Baryon and Lepton Number Violation in Gauge Extensions of the Standard Model

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Baryon and Lepton Number Violation in the Standard Model

The B+L Anomaly

- Baryon (B) and Lepton (L) number are symmetries of the Standard Model Lagrangian.

- The corresponding currents are classically conserved,

$$\begin{aligned} \partial_\mu j_B^\mu &= 0 = \partial_\mu j_L^\mu \\ \Rightarrow \quad Q_B &= \int d^3x\, j_B^0 \qquad \text{is time-independent.} \end{aligned}$$

- This is no longer true at the quantum level. With loops,

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = \frac{g_L^2}{32\pi^2}\, n_g\, F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu},$$

  where g_L and $F^a_{\mu\nu}$ correspond to $SU(2)_L$,
  and $n_g = 3$ is the number of generations.

Aside #1: Gauge Theory Vacua

- A non-Abelian gauge theory has many distinct vacua.

- Each vacuum can be labelled by an integer,
  called the $Chern-Simons$ number or $Pontryagin$ index,

$$N_{CS} = \frac{g^2}{32\pi^2} \int d^3x\, K^0,$$

  where K^0 is the 0-th component of

$$K^\mu = 2\,\epsilon^{\mu\nu\rho\sigma} \left(A^a_\nu \partial_\rho A^a_\sigma + \frac{g}{3}\, f^{abc} A^a_\nu A^b_\rho A^c_\sigma \right).$$

- N_{CS} is always an integer when A_μ is a vacuum configuration.
  ($\Rightarrow$ time-independent and with minimal energy.)

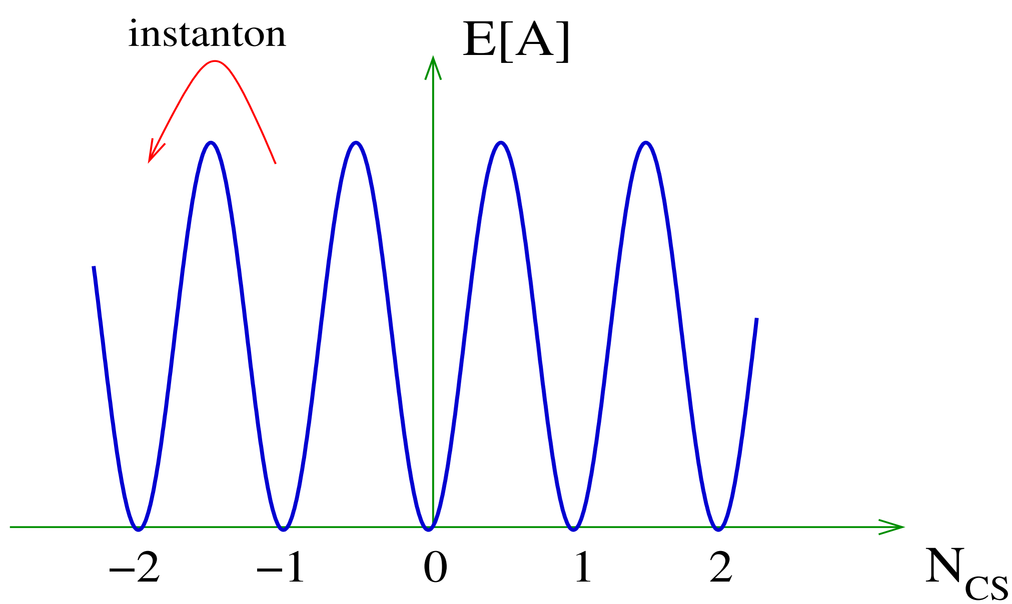
- One can also show that

$$\partial_\mu K^\mu = F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}.$$

  Recall that that $F\tilde{F}$ appears in the $(B+L)$ anomaly equations.

Aside #2: Instantons

- The energy barrier between different vacua has finite height.
- As a result, a system lying in one vacuum can tunnel to another vacuum.
- Instantons are the classical field configurations in Euclidean space that describe this tunnelling. [Belavin,Polyakov, Shvarts,Tyupkin '75]
- In each instanton transition, N_{CS} changes by one unit.



Instantons and (B+L) Violation

- Together,

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = \frac{g_L^2}{32\pi^2} n_g \, F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} = \frac{g_L^2}{32\pi^2} n_g \, \partial_\mu K^\mu,$$

$$N_{CS} = \int \frac{g_L^2}{32\pi^2} \, d^3x K^0$$

- Integrating these anomaly equations,

$$B(t_2) - B(t_1) = L(t_2) - L(t_1) = n_g \left[N_{CS}(t_2) - N_{CS}(t_1) \right].$$

- $SU(2)_L$ instantons are the source of $(B+L)$ violation in the SM.

