Unification without low-energy supersymmetry

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Based on work with J. Sayre and S. Willenbrock
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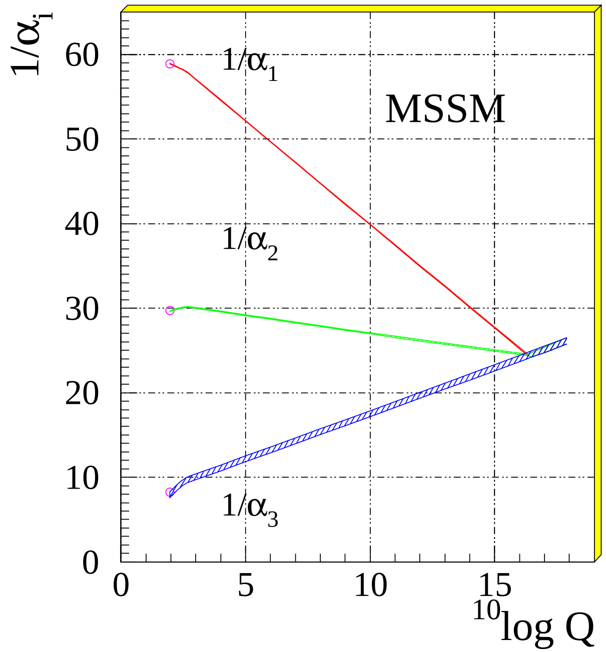
Motivation

SM gauge couplings converge at a high scale M_U but below the Planck scale.

$\rightarrow$ Embed SM in a more fundamental theory (Grand Unified Theory);

- order of M_U accounts for small neutrino masses and non-detection of proton decay.

[Amaldi, de Boer, Fürstenau 1991]



$\rightarrow$ SUSY SU(5)

SUSY GUTs provide
a beautiful framework,

low-energy supersymmetry
controls the gauge hierarchy.

Non-supersymmetric GUTs

Unification is possible without low-energy supersymmetry.

- Split Supersymmetry
- SM with at least five Higgs doublets.

[Adler 1998; Willenbrock 2003]

Models without supersymmetry are viable alternatives.

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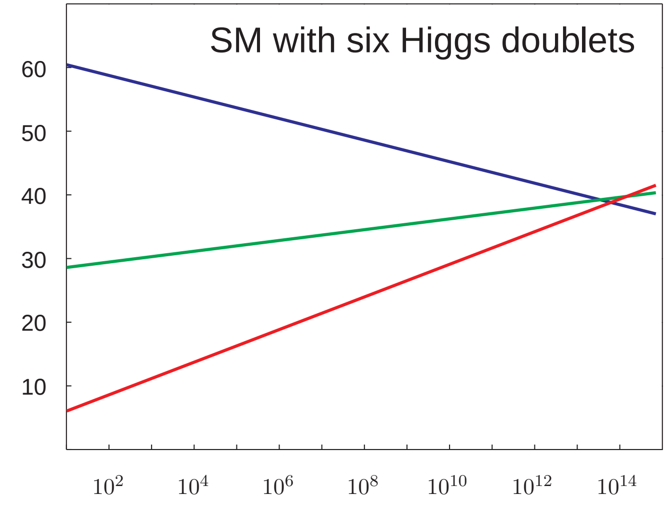

Trinified model $G_{TR} = SU(3)_C \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$

[Achiman, Stech 1978; de Rújula, Georgi, Glashow 1984; Babu, He, Pakvasa 1986]

- rank 6, subgroup of E_6;
- $\mathbb{Z}_3$ guarantess that gauge couplings conincide at M_U;
- no need for adjoint Higgs fields;
- up to five light Higgs doublets !

Is the non-supersymmetric minimal trinified model a viable candidate for a more fundamental theory at $M_U \sim 10^{14}$ GeV ?

Minimal Trinification

Gauge group: $G_{TR} = SU(3)_C \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$

Fermions: $(3, 3^*, 1) \oplus (3^*, 1, 3) \oplus (1, 3, 3^*) \equiv \psi_Q \oplus \psi_{Q^c} \oplus \psi_L$

$$\psi_Q = \begin{pmatrix} -d & u & B \end{pmatrix}, \qquad \psi_{Q^c} = \begin{pmatrix} \mathscr{D}^c \\ u^c \\ \mathscr{B}^c \end{pmatrix}, \qquad \psi_L = \begin{pmatrix} (\mathscr{E}) & (E^c) & (\mathscr{L}) \\ \mathscr{N}_1 & e^c & \mathscr{N}_2 \end{pmatrix}$$

In addition to the 15 SM fermions, there are 12 new fermions (as in E_6),

$$B\left(3, 1, -\tfrac{1}{3}\right), \qquad E^c\left(1, 2, \tfrac{1}{2}\right), \qquad \underbrace{\mathscr{N}_{1,2}\,(1, 1, 0)}_{\textit{sterile Neutrinos}},$$

doubling of $\left(3^*, 1, \tfrac{1}{3}\right)$, $\left(1, 2, -\tfrac{1}{2}\right)$,

$\rightarrow$ mixing between $\mathscr{D}^c$ and $\mathscr{B}^c$ as well as $\mathscr{L}$ and $\mathscr{E}$.

