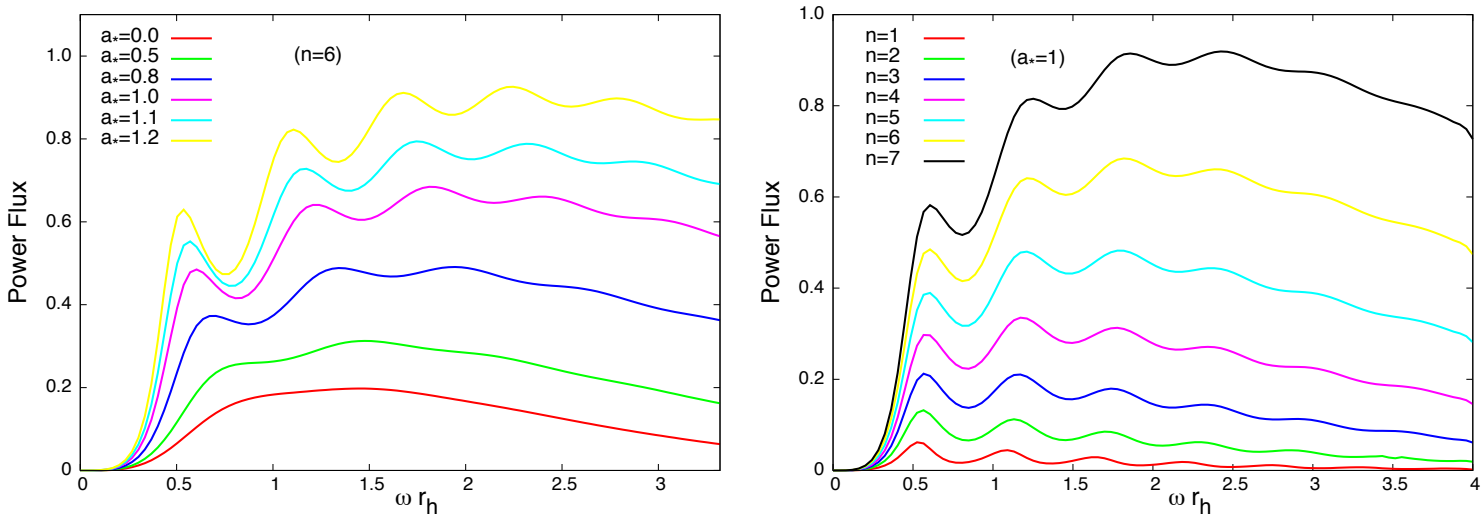


For scalar fields, as a or n increases, the energy emission rate is enhanced (Harris & P.K.; Duffy, Harris, P.K. & Winstanley; Ida, Oda & Park)

Kerr Black Holes

For Gauge Bosons, a similar behaviour was found:

(Casals, P.K. & Winstanley; Ida, Oda & Park)

