Symmetry Breaking From Flux Compactification

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Susy 06, Irvine 12-17/06

How to solve the hierarchy problem?

- SUSY, TECHNICOLOR, LITTLE HIGGS,…

- EXTRA DIMENSIONS:

Gauge-Higgs unification $H = A_i$ $i = 5, 6, \ldots$

(Manton 1979, Hosotani 1983,1989)

Gauge Symmetry Invariance

$m_H = 0$

What does Flux Compactification mean?

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BACKGROUND WITH A NON-TRIVIAL FIELD STRENGTH

$$F_{ij}\,(x_i) \neq 0 \qquad\qquad i,j=5,6,\,..$$

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- 4D CHIRALITY

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- 4D CHIRALITY
- SYMMETRY BREAKING?

SU(N) on $M_4 \times T^2$

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$$B_M(y + L_i) = \Omega_i \, B_M(y) \, \Omega_i^+ + i/g \, \Omega_i \, \partial_M \, \Omega_i^+$$

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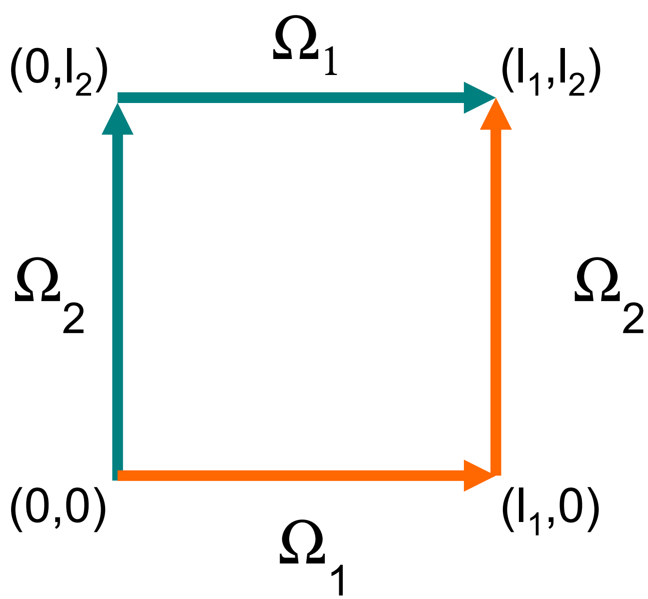

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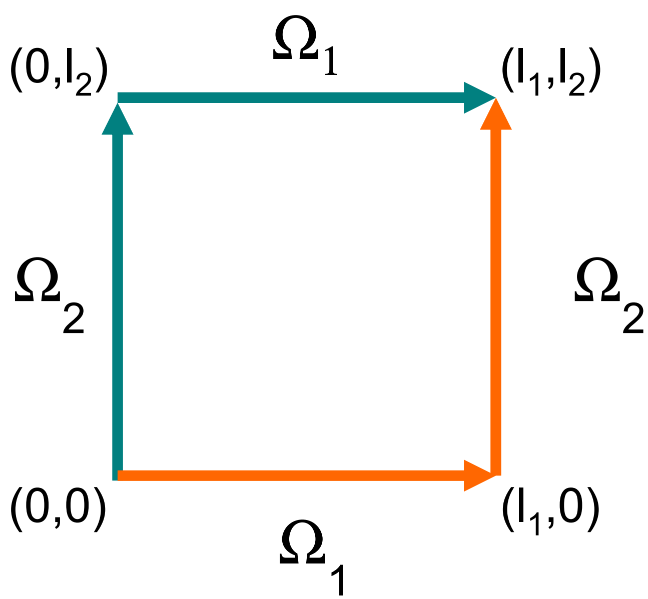
$$\Omega_1(y+L_2)\, \Omega_2(y) = e^{2\pi i\, m/N}\, \Omega_2\, (y+L_1)\, (y)\, \Omega_1(y)$$

m= 0,1, .., N-1

('t Hooft, 1979)



