THE BRANE AS A HIGGS DOMAIN WALL:

IDEAS AND ISSUES

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1. Background

2. $O(10)$ kinks: breaking to $SU(3) \otimes SU(2) \otimes U(1)^2$

3. Randall-Sundrum gravity

4. Prospects, issues, questions

1. Background

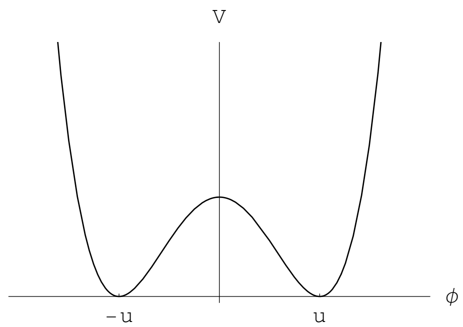
Rubakov and Shaposhnikov (1983) suggest that our universe might be a scalar field kink configuration or domain wall existing in a $4+1$-d spacetime.

But scalar fields can also spontaneously break internal symmetries.

So can we do something clever by combining brane-formation as a kink defect with the breaking of internal symmetries? Potentially, yes, via the "clash of symmetries".

1.1 The Z_2 kink

$$V = \lambda \left(\phi^2 - u^2 \right)^2$$



V has Z_2 discrete symmetry, $\phi \to -\phi$. Degenerate global minima at $\phi = -u$ and $\phi = +u$.

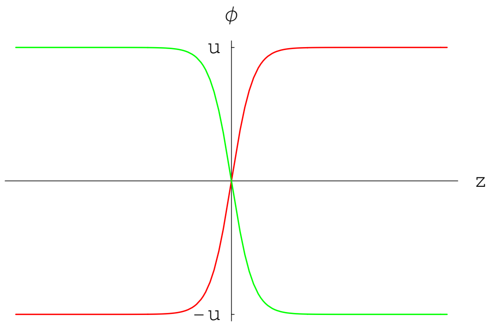
The Z_2 kink is a static 1-d (z say) solution of the classical Euler-Lagrange equations

$$\phi''(z) = \frac{dV}{d\phi}$$

that interpolates between the two global minima, the latter becoming boundary conditions at $z = \pm\infty$.

The kink and antikink:

$$\phi(z) = u \tanh(\sqrt{2\lambda}\, u\, z)$$
$$\phi(z) = -u \tanh(\sqrt{2\lambda}\, u\, z)$$



- Topologically stable because the global minima defining the b.c.'s are disconnected from each other (the vacuum "manifold" consists of just 2 points – signature of spontaneous discrete symmetry breakdown).
- Characteristic width $\Delta = \frac{1}{\sqrt{2\lambda}\, u}$.
- As $\lambda \to \infty$, smooth kink $\to$ step-function.
- In this limit, kink describes a $(d-1)$-dim interface embedded in the d-dim space. It is brane-like.

1.2 A $U(1)\otimes U(1)$ warm-up

Two complex fields Φ_1 and Φ_2 with Higgs potential

$$V=\lambda_1\left(\Phi_1^*\Phi_1+\Phi_2^*\Phi_2-u^2\right)^2+\lambda_2\,\Phi_1^*\Phi_1\,\Phi_2^*\Phi_2$$

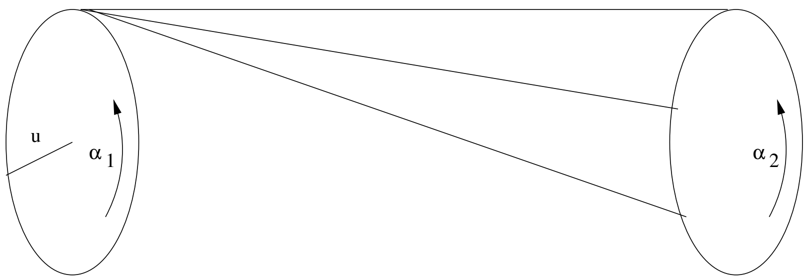
define a theory invariant under continuous global $U(1)\otimes U(1)$ transformations as well as the discrete symmetry $\Phi_1\leftrightarrow\Phi_2$.

For $\lambda_{1,2}>0$ global minima are

$$\text{Vac 1}:\ \Phi_1^*\Phi_1=u^2,\ \Phi_2=0, \qquad \text{Vac 2}:\ \Phi_1=0,\ \Phi_2^*\Phi_2=\mathrm{u}^2$$

Vacuum manifold is now two circles disconnected from each other and related by the spontaneously broken discrete symmetry:

$$\Phi_1\ =u\,e^{i\alpha_1},\ \Phi_2=0\ \text{ and }\ \Phi_1\ =0,\ \Phi_2=u\,e^{i\alpha_2}.$$



The straight lines are hypothetical different kink solutions for different phase boundary conditions at $z\to\pm\infty$. Continuous family of kinks??

