Gauge-Higgs Unification on Flat Space Revised

Giuliano Panico

ISAS-SISSA Trieste, Italy

The 14th International Conference on Supersymmetry and the Unification of Fundamental Interactions
Irvine, 12-17 June 2006

Based on Nucl. Phys. B739 (2006) 186 and hep-ph/0605292
with M. Serone and A. Wulzer

Outline

Why Extra Dimensions and GHU?

SM problems

- Hierarchy of fermion masses
- Stabilization of the electroweak scale

A possible solution

- TeV-sized extra dimensions compactified on $S^1/\mathbf{Z}_2$
- Higgs as a wilson line phase (GHU models)

Main features

- Masses generated by non-local effects
- Higgs mass protected by gauge invariance

Why Extra Dimensions and GHU?

SM problems

- Hierarchy of fermion masses
- Stabilization of the electroweak scale

A possible solution

- TeV-sized extra dimensions compactified on $S^1/\mathbf{Z}_2$
- Higgs as a wilson line phase (GHU models)

Main features

- Masses generated by non-local effects
- Higgs mass protected by gauge invariance

Problems of GHU Models on a Flat Space

Minimal realizations of GHU have common drawbacks.

[Scrucca, Serone and Silvestrini (2003)]

- Yukawa couplings constrained by gauge and Lorentz invariance
  - top mass too small
  - Higgs mass below experimental bounds
- Too low compactification scale

Some attempts to solve these problems:

- Large localized kinetic terms [Scrucca et al.]
  - Unwanted distorsions of wave functions
- Fermions in high rank representations [Cacciapaglia et al.]
  - Very low cut-off

Getting Larger Yukawa's

Break the Lorentz symmetry in the bulk: $SO(4,1) \rightarrow SO(3,1)$

- Fermions: $\overline{\Psi}\left[i \not{D}_4 - k D_5 \gamma^5\right] \Psi$
- Gauge fields: $-\frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} - \rho^2 \mathrm{Tr}\, F_{\mu 5} F^{\mu 5}$

- The top mass is increased: $m_t \lesssim k_t m_W$
- The Higgs effective quartic coupling gets larger

Getting Larger Yukawa's

Break the Lorentz symmetry in the bulk: $SO(4,1) \to SO(3,1)$

- Fermions: $\overline{\Psi}\left[i\,\not{D}_4 + k\,D_5\gamma^5\right]\Psi$
- Gauge fields: $-\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \rho^2\,\mathrm{Tr}\,F_{\mu 5}F^{\mu 5}$

- The top mass is increased: $m_t \lesssim k_t m_W$
- The Higgs effective quartic coupling gets larger

Getting Larger Yukawa's

Break the Lorentz symmetry in the bulk: $SO(4,1) \to SO(3,1)$

- Fermions: $\overline{\Psi} \left[i \not{D}_4 + k \not{D}_5 \gamma^5 \right] \Psi$
- Gauge fields: $-\frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \rho^2 \mathrm{Tr}\, F_{\mu 5} F^{\mu 5}$

- The top mass is increased: $m_t \lesssim k_t m_W$
- The Higgs effective quartic coupling gets larger

The "Mirror Symmetry"

Addressing the compactification scale problem

- Doubling part of the bulk fields: $\phi \to \phi_1,\ \phi_2$

  and imposing twisted boundary conditions

$$\phi_1(y \pm 2\pi R) = \phi_2(y), \quad \phi_1(-y) = \pm\phi_2(y)$$

- Requiring interchange symmetry: $\phi_1 \leftrightarrow \phi_2$

Twisted fields give periodic and antiperiodic fields $\phi_\pm = \dfrac{\phi_1 \pm \phi_2}{\sqrt{2}}$ with charges ± 1 under the "mirror" $\mathbf{Z}_2$

- An order of magnitude hierarchy between the EW scale and $1/R$ is completely natural

The "Mirror Symmetry"

Addressing the compactification scale problem

- Doubling part of the bulk fields: $\phi \to \phi_1, \ \phi_2$

  and imposing twisted boundary conditions

$$\phi_1(y \pm 2\pi R) = \phi_2(y), \quad \phi_1(-y) = \pm\phi_2(y)$$

- Requiring interchange symmetry: $\phi_1 \leftrightarrow \phi_2$

Twisted fields give periodic and antiperiodic fields $\phi_\pm = \dfrac{\phi_1 \pm \phi_2}{\sqrt{2}}$ with charges ± 1 under the "mirror" $\mathbf{Z}_2$

- An order of magnitude hierarchy between the EW scale and $1/R$ is completely natural