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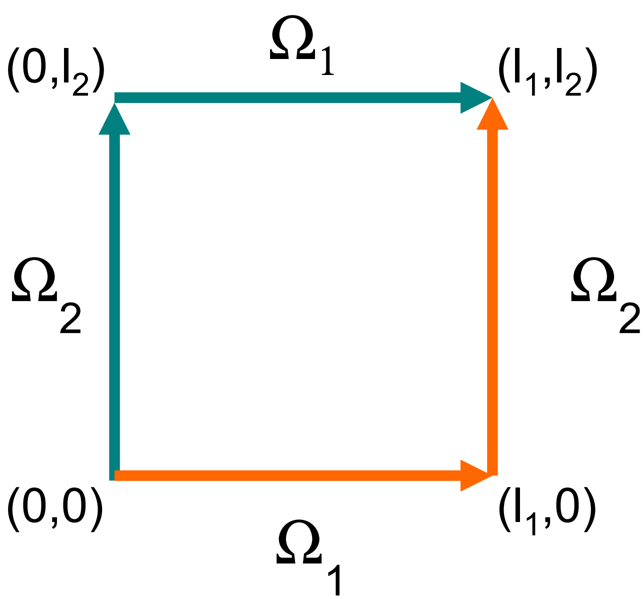

$$F_{12} = \frac{2\pi}{g \, \mathcal{A}} \, (m/N + k) \, \lambda$$

SU(N) Stable Minimum on T^2

$$A_M(x,y) = \begin{cases} A_\mu(x,y) & \mu=0,1,2,3 \\ B_i(y) + A_i(x,y) & i=5,6 \end{cases}$$

$$\mathcal{L}_{4D}^{scal} = - \tfrac{1}{2} \int_{T^2} \mathrm{Tr}\, F^{ij} F_{ij}$$

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- Negative Squared Mass

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(Nielsen-Olesen instability, 1979)

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$$\boxed{F_{ij} + \langle F_{ij}\rangle = 0}$$

“Symmetric” Gauge

Ω_i (y)

$B_i + \langle A_i \rangle$

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$U(y) \in SU(N)$

$\longleftrightarrow$

$B_i^{sym} = 0$

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⟷

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$0 \leq \alpha_i < 1$

$\alpha_i = \dfrac{k_i}{N}$

SU(2) Explicit Computation

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Kaluza Klein (n_1,n_2)

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Charged	**Landau Levels (j)**

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i) Gauge fixing and integration over T^2

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i) Gauge fixing and integration over T^2

tachyonic mass: $-\frac{4\pi}{\mathcal{A}} (m/2 + k)$

ii) Minimization with cubic and quartic interactions

4D Minimum

$$V_{4D}(\langle A_i \rangle) = -\,\mathcal{A}/2\,(F_{12})^2 \;=\; \frac{-\,8\,\pi^2}{g_{6D}^2\,\mathcal{A}}\,(m/N + k)^2$$