Examination of the Euler-Lagrange eqns for this (simple) case shows that the only solutions with physically sensible boundary conditions have $\alpha_{1,2}$ being constant.

But the lesson is:

If the disconnected portions of the vacuum space are non-trivial manifolds, then a rich family of kink solutions should generically be expected.

When $\lambda_2 = 4\lambda_1$, there are tanh-like analytic solutions for this model:

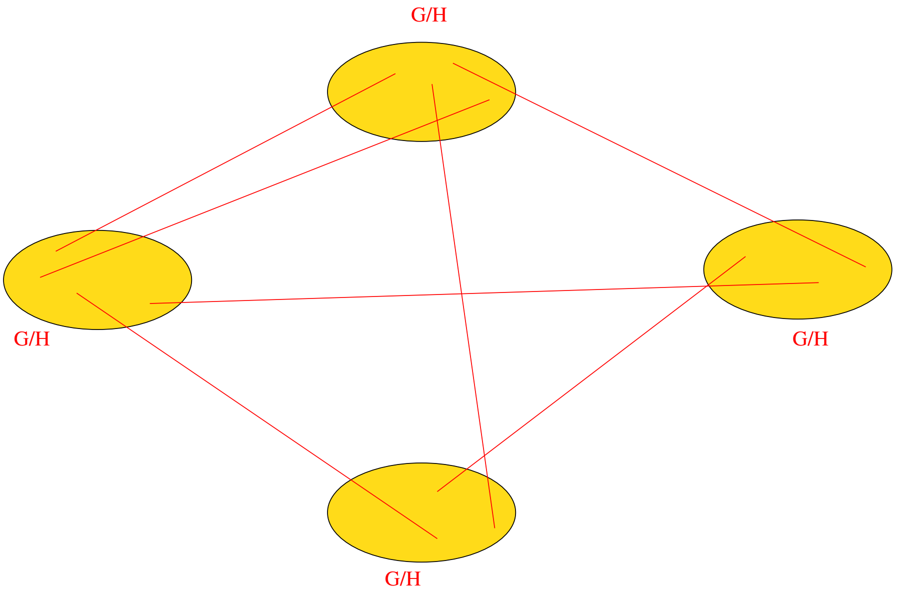
$$\begin{aligned} |\phi_1(z)| &= \frac{u}{2}\left(1 \pm \tanh(\sqrt{\lambda_1}\, u\, z)\right), \\ |\phi_2(z)| &= \frac{u}{2}\left(1 \mp \tanh(\sqrt{\lambda_1}\, u\, z)\right), \end{aligned}$$

with $\alpha_{1,2} =$ constant.

1.3 Clash of symmetries basic idea

Let a theory be invariant under a continuous group G that gets spontaneously broken down to H. Let it also have a discrete symmetry that is spontaneously broken. Suppose the action of the discrete symmetry on a vacuum state is definitely outside of G. Then:

The vacuum space is a discrete number of copies of the coset space manifold G/H.



The clash of symmetries mechanism relies on there being multiple possible embeddings of the H subgroup within G.

Consider, for instance, $SU(3) \to SU(2)$ breaking induced by a Higgs triplet. Different embeddings in this case are the I-spin, U-spin and V-spin $SU(2)$ subgroups of $SU(3)$. The VEVs are

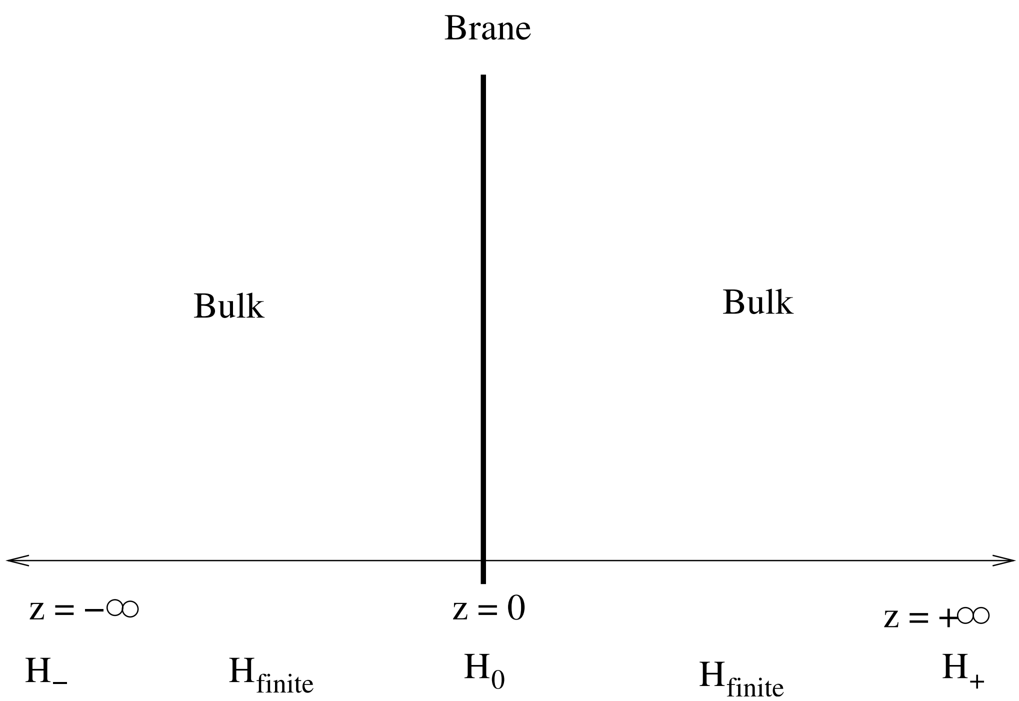
$$\begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}$$