- There are also QCD instantons, but they do not violate B or L.

The Instanton Rate

- Instantons describe tunnelling between distinct vacua.
- If there are no fermions, the tunnelling amplitude is

$$\mathcal{A} = \langle N_{CS} = n+1 |\, N_{CS} = n \rangle = \int [\mathscr{D}A\, \mathscr{D}\Phi]_n^{n+1}\, e^{-S_E[A,\Phi]}$$

- We can approximate this path integral by expanding to quadratic order about the classical instanton solution, ['t Hooft '76]

$$\mathcal{A} = \int d^4x_0 \int d\rho \int dU\, \frac{C}{g^8(\mu)} (\rho\mu)^{b_0} \rho^{-5}\, e^{-S_E[A_{cl},\Phi_{cl}]},$$

where μ is the $\overline{MS}$ scale, b_0 the 1-loop β-function coefficient, and

$$S_E[A_{cl}, \Phi_{cl}] = S_A + S_{Higgs} \simeq \frac{8\pi^2}{g^2(\mu)} + 2\pi^2 \langle\Phi\rangle^2 \rho^2.$$

- The total rate goes like $\Gamma_{inst} \propto |\mathcal{A}|^2 \propto \exp(-16\pi^2/g_L^2)$.

The Effect of Fermions

- Fermions coupled to A_μ change this picture drastically.

- The path integral now includes an integration over the fermion fields

$$\int [\mathscr{D}A\, \mathscr{D}\Phi]_n^{n+1} \to \int [\mathscr{D}A\, \mathscr{D}\Phi]_n^{n+1} [\mathscr{D}\bar{\psi} \mathscr{D}\psi],$$

  and there is a new term in the action,

$$S^E_{fermion} = -\int d^4x\; \bar{\psi} \not{D}\, \psi$$

  where $\not{D} \simeq \bar{\sigma}^\mu (\partial_\mu + igA^{cl}_\mu)$.

- Expanding ψ in eigenmodes, $\not{D} f_n = \lambda_n\, f_n$,

$$\psi(x) = \sum_n a_n\, f_n(x), \quad S^E_{fermion} = -\sum_n \lambda_n \bar{a}_n a_n, \quad \int [\mathscr{D}\psi] \to \prod_n \int da_n.$$

- If there are fermion zero modes, $\not{D} f_0(x) = 0 \cdot f_0(x) = 0$,

  the a_0 fermionic integral is unsaturated and the amplitude vanishes.

- The number of zero modes (in a one-instanton background) is

$$2\sum_r S(r), \qquad \text{where} \quad S(r)\,\delta^{ab} = tr(t_r^a t_r^b),$$

where the sum runs over chiral fermion representations.

- To get a non-zero amplitude, we must insert fermionic operators, one for each zero mode, in the path integral.

- Thus, the vacuum amplitude is zero, but there are non-zero fermion Green's functions due to the instanton.

- Equivalently, fermions are (destroyed) created in each (anti-)instanton transition.

Instantons in the Standard Model

- $SU(2)_L$ instantons violate B and L by $n_g = 3$ units each:

  $Q \to 3$ zero modes, $B = 1/3$, $L = 0$,

  $L \to 1$ zero mode, $B = 0$, $L = 1$.

- The SM $SU(2)_L$ instanton rate is unobservably small,

$$\Gamma_{inst} \propto e^{-16\pi^2/g_L^2} \sim 10^{-160}.$$

- B and L violation would be significant if g_L were larger.

A Gauge Extension of the SM:

$$SU(3)_c \times SU(2)_1 \times SU(2)_2 \times U(1)_Y$$

$SU(2)_1 \times SU(2)_2$

- We focus on the electroweak gauge group $SU(2)_1 \times SU(2)_2 \times U(1)_Y$. (*e.g.* topflavour, NCETC)

- $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ fermions:

$SU(2)$ doublets

$$Q_L^3 = \begin{pmatrix} t \\ b \end{pmatrix} = (\mathbf{2},\mathbf{1})_{1/6}, \qquad L_L^3 = \begin{pmatrix} \nu^\tau \\ \tau \end{pmatrix} = (\mathbf{2},\mathbf{1})_{-1/2}$$

$$Q_L^{1,2} = \begin{pmatrix} u \\ d \end{pmatrix} = (\mathbf{1},\mathbf{2})_{1/6}, \qquad L_L^{1,2} = \begin{pmatrix} \nu \\ e \end{pmatrix} = (\mathbf{1},\mathbf{2})_{-1/2}$$