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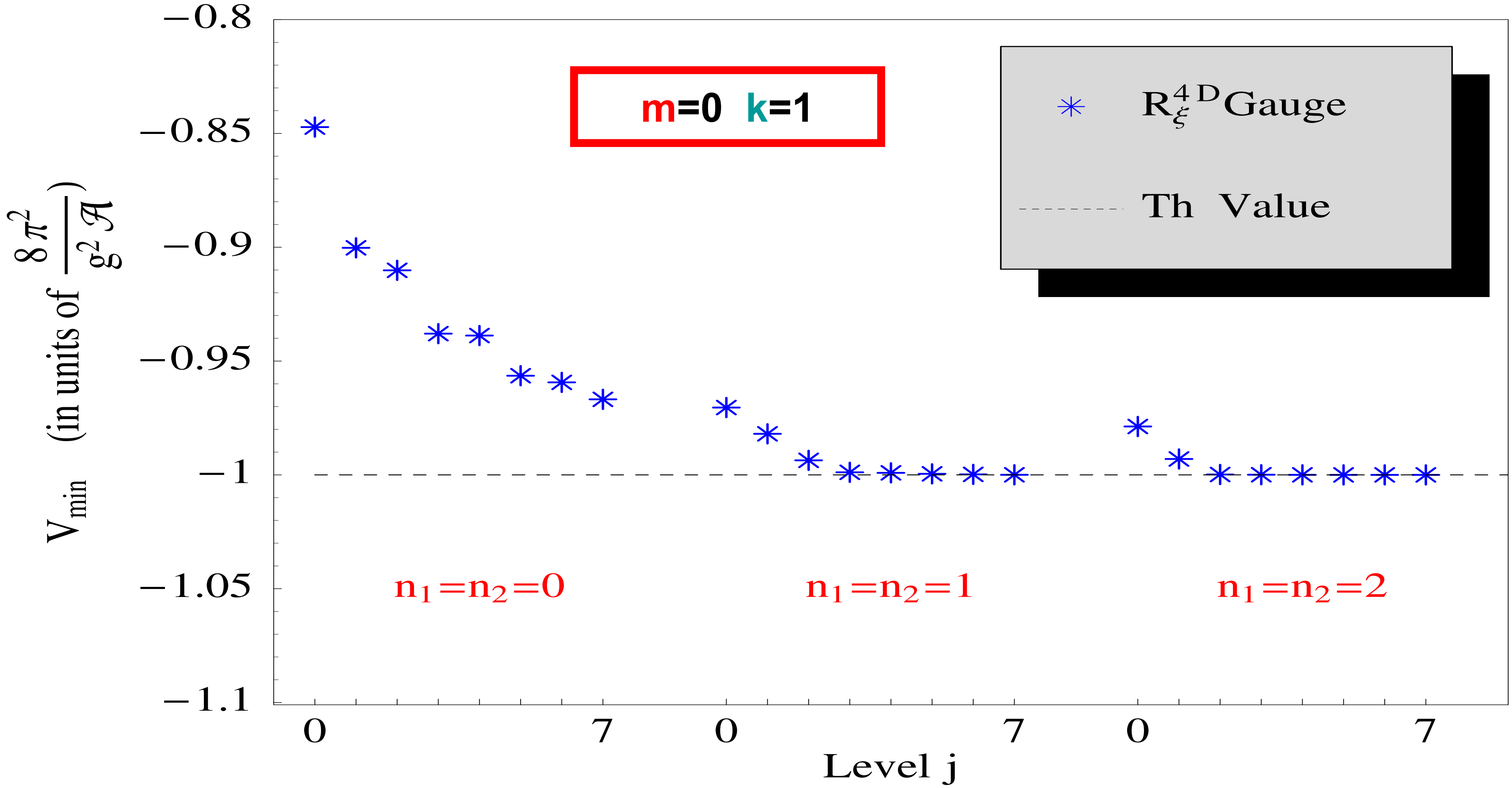

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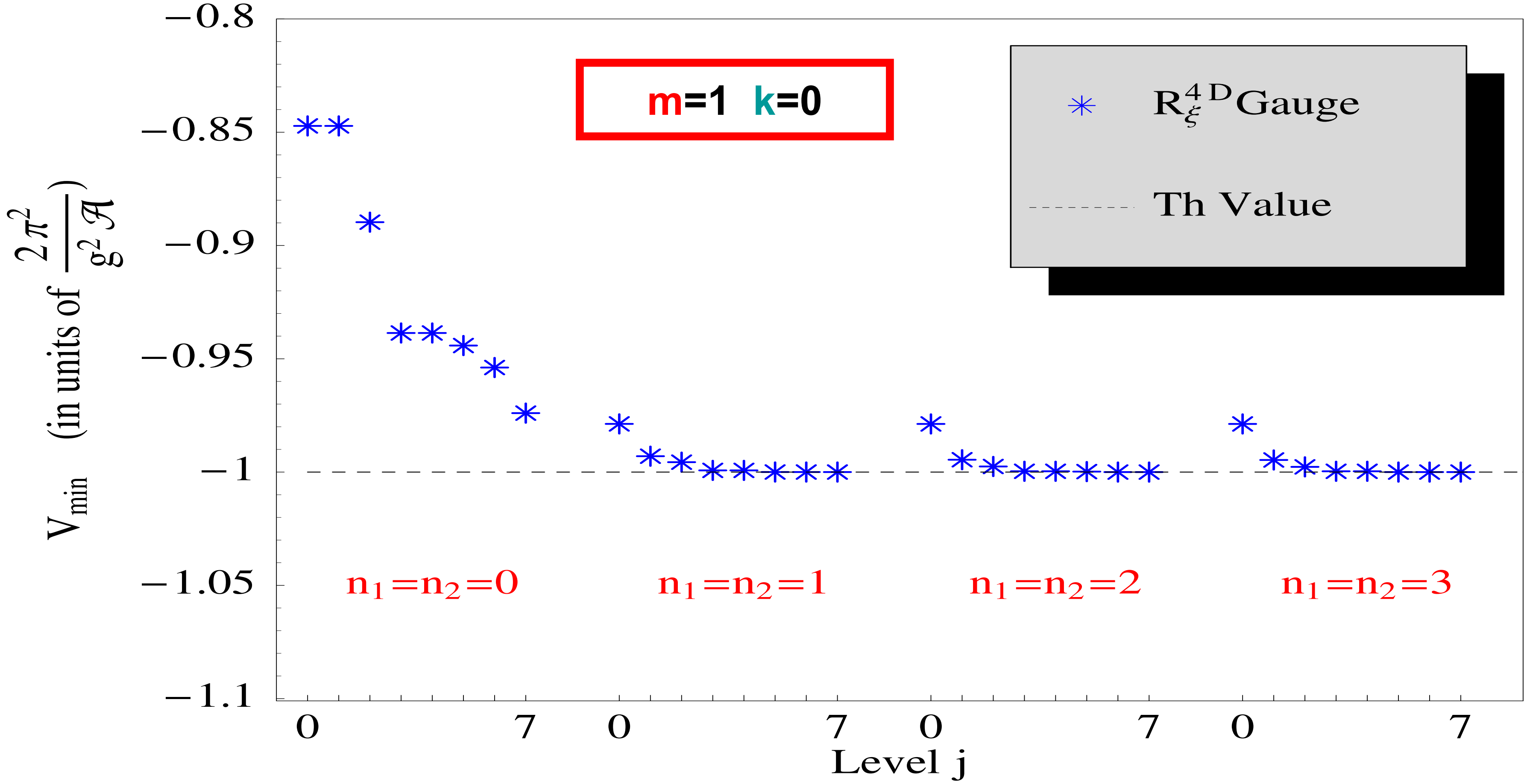

