Flowing to four dimensions

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Outline

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Motivations

Phenomenology of X-dim

- Symmetry breaking pattern generating masses $\approx$ TeV $\ll R^{-1}$, $\Lambda = M_\star$
- 5D : a bulk or a localised mass term has $[\mu^2] = 1$ or 2.
  $\Longrightarrow$ move to 6D : a localised mass parameter has $[\mu^2] = 0$

Features applicable to QCD in 4D

- Perturbative at UV
- Blowing up in the IR
- Generate light masses compared to the relevant scales

Theoretical motivations

- Dynamical generation of small masses in a theory which originally had only heavy degrees of freedom

Outline

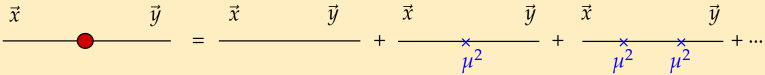
Action and propagator

Action

$$S = \int d^4x d^2y \left[|\partial_M \phi|^2 - \underbrace{\delta^2(\vec{y})}_{\text{"brane"}} \cdot \left(-\mu^2 |\phi|^2 + \frac{\lambda}{2} |\phi|^4 \right) \right]$$

Goldberger and Wise (2001)

Mixed representation : 4D momentum (k) x 2D coordinates ($\vec{x}$)



$$G_k^{(2)}(\vec{x},\vec{y}) = \overbrace{D_k(\vec{x},\vec{y})}^{\text{free 6D propagator}} + \frac{\mu^2}{1-\mu^2 D_k(\vec{0},\vec{0})} D_k(\vec{x},\vec{0}) D_k(\vec{0},\vec{y})$$

Introduce UV cut-off $\Lambda \Rightarrow D_k(\vec{0},\vec{0}) = \frac{1}{4\pi} \ln \frac{\Lambda^2}{k^2}$ for infinite X-dim.

The settings

Compact extra dimensions

Construct the theory on a disk with radius $R \gg \Lambda^{-1}$. Impose the Dirichlet boundary condition $\phi(r = R) = 0$. The behaviour of the propagator at the singularity becomes

$$D_k(\vec{0},\vec{0}) \sim \frac{1}{4\pi} \ln \frac{\Lambda^2}{k^2} \qquad k^2 \gg 1/R^2$$

$$D_k(\vec{0},\vec{0}) \sim \frac{1}{4\pi} \ln \Lambda^2 R^2 + \mathcal{O}(k^2 R^2) \qquad k^2 \ll 1/R^2$$

$\Longrightarrow$ Log scale dependence typical of the RG, true at the classical level.

RG flow at the classical level

Classical running

- Seen from the resummation

$$\mu^2(Q) = \frac{\mu^2(\Lambda)}{1 + \frac{\mu^2(\Lambda)}{2\pi} \ln \frac{Q}{\Lambda}}$$

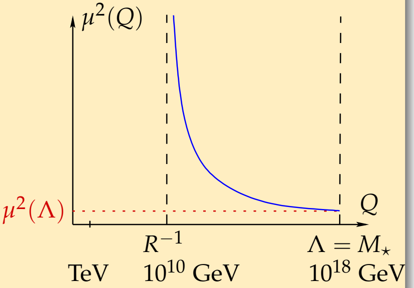
$\mu^2(\Lambda) \equiv \mu^2 \ll 1$: the coupling grows in the IR.

- If $\mu^2(R^{-1}) \ll 1 \to$ all $m \sim R^{-1}$.
- What if $\mu^2(R^{-1}) \to \infty$? i.e when

$$\mu^2 = \mu_c^2 = \frac{2\pi}{\ln \Lambda R} .$$

Our claim :
<u>a massless state in the spectrum.</u>

Fine-tuning $\mu^2 = \mu_c^2$



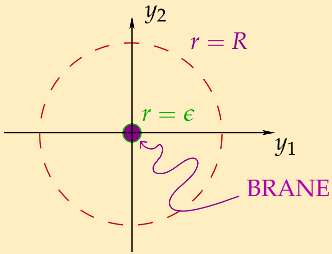
The running picture breaks down at low energies. We should work only with the bare coupling. Identify $\Lambda = 1/\epsilon$, ϵ : thickness of the resolved brane.

Outline

Resolving the brane

Regularised potential

$$V(\phi) = \frac{1}{\pi\epsilon^2}\left(-\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4\right) \quad \text{for } 0 \leq r \leq \epsilon\,,$$
$$V(\phi) = 0 \quad \text{for } \epsilon \leq r \leq R\,.$$



- Decomposition : $\phi(r,\theta) = e^{il\theta}\phi_l(r)$,
- KK zero mode $\sim$ s-wave solution $\phi_0 \equiv \phi$,
- Need to match the solutions at $r = \epsilon$.

Field equations

Equations of motion

$$\begin{cases} -p^2\phi - \triangle_2\phi + \left[-\frac{\mu^2}{\pi\epsilon^2}\phi + \frac{\lambda}{\pi\epsilon^2}\phi^3\right] = 0 & \text{for } 0 \leq r \leq \epsilon\,, \\ -p^2\phi - \triangle_2\phi = 0 & \text{for } \epsilon \leq r \leq R\,, \end{cases}$$

These are Schrödinger equations in 2D.

General solutions

• Drop the $\lambda\phi^4$ term ⇒ background $<\phi>=0$. Since $p^2 \ll \Lambda^2 = \epsilon^{-2}$,

$$\phi(r) = A \cdot J_0\left(\frac{\mu}{\sqrt{\pi}}\,\frac{r}{\epsilon}\right) \quad \text{for} \quad r \leq \epsilon\,,$$

$$\phi(r) = B\left[J_0(|p|r) - \frac{J_0(|p|R)}{N_0(|p|R)} \cdot N_0(|p|r)\right] \quad \text{for} \quad r \geq \epsilon\,,$$

when allowing for any sign of p^2.