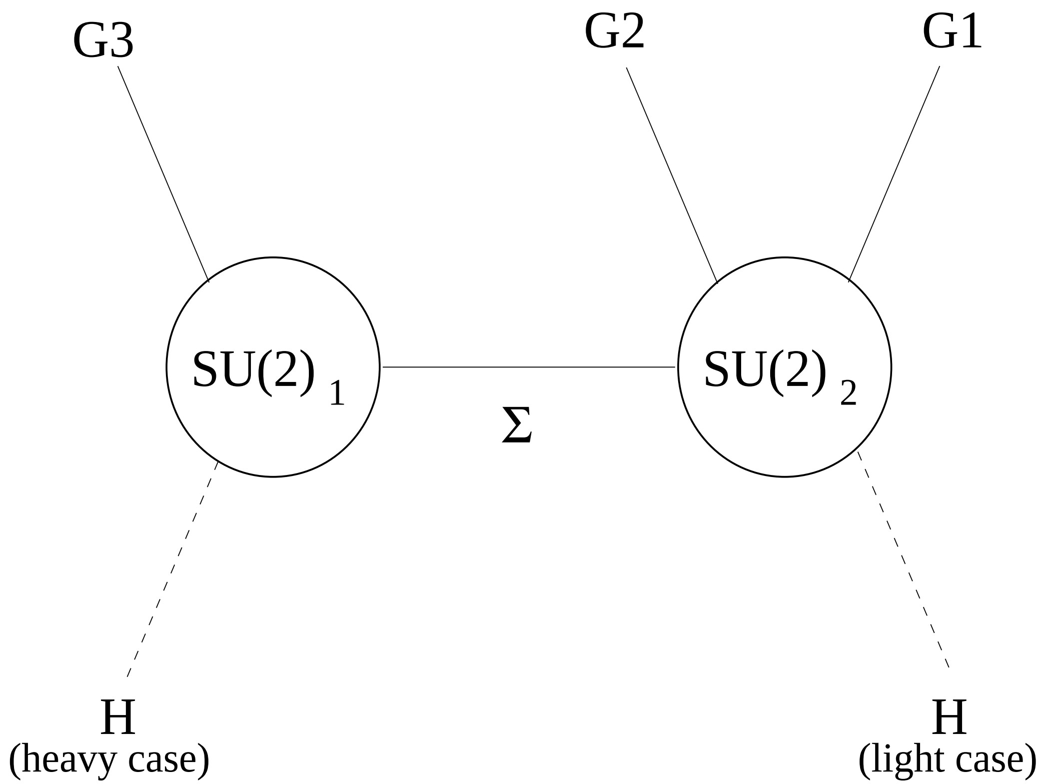
$SU(2)$ singlets

$$u_R^{1,2,3} = (\mathbf{1},\mathbf{1})_{2/3}, \quad d_R^{1,2,3} = (\mathbf{1},\mathbf{1})_{-1/3}, \quad e_R^{1,2,3} = (\mathbf{1},\mathbf{1})_{-1}.$$

- $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ scalars:

$$\Sigma = (\mathbf{2},\mathbf{2})_0,$$

$$H = (\mathbf{2},\mathbf{1})_{1/2} \quad \text{or} \quad (\mathbf{1},\mathbf{2})_{1/2}$$



G3
G2
G1
SU(2) 1
Σ
SU(2) 2
H
(heavy case)
H
(light case)

- $SU(2)_1 \times SU(2)_2 \to SU(2)_L$ when the bi-doublet field Σ gets a VEV,

$$\Sigma_{i\bar{j}} \to \left\langle \Sigma_{i\bar{j}} \right\rangle = u\,\delta_{i\bar{j}},$$

- The gauge couplings are related by

$$g_L = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}.$$

- $SU(2)_L \times U(1)_Y \to U(1)_{em}$ when the Higgs field H develops a VEV,

$$H \to \langle H \rangle = v \simeq 174 \text{ GeV}.$$

- Precision electroweak measurements give the constraint
  [Chivukula,Simmons,Terning '95; Malkawi,Tait,Yuan '96; Malkawi+Yuan '00]

$$u \gtrsim 2\text{–}3 \text{ TeV}.$$

Instantons in $SU(2)_1 \times SU(2)_2$

- In the extended gauge theory, $SU(2)_L$ instantons correspond to combined $SU(2)_1$-$SU(2)_2$ instantons.