The Gauge Lagrangian

Gauge group $SU(3)_w \times G_1 \times G_2$ with $G_i = U(1)_i \times SU(3)_{i,s}$

Non trivial $\mathbf{Z}_2$ orbifold projection on $SU(3)_w$

$$A_w^M(-y) = (-)^{\delta_{M,y}} P A_w^M(y) P^\dagger$$

$$P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Longrightarrow SU(3)_w \to SU(2)_L \times U(1)_w$$

Surviving gauge group at fixed points

- $y=0$: $SU(2)_L \times U(1)_w \times G_+$ (G_+ diag. subgr. of G_1 and G_2)
- $y=\pi R$: $SU(2)_L \times U(1)_w \times G_1 \times G_2$

The Fermion Lagrangian (Bulk)

$(\Psi, \widetilde{\Psi})$ fermion pairs in the bulk:
same quantum numbers and opposite parity

"Mirror" symmetry doubling

- $(\Psi_1, \widetilde{\Psi}_1)$ charged under G_1
- $(\Psi_2, \widetilde{\Psi}_2)$ charged under G_2

For each quark family

- $(\Psi^t_{1,2}, \widetilde{\Psi}^t_{1,2}) \Rightarrow \mathbf{\overline{3}}$ rep. of $SU(3)_w$
- $(\Psi^b_{1,2}, \widetilde{\Psi}^b_{1,2}) \Rightarrow \mathbf{6}$ rep. of $SU(3)_w$

both in the fund. rep. of $SU(3)_{1,2,s}$ and with $U(1)_{1,2}$ charge $+1/3$

The Fermion Lagrangian (Boundary)

Boundary fermions at $y = 0$

- one $SU(2)_L$ doublet: $Q_L = (t_L, \ b_L)^t$
- two $SU(2)_L$ singlets: t_R and b_R

They have $\mathbf{Z}_2$ “mirror” charge $+1 \Rightarrow$ couple only to Ψ_+ fields

Bulk-boundary couplings

$$
\begin{array}{ccccccccc}
\Psi^t_+, \widetilde{\Psi}^t_+ : & \overline{\mathbf{3}}_{1/3} & \rightarrow & \mathbf{2}_{1/6} & \oplus & \mathbf{1}_{2/3} & & \\
 & & & \updownarrow & & \updownarrow & & \\
 & & & Q_L & & t_R & & b_R \\
 & & & \updownarrow & & & & \updownarrow \\
\Psi^b_+, \widetilde{\Psi}^b_+ : & \mathbf{6}_{1/3} & \rightarrow & \mathbf{2}_{1/6} & \oplus & \mathbf{3}_{2/3} & \oplus & \mathbf{1}_{-1/3}
\end{array}
$$

The EW Symmetry Breaking

The 4D gauge group is the surviving group at $y = 0$:

$$SU(3)_{QCD} \times SU(2)_L \times U(1)_Y \times U(1)_X$$

($U(1)_Y$ is the diagonal subgroup of $U(1)_w$ and $U(1)_+$)

The extra $U(1)_X$ gauge group is broken by anomaly

- it is needed to get the correct Weinberg angle

The Higgs doublet

- identified with the $A_y^{4,5,6,7}$ components of the $SU(3)_w$ gauge group

The EW Symmetry Breaking

The 4D gauge group is the surviving group at $y = 0$:

$$SU(3)_{QCD} \times SU(2)_L \times U(1)_Y \times U(1)_X$$

($U(1)_Y$ is the diagonal subgroup of $U(1)_w$ and $U(1)_+$)

The extra $U(1)_X$ gauge group is broken by anomaly

- it is needed to get the correct Weinberg angle

The Higgs doublet

- identified with the $A_y^{4,5,6,7}$ components of the $SU(3)_w$ gauge group

The EW Symmetry Breaking

The 4D gauge group is the surviving group at $y = 0$:

$$SU(3)_{QCD} \times SU(2)_L \times U(1)_Y \times \cancel{U(1)_X}$$

($U(1)_Y$ is the diagonal subgroup of $U(1)_w$ and $U(1)_+$)

The extra $U(1)_X$ gauge group is broken by anomaly

- it is needed to get the correct Weinberg angle

The Higgs doublet

- identified with the $A_y^{4,5,6,7}$ components of the $SU(3)_w$ gauge group

The EW Symmetry breaking

- The Higgs potential is generated at one-loop level
  a non-trivial minimum is induced by bulk fermions

Higgs VEV: $\langle A_{w,y} \rangle = \frac{2\alpha}{g_5 R} t^7$ $\Rightarrow$ spontaneous breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$