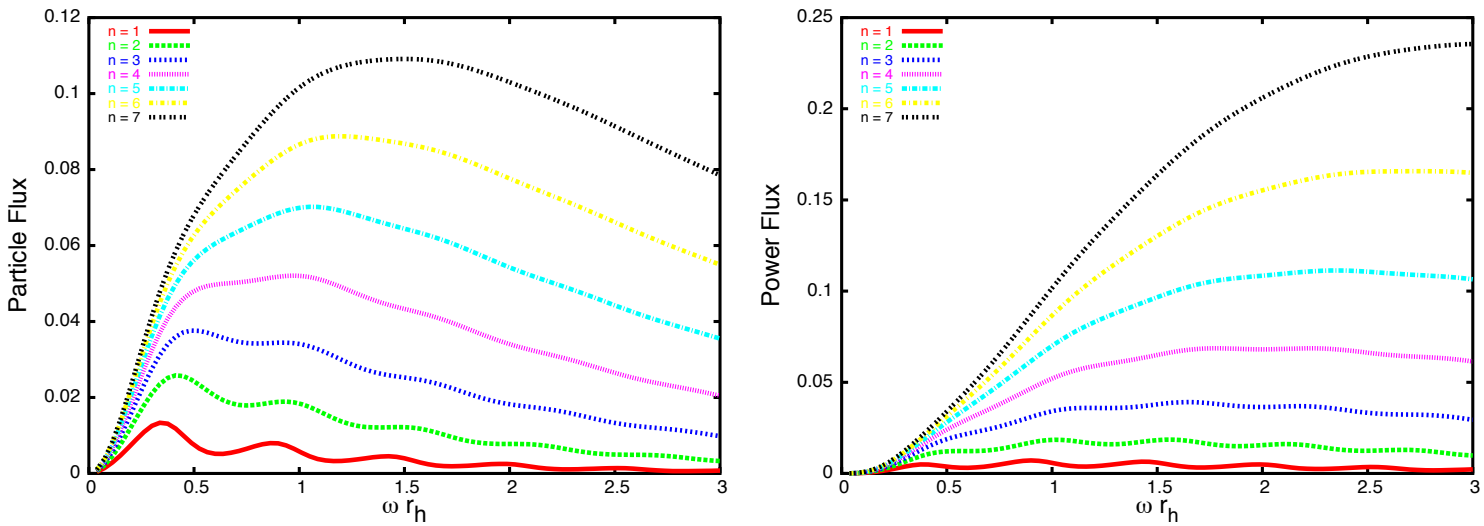


Again, as a or n increases, the energy emission rate is enhanced at all energies, particularly at the high-energy regime

Kerr Black Holes

Finally, for Fermions, our analysis has just been completed:

(Ida, Oda and Park; Casals, Dolan, P.K. & Winstanley)



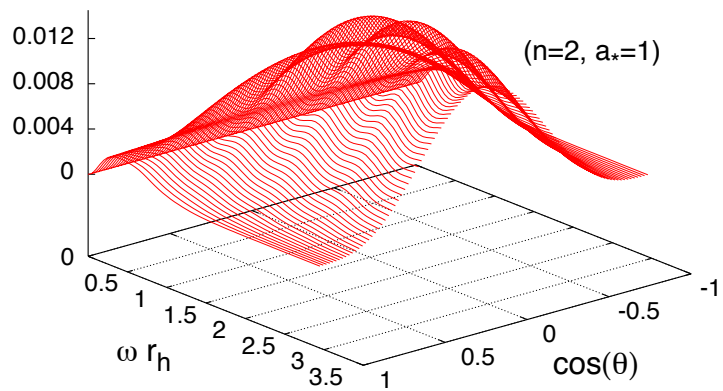
As n increases, both the number of particles and energy emission rates are again enhanced at all energies, particularly at the high-energy regime

Kerr Black Holes

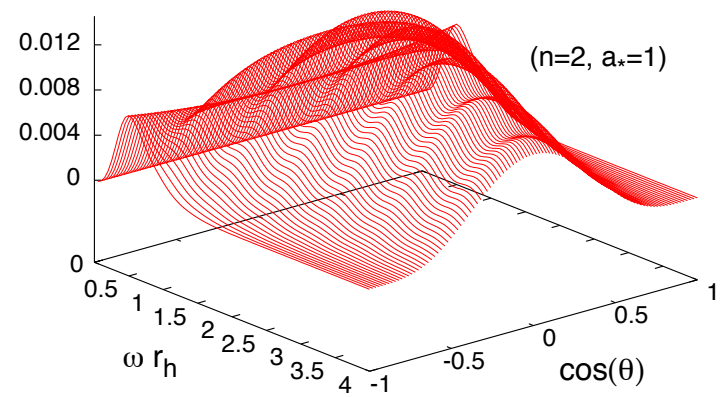
- Angular Distribution : For different species of fields, we find:

(Duffy, Harris, P.K., Winstanley; Casals, P.K., Winstanley; Casals, Dolan, P.K., Winstanley)

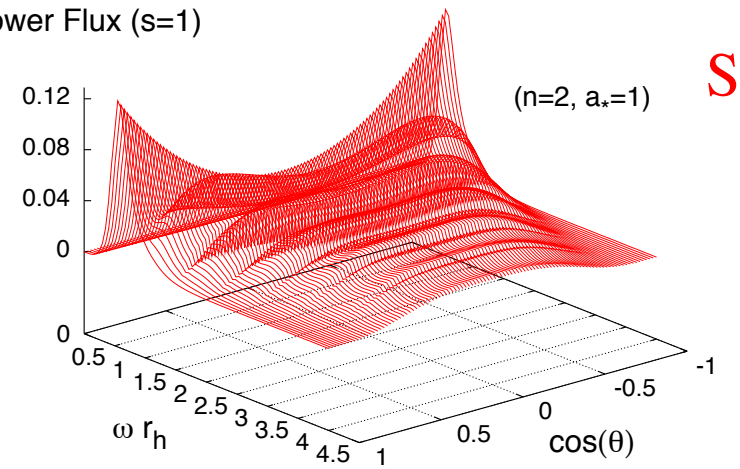
Power Flux (s=0)



Power Flux (s=1/2)



Power Flux (s=1)