- There are also new instantons for $SU(2)_1$ or $SU(2)_2$ alone, and these also violate B and L.

$$
\begin{aligned}
SU(2)_1: &\quad \Delta B = \Delta L = 1\,\Delta N_{CS},\ \Gamma \propto e^{-16\pi^2/g_1^2} \\
SU(2)_2: &\quad \Delta B = \Delta L = 2\,\Delta N_{CS},\ \Gamma \propto e^{-16\pi^2/g_2^2} \\
SU(2)_L: &\quad \Delta B = \Delta L = 3\,\Delta N_{CS},\ \Gamma \propto e^{-16\pi^2/g_L^2} = e^{-16\pi^2(1/g_1^2+1/g_2^2)}
\end{aligned}
$$

- Rates for B and L violation may be large if $g_1 \gg g_2$ or $g_2 \gg g_1$.

$SU(2)_1$ Instantons

Effective Operators

- $SU(2)_1$ couples only to the third generation fermions.
- The fermion Green's function due to an $SU(2)_1$ (anti-)instanton corresponds to the effective operator

$$\mathcal{O}_{eff} = \frac{\tilde{C}}{g_1^8} e^{-8\pi^2/g_1^2} \frac{1}{u^2} \epsilon^{abc} \Big[(t_L^a \cdot \tau_L)(t^b \cdot b_L^c) + (b_L^a \cdot \nu_L^\tau)(b_L^b \cdot t_L^c) \Big],$$

  where ϵ^{abc} contracts over colours, $\tilde{C}$ is dimensionless, and u is the $SU(2)_1$ breaking scale.

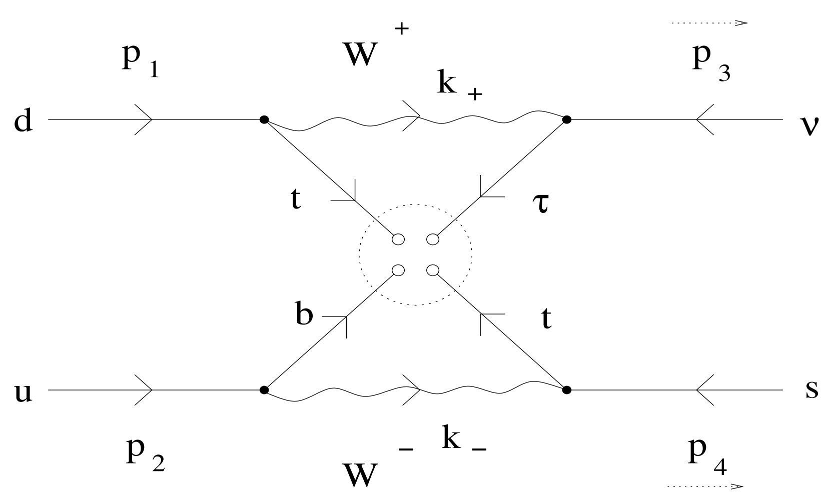
- This operator violates B and L by one unit each, and is invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- Physical consequences:
  - nucleon decay
  - $(B + L)$ violating scattering in colliders

Proton Decay

- A $\Delta B = \Delta L = 1$ operator is precisely what is needed to induce proton decay.

- However, the $SU(2)_1$ instanton operator couples only to third-generation fermions.

- Loops with $W^{\pm}$ gauge bosons connect these operators to the first and second generations. (The tree-level contribution is unknown.)