A clash-of-symmetries-style kink interpolates between global Higgs potential minima that leave differently embedded subgroups, H_- and H_+, unbroken asymptotically i.e. at $z = \pm\infty$.

At non-asymptotic points, $|z| < \infty$, the unbroken subgroup will be different again.

- Details depend on the specific case.
- Pure clash of symmetries has $H_{|z|<\infty} \equiv H_{\text{finite}} = H_- \cap H_+$.
- Sometimes $H_{z=0} \equiv H_0$ is a special case.

These configurations produce a spatially-dependent symmetry breaking pattern. We want to apply this to brane-world model building.



- Symmetry breaking pattern on brane is $G \to H_0$.
- If brane d.o.f.'s leak slightly into the bulk, then they also couple to $G \to H_{\rm finite}$.
- This would induce the hierarchical breakdown pattern $G \to H_0 \to H_{\rm finite}$.
- More bang for your symmetry breaking buck. Simpler Higgs sectors for SM extensions?

2. $O(10)$ kinks: breaking to $SU(3) \otimes SU(2) \otimes U(1)^2$.

Shin, Volkas, PRD 69, 045010 (2004)

This model has an $O(10)$ real adjoint Higgs field, the **45**. Write it as a 10×10 antisymmetric matrix Φ. The most general quartic potential has

$$\mathrm{Tr}(\Phi^2), \quad \mathrm{Tr}(\Phi^2)^2, \quad \text{and} \quad \mathrm{Tr}(\Phi^4)$$

terms only.

The cubic term $\mathrm{Tr}(\Phi^3)$, though invariant, is identically zero. This means there is an accidental discrete symmetry

$$\Phi \to -\Phi$$

It (essentially) will be used to ensure topological stability for the lowest energy kink in a given topological class.

Φ can always be brought by an $O(10)$ transformation to the standard form

$$\Phi = \mathrm{diag}(\, a_1\,\epsilon\,,\; a_2\,\epsilon\,,\; a_3\,\epsilon\,,\; a_4\,\epsilon\,,\; a_5\,\epsilon\,)$$

where $\epsilon = \mathrm{antidiag}(1, -1)$. For a range of parameters, the global minimum has

$$a_i^2 = \text{const.} \equiv a^2_{\mathrm{min}} \forall i$$

The stability group is $U(5)$, i.e. $O(10) \to U(5)$.

Let

$$\Phi(-\infty) = -\,a_{\text{min}}\,\text{diag}(\epsilon,\epsilon,\epsilon,\epsilon,\epsilon)$$

There are (essentially) three choices for the other boundary condition,

$$\Phi(+\infty) = \begin{cases} a_{\text{min}}\,\text{diag}(\epsilon,\epsilon,\epsilon,\epsilon,\epsilon) \\ a_{\text{min}}\,\text{diag}(\epsilon,\epsilon,\epsilon,-\epsilon,-\epsilon) \\ a_{\text{min}}\,\text{diag}(\epsilon,-\epsilon,-\epsilon,-\epsilon,-\epsilon) \end{cases}$$

giving three kinds of kink: symmetric, asymmetric and super-asymmetric.

The kink configurations are of the form

$$\Phi_k(z) = \alpha(z)\,\Phi(-\infty) + \beta(z)\,\Phi(+\infty)$$

leading to

$$H_{\text{finite}} = U(5), \quad U(3)\otimes U(2), \quad U(4)\otimes U(1)$$

respectively.

Since $U(3)\otimes U(2) \cong G_{SM}\otimes U(1)'$, asymmetric $O(10)$ kinks show some model-building promise.

Let's look at them in more detail.