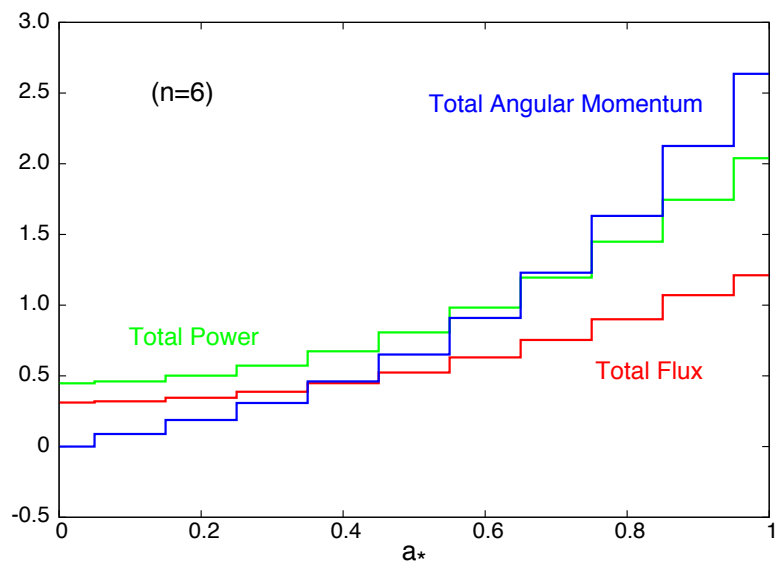


Scalars: emission on the equatorial plane (centrifugal potential)
Fermions-G. Bosons: emission parallel to the axis of rotation (spin-rotation coupling), then, on the equatorial plane

Kerr Black Holes

- Evolution in Time : For a realistic description, our results will be entered into the CHARYBDIS code

But, qualitatively, we observe:



(Duffy, Harris, P.K., Winstanley; Casals, P.K., Winstanley; Casals, Dolan, P.K., Winstanley)

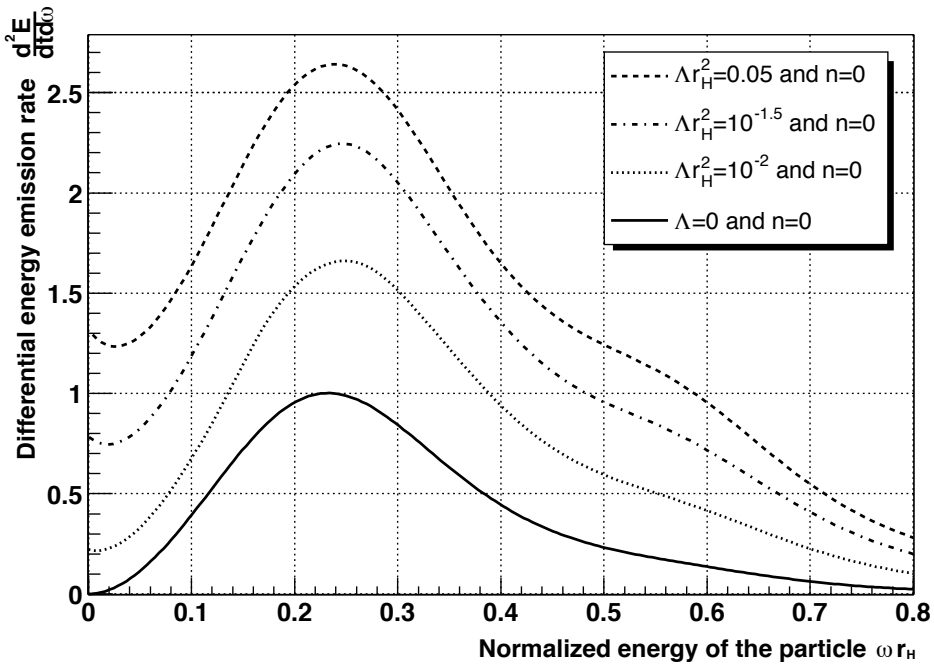
See next talk, by Seong Chan Park

Schwarzschild - de Sitter Black Holes

- In the presence of a Cosmological Constant $\Lambda > 0$, the geometrical background on the brane becomes: (P.K., Grain & Barrau)

$$ds^2 = -h(r)\, dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_2^2$$

where $h(r) = 1 - \frac{\mu}{r^{n+1}} - \Lambda\, r^2$. At the end, we get the spectrum



Conclusions

- Until today, we have not yet detected the Hawking radiation from a decaying black hole
- If Extra Dimensions exist, then
  - the production of small black holes becomes possible
  - the detection of Hawking radiation becomes more likely
- The emission spectra can help us determine the dimensionality of spacetime with possible signatures being the rate and type of the emitted radiation
- The spectrum will also depend on the cosmological constant of spacetime on which valuable information could also be extracted