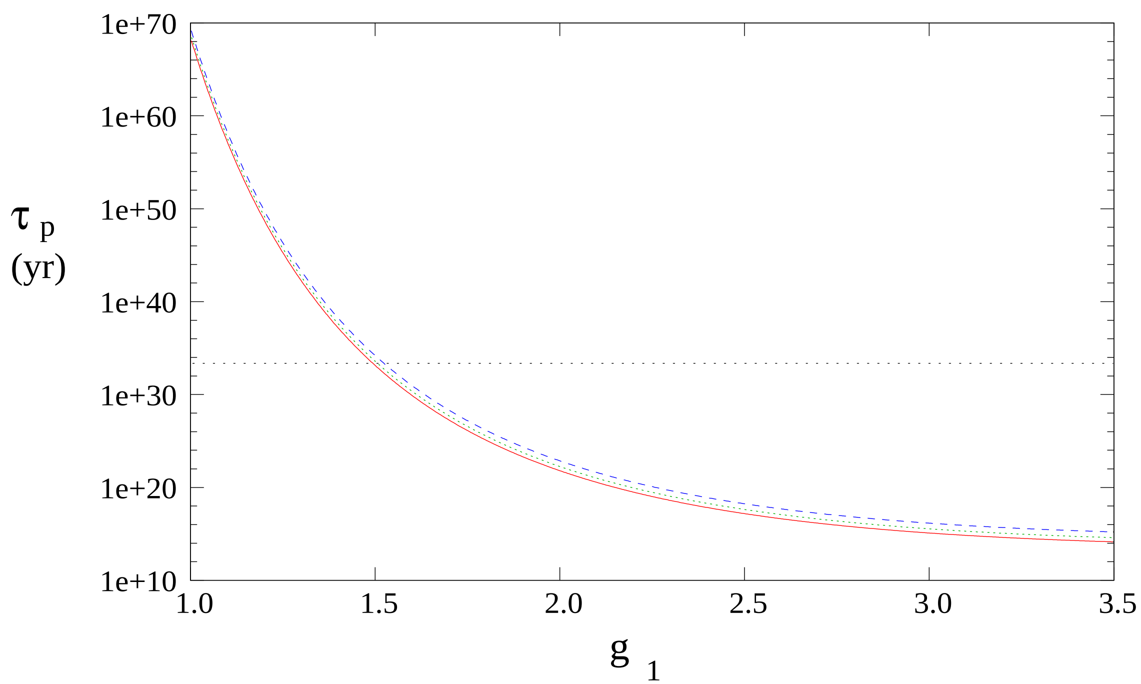


- The dominant decay mode is $p \to K^{+}\bar{\nu}^{\tau}$.

- The current lower bound on the proton lifetime is [Super-Kamiokande '05]

$$\tau_p \geq 2.3 \times 10^{33}\ yr \qquad (p \to K^+ \bar{\nu}^\tau,\ \ 90\%\ c.l.)$$

- For $u =$ 2 TeV, 3 TeV, 5 TeV the instanton induced decay rate is



- This translates into the limit $g_1 \lesssim 1.5$.

$(B+L)$-Violation at the LHC

- The instanton operator can also mediate scattering events.
- This operator involves only third generation fermions,

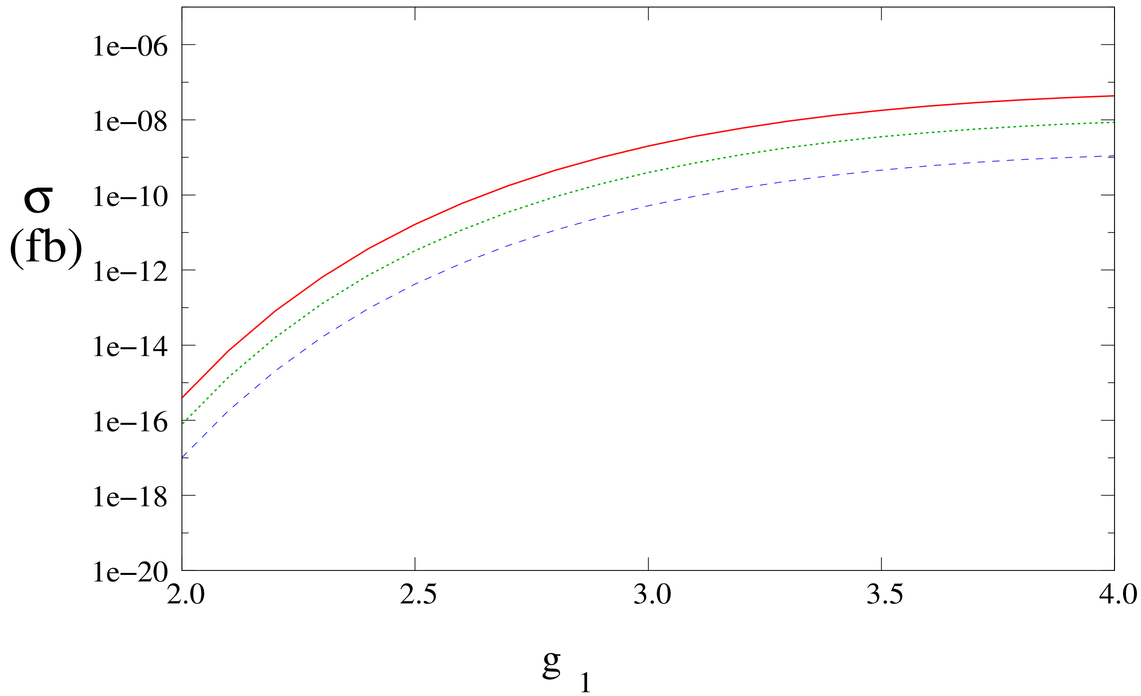
$$\mathcal{O}_{eff} \sim bbt\nu^{\tau} + ttb\tau.$$

- At the LHC, the initial state is pp.