- W boson mass: $m_W = \frac{\alpha}{R}$

cancellation between periodic and antiperiodic $\Rightarrow$ $\alpha \sim 0.1 \Leftrightarrow 1/R \sim 1\ \mathrm{TeV}$ completely natural

The Universal Parameters

Flavor-conserving new physics can be parametrized by:

[*e.g.* Han and Skiba(2005); Grojean et al.; Cacciapaglia et al.(2006)]

- Universal parameters: $\widehat{S}$, $\widehat{T}$, $\widehat{U}$, V, X, W and Y
- distortion of the Z^0 couplings: C_q, $\delta\varepsilon_q$, $\delta\varepsilon_b$

In Our Model:

Light quarks are nearly exactly localized

coupling to the Z^0 unperturbed $\Rightarrow$ C_q, $\delta\varepsilon_q$ negligible

b quark partially delocalized $\Rightarrow$ $Z\bar{b}_L b_L$ vertex modified by

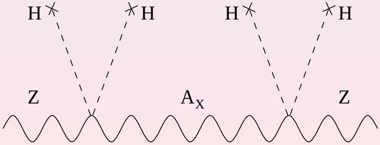
- Mixing with the A_X tower (anomaly)
- Mixing with fields with different $SU(2)_L$ charges

▸ $\delta\varepsilon_b$ is negligible for $1/R \gtrsim 3\ \mathrm{TeV}$

Universal parameters

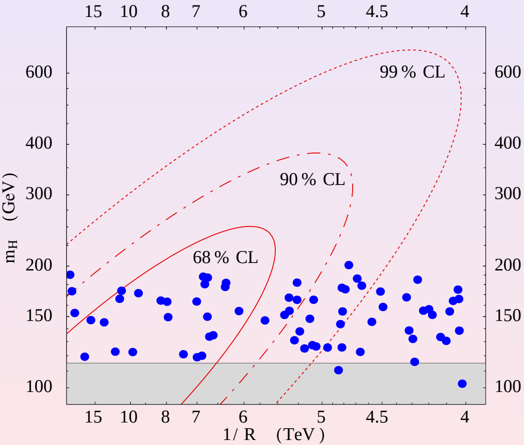
Relevant universal parameters in our model (compared to generic flat ED model [Barbieri et al. (2004)])

$$\begin{cases} \widehat{S} = \frac{2}{3}\pi^2\alpha^2 \left(= \frac{2}{3}\pi^2\alpha^2 \right) \\ \widehat{T} = \pi^2\alpha^2 \left(\gg \frac{1}{3}\tan^2(\theta_W)\pi^2\alpha^2 \right) \end{cases} \qquad \begin{cases} W = \frac{1}{3}\pi^2\alpha^2 \left(= \frac{1}{3}\pi^2\alpha^2 \right) \\ Y \simeq \frac{1}{3}\pi^2\alpha^2 \left(= \frac{1}{3}\pi^2\alpha^2 \right) \end{cases}$$



$\Rightarrow$ The large value of $\widehat{T}$ is explained by the distorsion of the ρ parameter

EWPT fit



- **Bounds**
  - Compatification scale $1/R \gtrsim 4 - 5\ \mathrm{TeV}$
  - Allowed Higgs masses up to $600\ \mathrm{GeV}$
- **Predictions (blue dots)**
  - The bounds can be satisfied with some tuning ($O(\mathit{few}\ \%)$)
  - The Higgs mass is in the range $100 - 200\ \mathrm{GeV}$

Summary

- Common problems of GHU models on 5D flat orbifolds can be solved by:
  - breaking the $SO(4,1)$ Lorentz symmetry in the bulk,
  - introducing a $\mathbf{Z}_2$ “mirror” symmetry.
- In this way a realistic model has been constructed which is compatible with the EW experimental measurements.
- The compactification scale has a bound $1/R \gtrsim 4 - 5\ \mathrm{TeV}$; the Higgs mass is predicted in the range $100 - 200\ \mathrm{GeV}$.

Outlook

- Some amount of fine-tuning is required to satisfy the EW precision tests. Is it possible to do better?
- Due to the mirror symmetry, the first Kaluza-Klein modes of the antiperiodic fields are stable.
  Can they provide a viable dark matter candidate?
- Find an appropriate way to introduce the flavour structure.

