Black Holes at Future Colliders:

Determination of BH evolution

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Based on Works:
Questions

Suppose that
+ We are living in a brane world in (4+n) dimensional spacetime.
+ Think of two particle collision with Trans–Planckian CM energy.

**Q1.** Estimate the production cross-section of Black Holes.

**Q2.** Briefly describe the Black Hole’s evolution.
**Good Answers**

**A1.** If the impact parameter is smaller than the Schwarzschild radius, BH will be formed. So the answer is as follows:

\[ \sigma \simeq \pi r_s (\sqrt{s})^2 \]

Good Answers

where the size of event horizon is \( r_s \sim (\frac{\sqrt{s}}{M_*})^{1/n+1} \frac{1}{M_*} \).

Thus the answer would be:

\[ \sigma \sim \pi \left(\frac{\sqrt{s}}{M_*}\right)^{2/n+1} \frac{1}{M_*^2} \]

**A2.** BH will lose its energy through Hawking radiation. Since Hawking radiation is essentially thermal, I expect that the signal would be spherically symmetric and every particle would be produced with an equal probability. Thus, roughly 10% leptons, 5% missing energy (neutrino) and 2% photon would be very specific signals of BH production at the LHC.

\[ \sigma \sim \pi \left(\frac{\sqrt{s}}{M_*}\right)^{2/n+1} \frac{1}{M_*^2} \]

\[ \text{Dimopoulos & Thomas 01, Giddings & Landsberg 01} \]
**Better Answers**

**A1.** BH has not only mass but also **angular momentum**.

\[
M = \sqrt{s}, \quad J = \frac{Mb}{2}
\]

\[
\sigma \approx \pi b_{\text{Max}}^2, \quad b \leq 2r_H(\sqrt{s}, J)
\]

\[
r_H(M, J) = r_S(M)/(1 + a_*^2)^{\frac{1}{n+1}}
\]

\[
a_* = (2 + n)J/2Mr_h
\]

\[
\sigma = \pi b_{\text{max}}^2 = 4 \left[1 + \left(\frac{n+2}{2}\right)^2\right]^{-\frac{2}{n+1}} \pi r_S^2
\]

~in good agreement with numerical studies by Giddings+Eardley02, Yoshino+Nambu02
A2. Most of black holes are produced with large angular momentum. 

- Geometry is not spherically symmetric. 
- Hawking radiation is not isotropic, not equally probable.

\[- \frac{d}{dt} \left( \frac{M}{J} \right) = \frac{1}{2\pi} \sum_{s,l,m} g_s \int d\omega \frac{\Gamma_{s,l,m}}{e^{\omega - m\Omega/T} + 1} \left( \omega \right) \]

\(\Gamma_{s,l,m}\) : The probability is not equal to every particle but crucially depends on spin and angular mode.

Anisotropic and nontrivial Hawking radiation is expected. We have to know this “greybody factor” to understand Hawking Radiation.

GOOD NEWS!!!

“greybody factor” for (4+n) dimensions rotating bh, for s=0, ½, 1 (i.e., all the SM particles localized on brane) is now available 😊

=> Ida, Oda, Park-I, II and III. (Also see Dr. Kanti’s nice works)
Caution:

Our calculation is valid only if

\[
\frac{1}{M_*} \ll r_H \ll r_C
\]

Higher Dimensional

Classical

\[
r_H \sim \left(\frac{\sqrt{s}}{M_*}\right)^{1/n+1} \frac{1}{M_*}
\]

⇒ \textbf{Trans-Planckian} Domain.

Large (or mildly warped) extra dimensions.
Greybody factor

\[-\frac{d}{dt} \left( \frac{M}{J} \right) = \frac{1}{2\pi} \sum_{s,l,m} g_s \int d\omega \frac{\Gamma_{s,l,m}}{e^{\omega - m\Omega/T} + 1} \left( \frac{\omega}{m} \right)\]

= Absorption Probability of (s, l, m) wave mode by BH.

= Modification factor to take the curved geometry NH into account.