  The proton PDF has a very small component of b and $\bar{b}$.
- The dominant scattering channel is $bb \to \bar{t}\bar{\nu}^{\tau}$.

  $\Rightarrow$ new channel for single top production.

- For $u =$ 2 TeV, 3 TeV, 5 TeV, the rate for $pp \to \bar{t}\,\bar{\nu}^{\tau}$ is



$SU(2)_2$ Instantons

$SU(2)_2$ Effective Operators

- $SU(2)_2$ couples to the first and second generation fermions.

- $\Delta B = \Delta L = 2$.

  3 Q zero modes, 1 L zero mode for each generation.

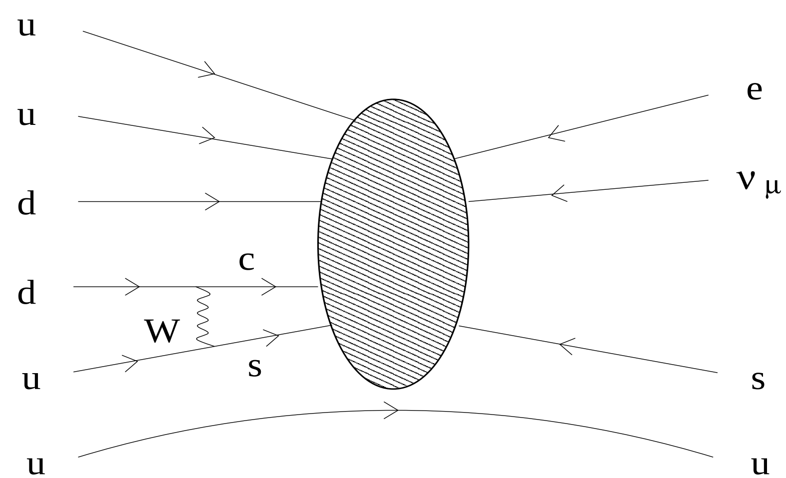
$$\mathcal{O}_{eff} = \frac{\tilde{C}'}{g_2^8}\, e^{-8\pi^2/g_2^2(\mu)}\, \frac{1}{u^8} \cdot$$
$$\cdot \Big[(u_L u_L d_L e_L)(c_L c_L s_L \mu_L) + (u_L d_L d_L \nu_L^e)(c_L s_L s_L \nu_L^\mu) + \ldots \Big].$$

- Note the further suppression by a factor of $1/u^8$.

- Potential effects:
  - Di-Nucleon Decay
  - $(B + L)$ violation at the LHC

Di-Nucleon Decay

- Since the $SU(2)_2$ instanton violates B by two units, it does not mediate proton decay.
- Instead, these instantons can induce di-nucleon decay.