Without quartic term

Light mass state

Asking for $|p^2| \ll R^{-2}$, one finds

$$p^2 \equiv m^2_{(4)} = \frac{8\pi}{R^2} \frac{\mu_c^2 - \mu^2}{\mu_c^4} = \frac{8\pi}{R^2} \frac{1}{\mu^2(R^{-1})} \quad .$$

• The zero mode is massless at the critical coupling. The existence of a massless mode is seen as a pole in the propagator at $p^2 = 0$ and $\mu^2 = \mu_c^2$ since

$$\frac{\mu_c^2}{1 - \mu_c^2 D_p(\vec{0}, \vec{0})} \sim \mathcal{O}\left(\frac{1}{p^2 R^2}\right) \quad \text{for } p^2 \ll 1/R^2 \, .$$

• $\mu^2 = \mu_c^2$ is the point of second order phase transition.

Without quartic term

Light mass state

- We recover $m^2 \gg R^{-2}$ for $\mu^2(R^{-1}) \ll 1$.
- For $\mu^2(\Lambda) \gtrsim \mu_c^2$, tachyon $\Rightarrow$ consider the $\lambda\phi^4$ term.

From the boundary condition

Qualitatively, a Dirichlet BC at $r = R \Longleftrightarrow$ a Scherk-Schwartz BC.

Naively at least : $m^2_{(4),\text{naive}} = \frac{z_0^2}{R^2} - \frac{\mu^2}{\pi R^2}$

With quartic term

Symmetry breaking : $\mu^2 > \mu_c^2$

• The field varies only slightly inside the brane $V'(\phi_c) \approx V'(\phi_0)$ for $r \leq \epsilon$:

$$\begin{aligned} \phi_c(r) &= \phi_0 + \frac{1}{4} V'(\phi_0) \cdot r^2 \quad \text{inside ,} \\ \phi_c(r) &= \phi_0 \cdot \frac{\ln R/r}{\ln R/\epsilon} \quad \text{outside .} \end{aligned}$$

• Matching at $r = \epsilon$: $\phi_0^2 = \dfrac{\mu^2 - \mu_c^2}{\lambda}$.

• Mass of the fluctuation $\phi(r,p) = \phi_c(r) + \xi(r,p)$ assuming $m_\xi^2 \ll R^{-2}$:

$$m_\xi^2 = \frac{16\pi}{R^2} \frac{\mu^2 - \mu_c^2}{\mu_c^4} = 2\,|m^2_{\text{tachyon}}|$$

Outline

4D effective theory

Technically

- Decompose $\phi(x^\mu, r) = \sigma(x^\mu) \cdot \zeta(r)$, with ζ the general solution.
- Impose $\int \zeta^2 d^2y = 1$ so that σ has canonical kinetic term.
- Resolve the brane : $\zeta(0) = \zeta(\epsilon)$.

Integration over the X-dim

The 4D effective potential for σ is

$$V_{eff}(\sigma) \; = \; \frac{m^2_{(4)}}{2}\sigma^2 + \frac{\lambda_{(4)}}{4}\sigma^4 \quad ,$$

with $\lambda_{(4)} = \frac{64\pi^2}{\mu_c^8}\, \frac{\lambda}{R^4}$ and $m^2_{(4)}$ found before.

For $\mu^2 > \mu_c^2$, the 4d field develops a vev $< \sigma^2 >= -\frac{m^2_{(4)}}{\lambda_{(4)}}$
and its fluctuation's mass is $m^2_{\zeta} = -2m^2_{(4)}$.
Therefore, the 6D $\leftrightarrow$ 4D equivalence exists in the broken phase.

Outline

KK heavy states

Solutions

• Solve the Schrödinger eq. for $l \neq 0$, at $\mu^2 \leq \mu_c^2$, above $< \phi >= 0$ assuming positive squared mass p^2:

$$\begin{aligned} \phi_l(r) &= A\, J_l\left(\sqrt{p^2 + \frac{\mu^2}{\pi\epsilon^2}}\, r\right) \qquad \text{inside}\,, \\ \phi_l(r) &= B\left[J_l(pr) - \frac{J_l(pR)}{N_l(pR)} N_l(pr)\right] \quad \text{outside}\,. \end{aligned}$$

Matching at $r = \epsilon$, with $M\epsilon \ll 1$ and $\mu^2 \ll 1$

$$\frac{2}{\pi}\,\frac{J_0(MR)}{N_0(MR)} \cdot \left\{\ln\frac{MR}{2} + \frac{\mu_c^2 - \mu^2}{\mu_c^2}\,\ln\Lambda R\right\} = 1\,.$$

At the critical coupling, the KK masses have *no dependence on* Λ

The return of the running

Expression at the pole

$$\frac{J_0(MR)}{N_0(MR)} = \frac{1}{4}\,\mu^2(Q=M)$$

See also :

$$\frac{\mu_c^2}{1-\mu_c^2 D_p(\vec{0},\vec{0})} = \frac{2\pi}{\ln(pR)} \quad \text{for } p^2 \gg 1/R^2\,.$$

Precisely at the critical coupling , all physical quantities become independent on the UV cut-off.

Outline

In a nutshell

Open questions and prospects

- A new class of models which provide a new way to get small masses. Possible application to *electroweak* or *supersymmetry* breaking ?
- Similar light modes for chiral fermions ? $\Longrightarrow$ **yes**
- Incorporate gravity
- How to drive parameters near a critical point ? What would be the *dynamics* or the *symmetries* of such a mechanism ?
- Are such models dual to Nambu-Jona-Lasinio theories ? $\Longrightarrow$ **work in progress**