Looks not that black to me. It is really Grey!
Brief History of Greybody Factor for Rotating BH

**Teukolsky equation**

= Wave equation for general (s,l,m) wave for 4D Kerr BH (1972,1973) : even before Hawking’s paper!

⇒ Generalized to (4+n) for brane fields. Ida, Oda, Park-1 (PRD 03’).

**Solution to Teukolsky equation**

: Analytic and Numerical methods was developed by Teukolsky-Press, Starobinsky, Unruh, Page in 1973-1976.

⇒ Analytic (5D): Ida, Oda, Park-1

⇒ Numerical ((4+n)D):

  s=0 Ida, Oda, Park-II (PRD 05’),
  Harris, Kanti (PLB 06’), Duffy, Harris, Kanti, Winstanley (JHEP 05’)
  s=1/2,1 Ida, Oda, Park-III (PRD 06’)
  Casals, Kanti, Winstanley (for s=1) (JHEP 06’)

**Hawking radiation and its evolution**

: Hawking 1975, Page 1976 (4D)

⇒ Ida, Oda, Park-III (PRD 06’)((4+n)D) including all the SM fields.
\[ ds^2 = g^{(4)}(r, \vartheta) + r^2 \cos^2 \vartheta \ d\Omega_n^2 \]

\[ g^{(4)}(r, \vartheta) = \begin{pmatrix}
  \Delta - a^2 \sin^2 \vartheta & (\Delta - r^2 - a^2)a \sin^2 \vartheta & 0 & 0 \\
  \Sigma & 0 & 0 & 0 \\
  \ast & 0 & 0 & \Delta \\
  0 & 0 & \Sigma & 0 \\
  0 & 0 & 0 & \Sigma \\
\end{pmatrix} \]

\[ \Sigma = r^2 + a^2 \cos \vartheta \]

\[ \Delta = r^2 \left( 1 - \frac{\mu}{r^{n+1}} + \frac{a^2}{r^2} \right) \]
Ida, Oda, Park-I (PRD 03')

Teukolsky eq. for brane fields in D=4+n

\[
\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{dS}{d\theta} \right) \left( (s-a \omega \cos \theta)^2 - (s \cot \theta + mc \csc \theta)^2 - s(s-1) + A \right) aS = 0
\]

\[
\Delta^{-s} \frac{d}{dr} \left( \Delta^{s+1} \frac{dR}{dr} \right) + \left[ \frac{K^2}{\Delta} + s \left( 4i \omega r - i \frac{\Delta_s K}{\Delta} + \Delta_{rr} - 2 \right) + 2m \omega - (a \omega)^2 - A \right] R = 0
\]

Greybody factors in D=5 (analytic)

\[
\Gamma_{l,m} = 1 - \left| \frac{Y_{out} Z_{out}}{Y_{in} Z_{in}} \right|^2 = \left[ \frac{(l - \delta)(l + \delta)}{(2l)(2l + 1)} \right]^2 R\Sigma \left\{ \left( -i \tilde{Q} - i \right)_{2l+1} (4i \tilde{\omega})^{2l+1} \right\}
\]

**Greybody factors** in low frequency expansion.

**NH limit:**

\[ \omega \xi \ll 1 \]

**FF limit:**

\[ \xi \gg 1 + |Q| \]

Matching here!

Overlapping region

\[ |Q| \ll \xi \ll 1/\omega \]
Numerical Calculation

1. Near Horizon
   BC: Imposing Purely Incoming

2. Numerical Integration Of Teukolsky equation.

3. Read out Ingoing & outgoing wave

Ida, Oda, Park –II (PRD 05’) : s=0
Ida, Oda, Park –III (PRD 06’) : s=1/2,1
Two main difficulties:

1. Imposing “purely” incoming BC at NH

\[ R_{in}^{NH} \simeq 1 \]
\[ R_{out}^{NH} \simeq e^{2ikr^*}(r - r_H)^s \]

Outgoing wave contamination is growing faster than the value we want to calculate!
Idea:

\[ R_{in}^{NH} \simeq 1 + c_1 (r - r_H) + c_2 (r - r_H)^2 + \cdots \]

\[ R_{out}^{NH} \simeq e^{2ikr_H} (r - r_H)^s (1 + b_1 (r - r_H) + \cdots) \]