4D Spectrum (L_1=L_2=1)

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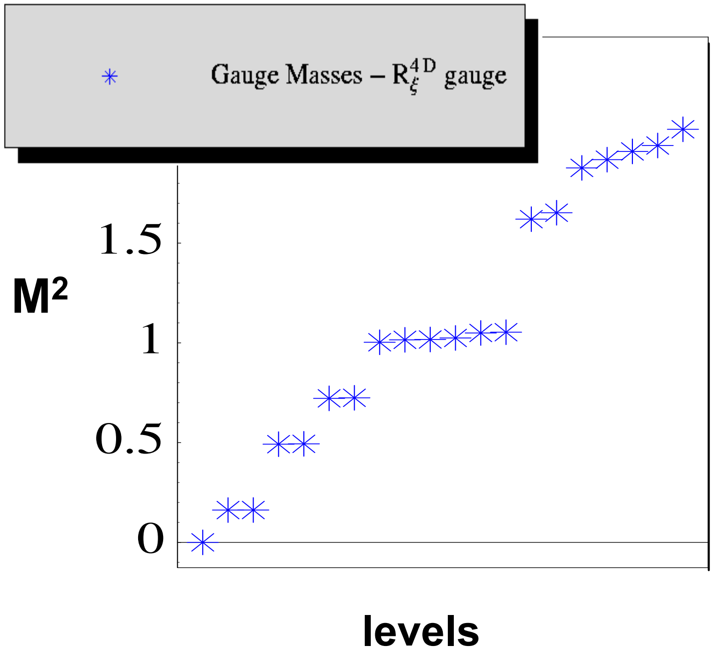

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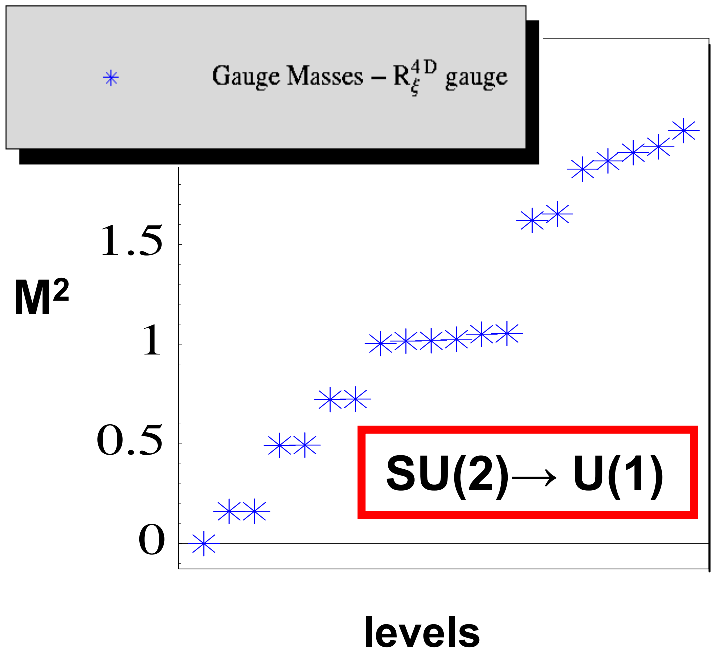