Breaking of G_{TR}

Breaking to G_{SM} by a pair of $\Phi_{L}\ (1,3,3^*) = \begin{pmatrix} (\phi_1) & (\phi_2) & (\phi_3) \\ S_1 & S_2 & S_3 \end{pmatrix}$ with

$$\langle \Phi_L^1 \rangle = \begin{pmatrix} \begin{pmatrix} \ \\ \ \end{pmatrix} & \begin{pmatrix} \ \\ \ \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ 0 & 0 & v_1 \end{pmatrix} \text{ and } \langle \Phi_L^2 \rangle = \begin{pmatrix} \begin{pmatrix} \ \\ \ \end{pmatrix} & \begin{pmatrix} \ \\ \ \end{pmatrix} & \begin{pmatrix} \ \\ \ \end{pmatrix} \\ v_2 & 0 & 0 \end{pmatrix}$$

v_1 and v_2 break G_{TR} to different $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$

Of the six Higgs doublets, one linear combination is eaten by the gauge bosons that acquire unification-scale masses.

If the remaining five doublets have electroweak-scale masses, then gauge-coupling unification results at $M_U \simeq 10^{14}$ GeV without supersymmetry.

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Yukawa couplings:

$$Y = \psi_{Q^c}\,\psi_Q\,\left(g_1\,\Phi_L^1 + g_2\,\Phi_L^2\right) + \tfrac{1}{2}\,\psi_L\,\psi_L\,\left(h_1\,\Phi_L^1 + h_2\,\Phi_L^2\right)$$

$\rightarrow$ Heavy states: $B^c = \ \ c_\alpha\,\mathscr{D}^c + s_\alpha\,\mathscr{B}^c$, $\quad E = -s_\beta\,\mathscr{E} + c_\beta\,\mathscr{L}$, $\quad \tan\alpha = \frac{g_1 v_1}{g_2 v_2}$

massless: $d^c = -s_\alpha\,\mathscr{D}^c + c_\alpha\,\mathscr{B}^c$, $\quad L = \ \ c_\beta\,\mathscr{E} + s_\beta\,\mathscr{L}$, $\quad \tan\beta = \frac{h_1 v_1}{h_2 v_2}$

$N_1 = \ \ s_\beta\,\mathscr{N}_1 - c_\beta\,\mathscr{N}_2$, $\quad N_2 = -c_\beta\,\mathscr{N}_1 - s_\beta\,\mathscr{N}_2$

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For simplicity, we choose $n_{1,2,3} = 0$ here.

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light fermion masses

$$m_u = g_1 u_2\,, \quad m_d \simeq g_1 u_1\, s_\alpha\,,$$

$$m_{\nu,N_1} = h_1 u_2\,, \quad m_e \simeq h_1 u_1\, s_\beta\,, \quad m_{N_2} \simeq \frac{h_1^2 u_1 u_2\, s_\beta}{\sqrt{h_1^2 v_1^2 + h_2^2 v_2^2}}\,.$$

No relation between the masses of the quarks and leptons; the minimal model is sufficient to describe the masses of the quarks and charged leptons.

Breaking of G_{TR}

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$$\langle \Phi^1_{L} \rangle = \left(\begin{pmatrix} u_1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ v_1 \end{pmatrix} \right) \text{ and } \langle \Phi^2_{L} \rangle = \left(\begin{pmatrix} n_1 \\ 0 \\ v_2 \end{pmatrix} \begin{pmatrix} 0 \\ n_2 \\ 0 \end{pmatrix} \begin{pmatrix} n_3 \\ 0 \\ 0 \end{pmatrix} \right)$$