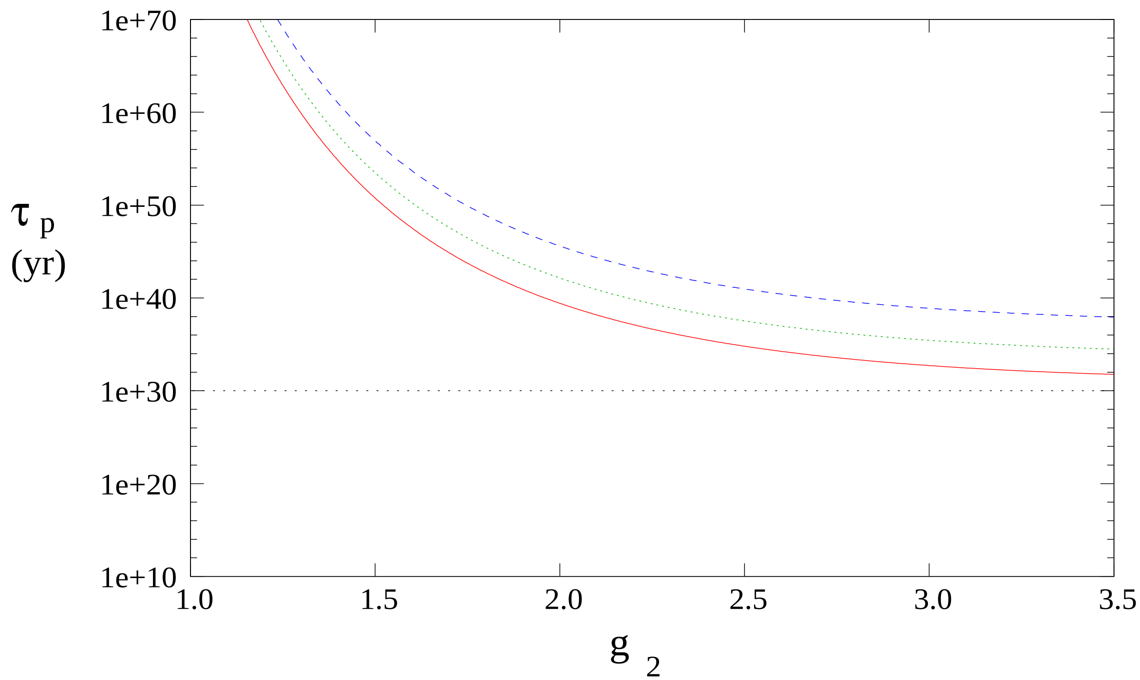
- The decay rate goes like

$$\Gamma_{pp} \sim m_p \left(\frac{m_p}{u}\right)^{16} e^{-16\pi^2/g_2^2}.$$

- The strongest limits on such a decay come from the Fréjus experiment, which looked for decaying nuclei in ^{56}Fe.
- Their limit is [Fréjus '91]

$$\tau_{pp} \geq 10^{30}\ yr.$$

- A more careful estimate of the di-nucleon lifetime gives



Events at the LHC

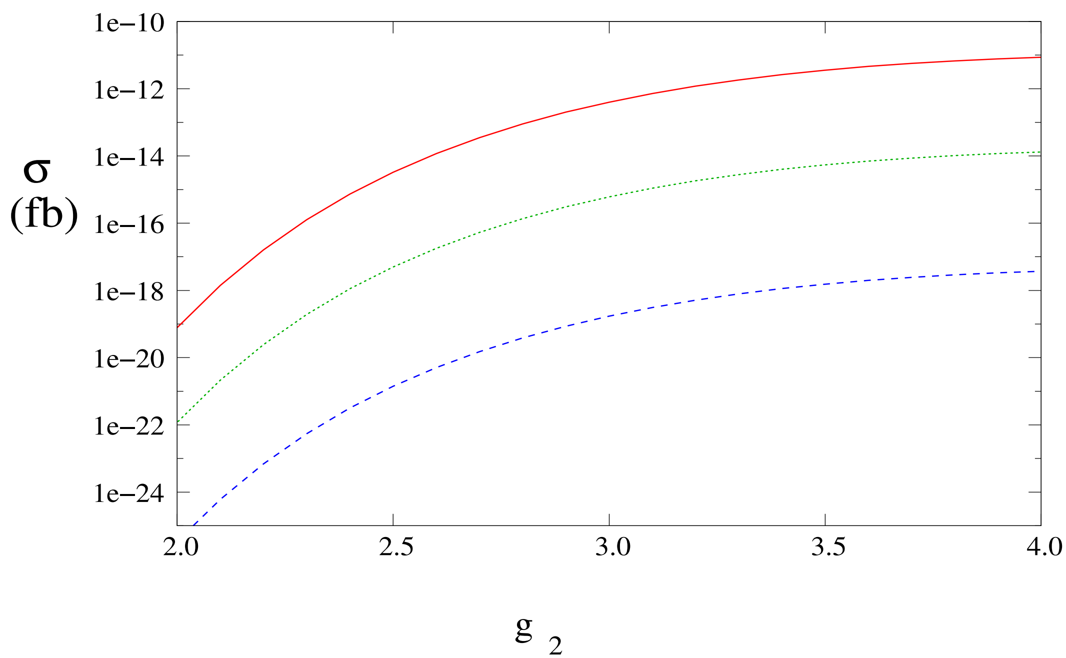
- $SU(2)_2$ instantons also mediate $(B+L)$ violating collider events.

$$e.g. \qquad u\,u \rightarrow \bar{d}\,\bar{c}\,\bar{s}\,\bar{s}\,e^{+}\,\mu^{+}.$$

- There is much less suppression due to the proton PDF in this case.
- There is a suppression by powers of u,

$$\sigma_{parton} \propto \frac{1}{s}\left(\frac{\sqrt{s}}{u}\right)^{16} e^{-16\pi^2/g_2^2}.$$