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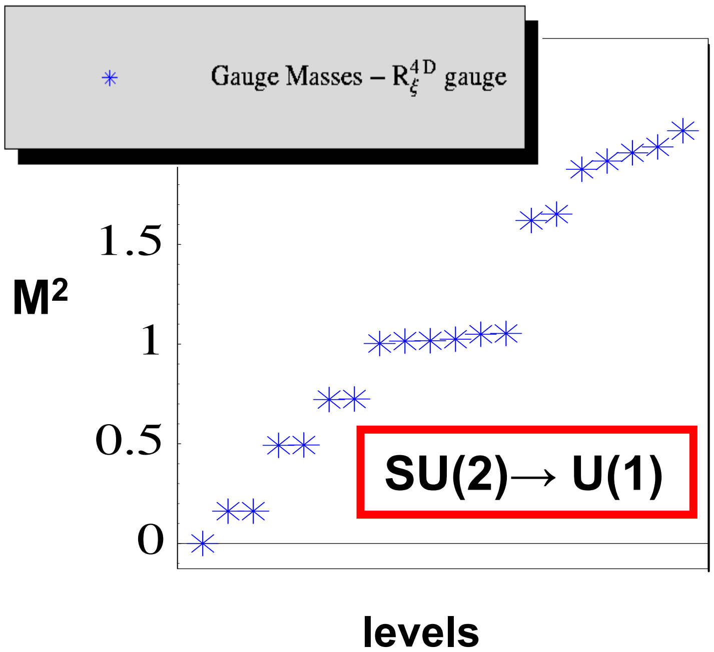

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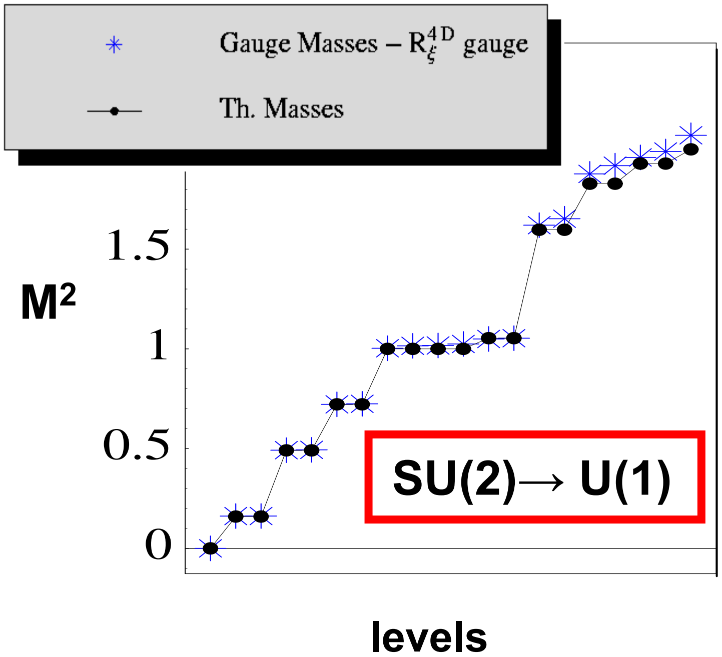