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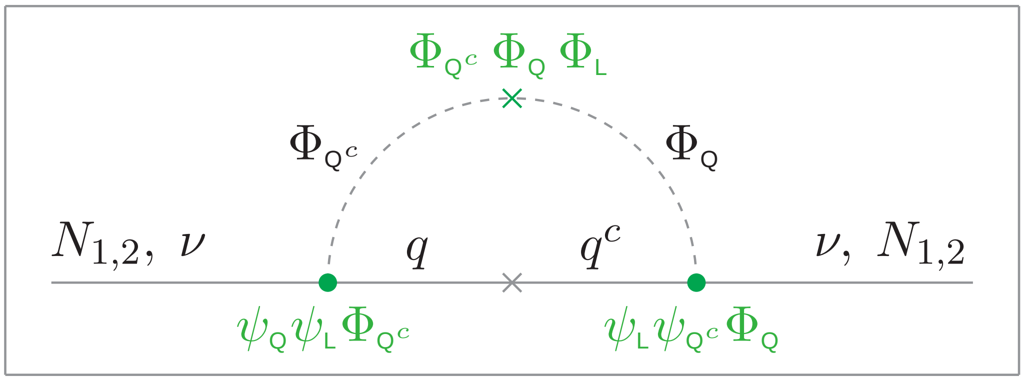
The active neutrino, ν, together with N_1 forms a Dirac state, whereas the sterile N_2 receives a small Majorana mass.

Correction via one-loop diagrams!

Radiative Seesaw Mechanism

One-loop diagrams appear due to the coupling of the neutral fermions to color-charged Higgs bosons and the cubic couplings of the Higgs fields.

$\Phi_{\mathsf{L}}\ (1,3,3^*)$
$\Phi_{\mathsf{Q}}\ (3,3^*,1)$
$\Phi_{\mathsf{Q}^c}\ (3^*,1,3)$



$$\mathscr{L}_q = g\left(\psi_{\mathsf{Q}^c}\,\psi_{\mathsf{Q}}\Phi_{\mathsf{L}} + \psi_{\mathsf{L}}\,\psi_{\mathsf{Q}^c}\Phi_{\mathsf{Q}} + \psi_{\mathsf{Q}}\,\psi_{\mathsf{L}}\Phi_{\mathsf{Q}^c}\right) + \text{h.c.}$$

$$\mathscr{L}_h = m^2\left(\Phi^*_{\mathsf{Q}}\Phi_{\mathsf{Q}} + \Phi^*_{\mathsf{Q}^c}\Phi_{\mathsf{Q}^c} + \Phi^*_{\mathsf{L}}\Phi_{\mathsf{L}}\right) + \left[\gamma_1\Phi_{\mathsf{Q}^c}\Phi_{\mathsf{Q}}\Phi_{\mathsf{L}} + \gamma_2\left(\Phi_{\mathsf{L}}\Phi_{\mathsf{L}}\Phi_{\mathsf{L}} + \text{cyclic}\right) + \text{h.c.}\right]$$

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Dominant graphs:

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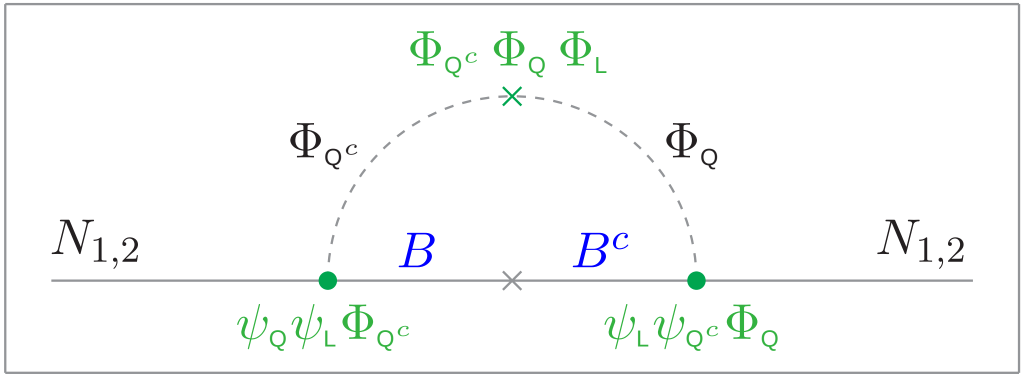

$\longrightarrow$ mass matrix for neutrinos (ν, N_1, N_2)

$$M_N \simeq \begin{pmatrix} 0 & -h_1 u_1 & 0 \\ -h_1 u_1 & s_{\alpha-\beta} c_\beta\, g^2 F_q\,(B) & (s_{2\beta} s_\alpha - c_\alpha)\, g^2 F_q\,(B) \\ 0 & (s_{2\beta} s_\alpha - c_\alpha)\, g^2 F_q\,(B) & c_{\alpha-\beta} s_\beta\, g^2 F_q\,(B) \end{pmatrix},$$

$F_q\,(q) \propto m_q$
(loop integral)