Asymmetric kinks have

$$a_1 = a_2 = a_3 \equiv f(z), \qquad a_4 = a_5 = g(z)$$

where f is an odd function with b.c.'s $f(-\infty) = -f(+\infty) = -a_{\min}$, while g is even with $g(\pm\infty) = -a_{\min}$.

So, $f(0) = 0$. This means that at the centre of the wall we have

$$H_0 = O(6) \otimes U(2) \cong SU(4) \otimes SU(2) \otimes U(1)$$

leading to the prospect of the hierarchical cascade

$$SO(10) \to SU(4) \otimes SU(2) \otimes U(1) \to G_{SM} \otimes U(1)'.$$

If the leakage of brane d.o.f.'s off the wall is exponentially small, we expect the hierarchy to be large.

When the coefficient of $\mathrm{Tr}(\Phi^2)^2$ is zero, there is a simple analytic solution: $f(z) = a_{\min} \tanh(\mu z)$, $g(z) = -a_{\min}$. For that special point, the super-asymmetric kink has the lowest energy density, not the asymmetric kink, unfortunately. We don't know if there is a parameter region where the asymmetric kink has the lowest energy and is thus stable. Also: why quartic potentials?

3. Global $U(1) \otimes U(1)$ and RS-like gravity

Dando, Davidson, George, Volkas, Wali, PRD 72, 045016 (2005)

$$S = \int \left[-\frac{\kappa}{2} R - g^{AB} \left(\overline{\Phi}^1_{,A} \Phi^1_{,B} + \overline{\Phi}^2_{,A} \Phi^2_{,B} \right) - V \right] \sqrt{-g}\, d^5x$$

The game we played: inverse problem: what potential V gives

$$\phi_1(w) = \frac{u}{\sqrt{2}} \sqrt{1 + \tanh \beta w}$$

$$\phi_2(w) = \frac{u}{\sqrt{2}} \sqrt{1 - \tanh \beta w}$$

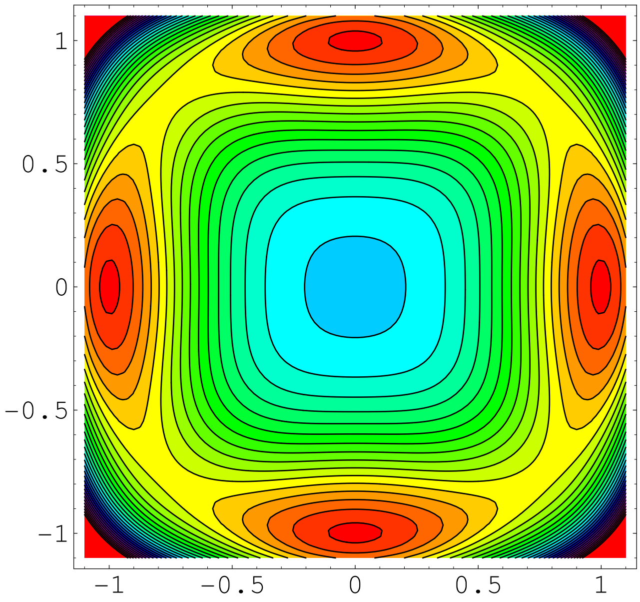
$$f(w) = -\frac{u^2}{12\kappa} \ln \left(\cosh \beta w \right)$$

where f specifies the warped metric

$$ds_5^2 = dw^2 + e^{2f(w)}\, ds_4^2$$

and the brane is Minkowski?

The answer is a certain sextic potential.



Contour plot of sextic potential for a representative parameter point.

The Randall-Sundrum limit is obtained thus:

$$e^{2f(w)} = \left[(\cosh \beta w)^{-\frac{1}{\beta}} \right]^{\frac{u^2 \beta}{6\kappa}}, \quad \lim_{\beta \to \infty} (\cosh \beta w)^{-\frac{1}{\beta}} = e^{-|w|}$$

So

$$\lim_{RS} e^{2f(w)} = e^{-\frac{u^2\beta}{6\kappa}|w|}$$

requires

$$\beta \to \infty, \quad u \to 0, \quad \text{s.t.} \quad u^2 \beta \to \text{finite}$$

4. Prospects, issues, questions

What do we want eventually? A scalar-field kink brane world model with a phenomenologically successful 4-d effective theory on the domain wall. Hopefully, we can also use the scalar field that forms the brane to also perform some useful internal symmetry breaking, e.g. GUT breaking.

Question: How do we simultaneously localise fermions, gravitons, gauge bosons and perhaps also Higgs bosons on the domain wall? It is known how to do fermions and gravitons simultaneously. There are suggested mechanisms for gauge bosons. Which should we choose?

Question: Localised fermions will interact with each other through the exchange of quanta associated with excitations of the kink. What effects are possible. Damien George and I are looking at this.

The model should also give successful cosmology. Tracy Slatyer and I are looking at the basics of this.