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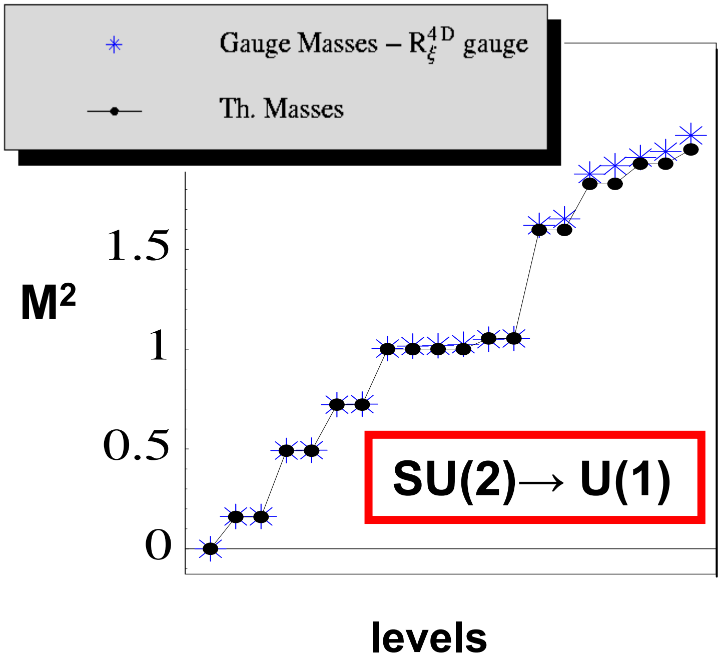

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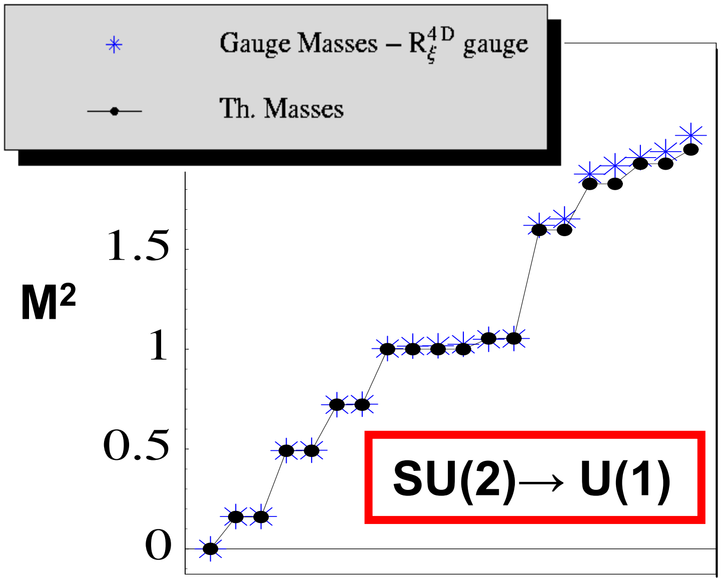