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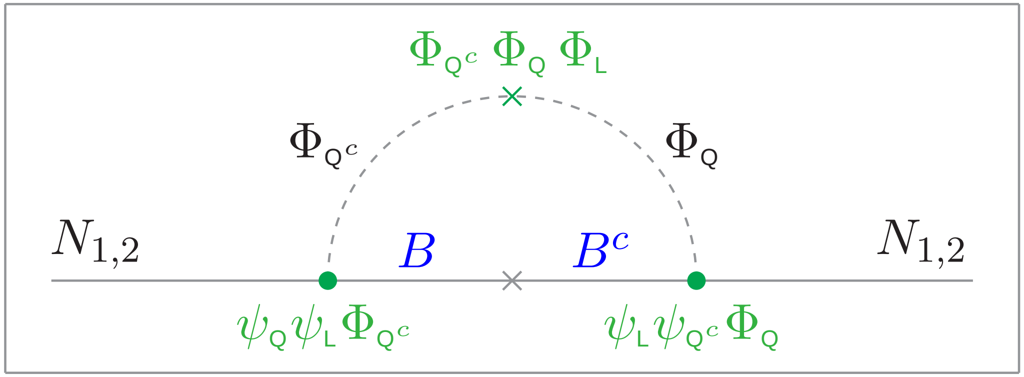

This mechanism is absent in models with low-energy supersymmetry.

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(loop integral)

sterile neutrinos obtain masses $\lambda_N \sim F_q(B) \sim \mathcal{O}(M_U)$,

active neutrino is light, $\lambda_\nu \sim \dfrac{(h_1 u_1)^2}{g^2 F_q(B)} \simeq 0.1\,\mathrm{eV}$!

Neutrino Hierarchy

The neutrino hierarchy is related to the couplings in the quark sector.

The dominant 1-loop contributions are those with the heaviest quark, B_3.

$\longrightarrow$ three-generational mass matrix for the sterile neutrinos (both N_1 and N_2), $\mathrm{M}^N \sim \left(g^{3i}\, g^{j3} + g^{i3}\, g^{3j}\right) F_{B_3}$

[Lola, Ross 1999]

Assume $$g \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \epsilon^2 \simeq \frac{m_c}{m_t}.$$

$$\Longrightarrow \qquad m_3^N \sim m_2^N \sim F_{B_3} \sim 10^{12}\,\text{GeV}\,, \qquad m_1^N \sim \epsilon^4 F_{B_3} \sim 10^8\,\text{GeV}\,.$$

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Light neutrinos: eigenvalues are proportional to $\frac{h^2}{g^2}$ due to the common loop-integral

$\longrightarrow$ hierarchy is determined by the hierarchy of h

$\longrightarrow$ quasi-degenerate masses or a normal hierarchy.

Proton Decay

Quarks and leptons in different multiplets. $\longrightarrow$ No proton decay via gauge bosons.

Instead, proton decay is mediated by Φ_{Q^c} and Φ_{Q}.

These dimension-six operators are suppressed by small Yukawa couplings,

$$\left[(g\,\hat{s}_\beta)\,h\,QQQL + g\left(-\hat{s}_\alpha^\top h\right) d^c u^c e^c u^c\right] - \left[g^* h\,QQe^{c*}u^{c*} + (g\,\hat{s}_\beta)\left(-\hat{s}_\alpha^\top h\right)^* d^{c*}u^{c*}QL\right]$$

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Lifetime: $\tau \simeq \left(\frac{1}{g\,h}\right)^2 \times 10^{28}$ years.

$$g \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \epsilon^2 \simeq \frac{m_c}{m_t},$$

mode	dominant coeff.	exp. limit [y]
$\bar{\nu}K^+$	$g^{23}h^{11}$	2.2×10^{33}
$\bar{\nu}\pi^+$	$g^{13}h^{11}$	2.5×10^{31}
μ^+K^0	$g^{12}h^{12}$	1.4×10^{33}

The decay width of $p \rightarrow \bar{\nu}K^+$ is close to the experimental limit.

Summary

- The minimal trinified model, $G_{TR} = SU(3)_C \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$ is an interesting candidate for non-supersymmetric unification.

  Breaking is achieved by only two $\Phi_L\,(1, 3, 3^*)$ representations which include five Higgs doublets.

- Sterile Neutrinos become massive with $M \gg M_{EW}$ via radiative seesaw mechanism; at M_{EW}, only the SM fermions remain.

- No need to introduce intermediate scales, additional Higgs fields, or higher-dimensional operators.

- Proton decay is mediated by color-charged Higgs bosons.

  The decay mode $p \to \bar{\nu} K^+$ is dominant. ($\nearrow$ SUSY models with dim-5 ops.)

- Possibility to verify model:
  - no SUSY particles at TeV scale;
  - detection of $p \to \bar{\nu} K^+$ as dominant decay mode (future experiments aim to reach 10^{35} years).

- Results are also valid for SUSY models, where scalars are as heavy as M_U.
  - Proton decay is unobservable.