Cut-off

One-loop polarization corrections for the gauge fields compared with the tree level couplings (Pauli-Villard regularization)

$$
\begin{aligned}
(\Lambda_s^{(\mu)} R)\frac{\alpha_s}{6}\frac{1}{2}\left(\frac{12}{k_t}+\frac{24}{k_b}\right) \sim 1 \quad &\Rightarrow \quad \Lambda_s^{(\mu)} \sim \frac{6}{R}\,,\\
(\Lambda_s^{(5)} R)\frac{\alpha_s}{6}\frac{1}{2}\left(12 k_t+24 k_b\right) \sim \rho_s^2 \quad &\Rightarrow \quad \Lambda_s^{(5)} \sim \frac{3\rho_s^2}{R}\,,\\
(\Lambda_w^{(\mu)} R)\frac{\alpha_w}{6}\frac{1}{2}\left(\frac{12}{k_t}+\frac{60}{k_b}\right) \sim 1 \quad &\Rightarrow \quad \Lambda_w^{(\mu)} \sim \frac{5}{R}\,,\\
(\Lambda_w^{(5)} R)\frac{\alpha_w}{6}\frac{1}{2}\left(12 k_t+60 k_b\right) \sim 1 \quad &\Rightarrow \quad \Lambda_w^{(5)} \sim \frac{4}{R}\,.
\end{aligned}
$$

For $k_t \simeq 2-3$ and $k_b \simeq 1$ the cut-off is $\Lambda \sim \frac{4}{R}$

Is Our Model Really 5*D*?

The Lorentz symmetry breaking can have a simple origin

We introduce an axion-like field Φ

- invariant under the shift $\Phi \to \Phi + 2\pi$
- with periodicity conditions $\Phi(y + 2\pi R) = \Phi(y) + 2\pi$

The background configuration is $\Phi_0(y) = \dfrac{y}{R}$

▶ This generates a spontaneous symmetry breaking of the $SO(4,1)/SO(3,1)$ Lorentz symmetry.

The Lorentz-violating factors can be reinterpreted as

$$\frac{\alpha}{f_\Phi^2}\,\partial_M\Phi\partial_N\Phi\,\overline{\Psi}\gamma^M D^N \Psi$$

Fine-Tuning and Sensitivity

- Using the logarithmic derivative to quantify the fine-tuning [Barbieri and Giudice (1988)]

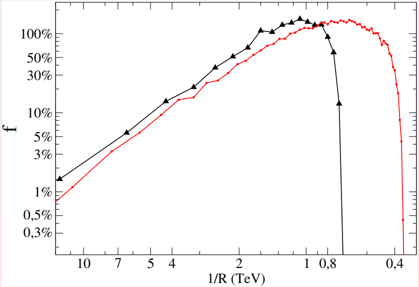
$$C \equiv \mathrm{Max}\left\{\left|\frac{\partial \log \alpha}{\partial \log i}\right|\right\}$$

at $1/R \sim 5\ \mathrm{TeV}$ one finds a fine-tuning $f = 1/C \sim O(1\%)$

- With a more refined procedure [Anderson and Castano (1995)]

$$f = \frac{\alpha\rho(\alpha)}{\langle\alpha\rho(\alpha)\rangle}$$

a tuning of $O(10\%)$ is found



Embedding Leptons

Bulk fields

- $(\Psi^l_{1,2}, \widetilde{\Psi}^l_{1,2}) \Rightarrow \mathbf{3}$ rep. of $SU(3)_w$
- $(\Psi^\nu_{1,2}, \widetilde{\Psi}^\nu_{1,2}) \Rightarrow \overline{\mathbf{6}}$ rep. of $SU(3)_w$

Localized fields (y=0)

- doublet: $L_L \equiv (e_L, \nu^t_L)$
- singlets: e_R and ν_R

Bulk-boundary couplings

$$
\begin{array}{ccccccccc}
\Psi^l_+, \widetilde{\Psi}^l_+ : & \mathbf{3}_{-2/3} & \rightarrow & \mathbf{2}_{-1/2} & \oplus & \mathbf{1}_{-1} & & \\
 & & & \updownarrow & & \updownarrow & & \\
 & & & L_L & & e_R & & \nu_R \\
 & & & \updownarrow & & & & \updownarrow \\
\Psi^\nu_+, \widetilde{\Psi}^\nu_+ : & \overline{\mathbf{6}}_{-2/3} & \rightarrow & \mathbf{2}_{-1/2} & \oplus & \mathbf{3}_{-1} & \oplus & \mathbf{1}_0
\end{array}
$$

The Effective Potential

Contributions to the effective potential (massless fermions)

- periodic: $$V_P(\alpha) = \frac{3k_P^4}{8\pi^6 R^4} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(2\pi n q \alpha)$$
- antiperiodic: $$V_A(\alpha) = \frac{3k_A^4}{8\pi^6 R^4} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5} \cos(2\pi n q \alpha)$$

- The leading cosine term cancels out between periodic and antiperiodic contributions if they have $k_P = k_A$.