Get rid of this part!

\[ \tilde{R} = R - (1 + c_1 (r - r_H)) \]

Now, we don’t worry about the outgoing wave contamination. Numerical integration is done for \( \tilde{R} \):

\[ \hat{L} \tilde{R} = 0 \]

\[ \hat{L} \tilde{R} = g \]

\[ g = -\hat{L} (1 + c_1 (r - r_H)) \]
2. Separation of Ingoing and Outgoing parts at FF

\[ R_{\text{FF}} = R_{\text{in}}^{\text{FF}} + R_{\text{out}}^{\text{FF}} \]

\[
\begin{align*}
R_{\text{in}}^{\text{FF}} &\sim r^{2s-1} \\
R_{\text{out}}^{\text{FF}} &\sim e^{2ikr_*/r}
\end{align*}
\]

\[ \begin{array}{ll}
s=0: \text{(In) } \sim \text{ (Out)} \\
s=1/2: \text{(In) } > \text{ (Out)} \\
s=1: \text{(In) } >> \text{ (Out)}
\end{array} \]

Numerically difficult to separate since there is a big hierarchy!
**Idea:** Analytically expand the solution

\[ R_{in}^{FF} = r^{2s-1}(1 + d_1/r + d_2/r^2 + \cdots) \]

\[ R_{out}^{FF} = e^{2iwr_\ast} \frac{1}{r}(1 + f_1/r + f_2/r^2 + \cdots) \]

We can always find the same order terms.

⇒ By comparing them, we can safely separate In and Out parts.

**Point:** Separation of the out-going and in-going wave sol.

⇒ using $\chi^2$ fitting with higher order terms.
Results Part-1 : Greybody factor, Hawking Radiation

1. Scalar
2. Fermion
3. Vector
Higgs-I : S-wave, Various Dimensions

$\Gamma(s=0, l=m=0, D=4, 6, 8, 10)$

$l=m=0, a=0$

\[ \sigma \simeq 4\pi r_H^2 \]

Area of Black hole event horizon

After angle integration and scaling $w^{-2}$
Higgs-II : S-wave (l=0), Rotating hole (a=0, .3,.6,.9)

**NOTE:** low energy enhancement appears in S-wave mode when BH is highly rotating.
$S = 1/2, D = 5$

Figure 5: Complete spectra for $D=5$. ($\alpha = 0, 0.9$ and $1.5$ from left to right columns.)
$D = 10, s = 1/2$

Figure 6: Complete spectra for $D=10$. ($a_\star = 0, 0.9$ and $1.5$ from left to right columns.)
$D = 5, s = 1$
$D = 11, s = 1$
Time evolution of 5d, 10d bh

Note: \((g_s = 4, g_f = 90, g_v = 24)\) is used with \(T_{bh} > m_{\text{top}}, m_{\text{Higgs}}\)
$$\alpha(a_s) \equiv -r_n^m \frac{d \ln M}{dt},$$

$$\beta(a_s) \equiv -r_n^m \frac{d \ln J}{dt},$$
Emission of M, 10 D

Emission of J, 5d
Time Evolution, 5D

In 5D, \( a_h < 1.5 \) or \( a_s < 0.832 \)
I will take \( a_s(\text{ini}) = 0.832 \)
And \( a_s(\text{final}) = 0.01 \) for spin-down phase

Mass vs Angular Momentum

Angular momentum vs. Time
In 10D, $a_h < 4$ or $a_s < 2.67$
So I will take $a_s(\text{ini}) = 2.67$
and $a_s(\text{final}) = 0.01$ for spin-down phase
Summary and Perspectives

1. **Greybody factors** for (4+n)Dim’ BH for brane localized SM fields are now available.

2. **Most of Energy is lost** during the “Spin Down Phase” (~70% 5D, ~60% 10D) → Anisotropic, Non-equally probable emission.

2. We can start to think of **Collider phenomenology** more seriously.