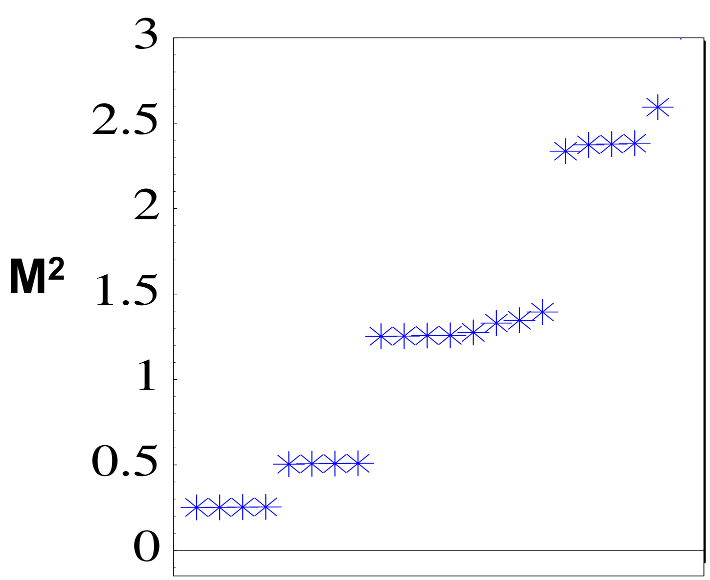

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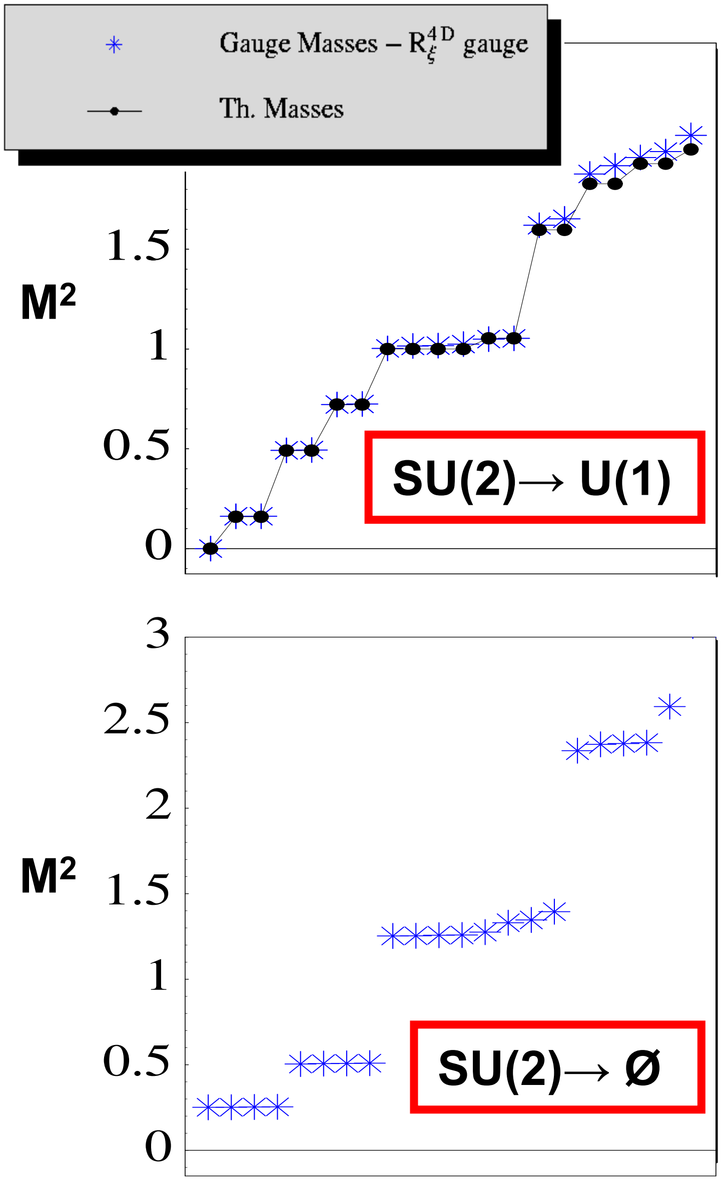

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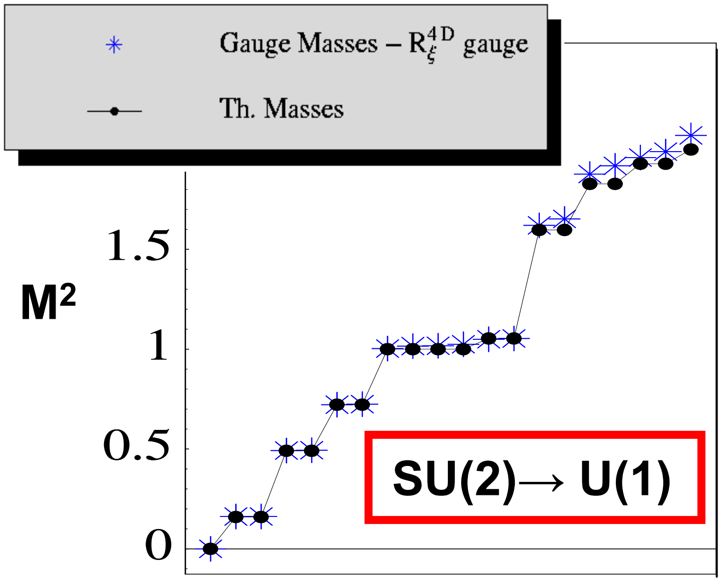