- For $u =$ 2 TeV, 3 TeV, 5 TeV, the rate is



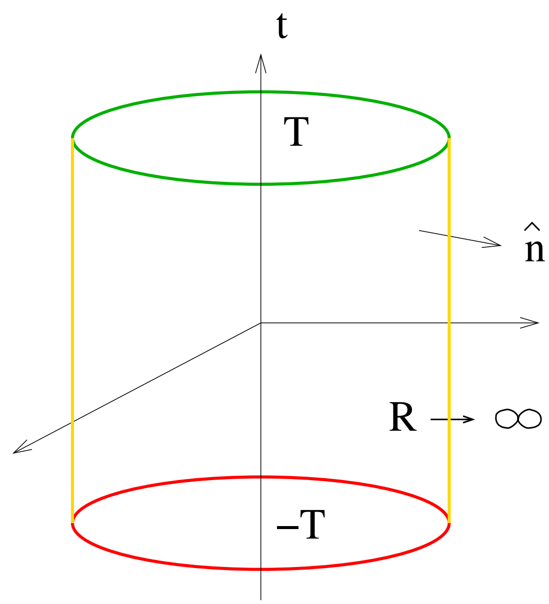
Summary

- Extending the gauge group of the SM can lead to interesting new non-perturbative phenomena.
- Nucleon decay limits put constrain the gauge coupling of an extended gauge group coupling to a single SM generation.
- These constraints can be avoided by coupling more light fermions to the extended gauge group.
- Collider cross-sections seem to be unobservably small if the extended gauge structure is broken at scale $u \gtrsim$ TeV.
- We have considered only exclusive collider processes. It has been argued that multi-instanton inclusive rates could be much larger. [Ringwald '89, Espinosa '90]

Extra Slides

Instantons and (B+L) Violation

- Consider integrating the anomaly equation over a 4-cylinder with axis along the time direction and infinite radius.



$$\int_{cyl} d^4x \; \partial_\mu j^\mu_{B,L} = \frac{g_L^2}{32\pi^2} n_g \int_{cyl} d^4x \; F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu} = \frac{g_L^2}{32\pi^2} n_g \int_{cyl} d^4x \; \partial_\mu K^\mu$$

- In $A^0 = 0$ gauge, we find using the divergence theorem

$$\begin{aligned} L.H.S. &= \int_{-T}^{T} dt \int d^3x\, \partial_\mu j^\mu \\ &= \int_{top} d^3x\, j^0(T) - \int_{bottom} d^3x\, j^0(-T) + \int_{side} da\, \hat{n} \cdot \vec{j} \\ &= Q(T) - Q(-T) + 0 \end{aligned}$$

$$\begin{aligned} R.H.S. &= \int_{-T}^{T} dt \int d^3x\, \partial_\mu K^\mu \\ &= \int_{top} d^3x\, K^0(T) - \int_{bottom} d^3x\, K^0(-T) + \int_{side} da\, \hat{n} \cdot \vec{K} \\ &= N_{CS}(T) - N_{CS}(-T) + 0 \end{aligned}$$

- Equating both sides,

$$\Delta Q_B = \Delta Q_L = n_g\, \Delta N_{CS}$$

$\Rightarrow$ $SU(2)_L$ instantons are responsible for violating $(B + L)$!

* (There are also QCD instantons, but these do not violate B or L.)

Fermion Zero Modes

- Let us expand the fermions in eigenmodes of $\not{D}$,

$$\psi(x) = \sum_n a_n f_n(x),$$
$$\bar{\psi}(x) = \sum_m b_m g_m(x),$$

where a_n and b_n are Grassmann variables and

$$\not{D} f_n(x) = \lambda_n g_n(x), \qquad \int d^4x\, f_n f_m = \delta_{mn},$$
$$\bar{\not{D}} g_n(x) = -\lambda_n f_n(x), \qquad \int d^4x\, g_n g_m = \delta_{mn}.$$

- There is a one-to-one correspondence between the f_n and g_n modes unless one or more of the f_n or g_n eigenvalues is zero.

- Suppose there is one f_0 mode with $\lambda_0 = 0$, but no g_0 eigenmode.

- The fermion path integral measure is given by

$$\int [\mathscr{D}\bar{\psi}][\mathscr{D}\psi] = \prod_{m,n} \int da_n \, db_m,$$

while the action becomes

$$\begin{aligned} S_{fermion} &= -\int d^4x \; \bar{\psi} \not{D} \, \psi \\ &= -\sum_{m,n} b_m \, a_n \int d^4x \; g_m \, \lambda_n \, g_n \\ &= -\sum_{m} b_m \, \lambda_m \, a_m. \end{aligned}$$

- Using the rules of Grassmann integration, $\int d\xi = 0$, $\int d\xi \, \xi = 1$, we get

$$\int [\mathscr{D}\bar{\psi}\mathscr{D}\psi] \, e^{-S_{fermion}} = \int da_0 \cdot \prod_{n \geq 1} \lambda_n.$$

- This vanishes! There are no fermions to saturate the a_0 integral. The result would be non-zero if we had inserted a $\psi(x)$ field.