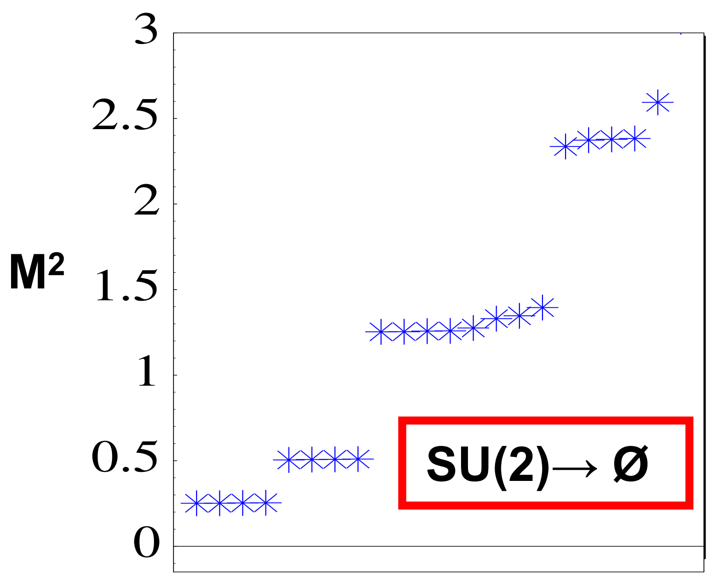

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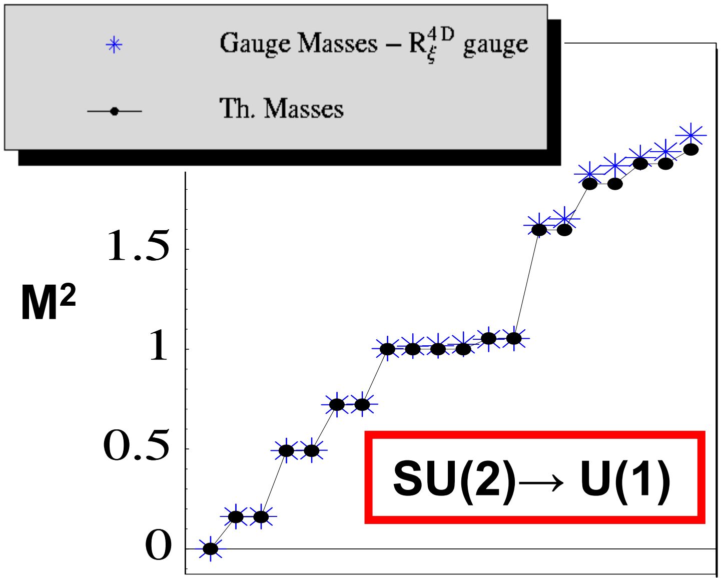

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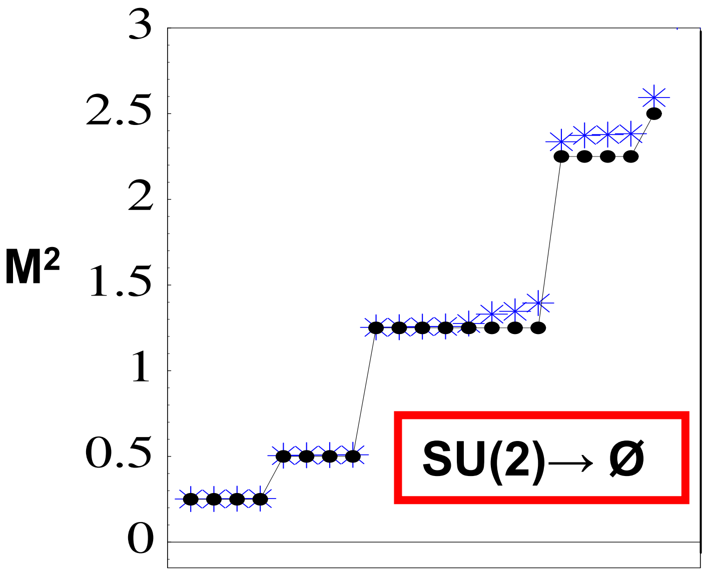

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- Gauge Equivalence

Flux Compactification ⟷ **Constant Boundary Conditions**

(SS or Twisted b.c.)

m=0   **$m \neq 0$**

To Improve….

- 4D Chirality: SU(N) → U(N)
- Symmetry breaking pattern…
- Degeneration 4D gauge-scalar sector

$$M^2(A_\mu)=M^2(A_i)$$

New mechanism to distinguish A_μ-A_i

Orbifold?? $T^{N>2}$?? … …