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Patterns of supersymmetry breaking in moduli-mixing racetrack model

Hiroyuki Abe

(YITP, Kyoto)

Based on articles

- Phys.Rev.D73 (2006) 046005 (hep-th/0511160)
- Nucl.Phys.B742 (2006) 187 (hep-th/0512232)
- hep-th/0606095

with Tetsutaro Higaki and Tatsuo Kobayashi

I. Introduction

Candidate of unified theory of gravity, gauge and matter fields:

(higher-dimensional) SUGRA, superstring/M-theory

The 4D effective theory has many moduli superfields $\Phi^I = (\phi,\, \chi,\, F^\phi)^I$.

$$\langle \phi \rangle \quad \longrightarrow \quad \left\{ \begin{array}{rcl} M_{Pl} & : & \text{Planck scale}, \\ g_a & : & \text{gauge couplings}, \\ Y_{ijk} & : & \text{Yukawa couplings}, \\ \vdots & & \end{array} \right.$$

$$\langle F^\phi \rangle \quad \longrightarrow \quad \left\{ \begin{array}{rcl} M_a & : & \text{gaugino masses}, \\ m_i & : & \text{squark, slepton masses}, \\ A_{ijk} & : & A\text{-terms}, \\ \vdots & & \end{array} \right.$$

Thus, moduli stabilization and its effect on SUSY breaking is quite relevant to particle phenomenology/cosmology.

Sources of moduli potential (in Type IIB $O3/O7$ on CY_3 with $h_{1,1} = 1$)

Gaugino condensation: $SU(N_a)$ SYM, $1/g_a^2 = Re\, f_a$

$$W_{GC} \;=\; N_a e^{-8\pi^2 f_a / N_a} \quad \longleftarrow \begin{cases} f_{D7} = T & : \text{Kähler modulus} \\ f_{D3} = S & : \text{dilaton} \end{cases}$$

Three-form flux: $G_3 = F_3 - 2\pi i S H_3$

$$W_{flux} \;=\; \int_{CY_3} G_3 \wedge \Omega \qquad (\Omega : \text{ holomolphic three-form})$$

$$= \; f^{RR}(U^\alpha) + S\, f^{NS}(U^\alpha) \quad \longleftarrow \begin{cases} S & : \text{dilaton} \\ U^\alpha & : \text{complex structure moduli} \end{cases}$$

$f^{RR,NS}(U^\alpha)$: functions determined by flux

Gukov, Vafa, Witten, Nucl.Phys.B584 (2000) 69

Similarly, moduli potential can arise also in other models (e.g. IIA, hetero)

Two stage stabilization (KKLT-type)

Kachru, Kallosh, Linde, Trivedi, Phys.Rev.D68 (2003) 046005

Choi, Falkowski, Nilles, Olechowski, Pokorski, JHEP 0411 (2004) 076, ...

H.A., Higaki, Kobayashi, hep-th/0606095

By fine-tuning flux such that

$$w_0 \;=\; \langle W_{flux} \rangle \;\ll\; 1 \qquad (M_{Pl} = 1 \text{ unit})$$

while keeping $\langle \partial_S \partial_{U^\alpha} W_{flux} \rangle, \quad \langle \partial_{U^\alpha} \partial_{U^\beta} W_{flux} \rangle \;\sim\; 1$

we find ...

- Moduli S and U^α can be stabilized by flux and receive heavy mass ~ 1
- Remaining T is stabilized by GC with mass $\sim m_{3/2} \sim w_0 \sim \langle W_{GC} \rangle \ll 1$

We first integrate out S, U^α, and derive the effective action of T.

II. 4D effective SUGRA for single modulus

Stabilization at SUSY AdS minimum: $F^T = -e^{K/2} K^{\bar{T}T}(W_T + K_T W) = 0$

$$V \;=\; K_{I\bar{J}} F^I \bar{F}^{\bar{J}} - 3e^K |W|^2 \quad\rightarrow\quad -3m_{3/2}^2 \;<\; 0$$

Flux + GC: $K \;=\; -3\ln(T+\bar{T}), \qquad W \;=\; w_0 - Ae^{-aT}$

$$at \;\simeq\; \ln(A/w_0) \;\sim\; 4\pi^2, \qquad \sigma_t = 0, \qquad (T = t + i\sigma_t)$$

$$A \;\approx\; 1, \quad m_{3/2} \;\simeq\; w_0 \;\sim\; 10^{-14} \quad \text{(fine-tuning)}$$

Racetrack : $K \;=\; -3\ln(T+\bar{T}), \qquad W = Ae^{-aT} - Be^{-bT}$

$$(a-b)t \;\simeq\; \ln(aA/bB) \;\longrightarrow\; at \;\approx\; bt \;\approx\; 4\pi^2$$

$$m_{3/2} \;\simeq\; Ae^{-aT} \;\sim\; 10^{-14}, \qquad |a-b| \;\equiv\; \frac{a+b}{\ln \frac{M_{Pl}}{m_{3/2}}} \quad \text{(fine-tuning)}$$

Uplifting AdS to dS (Minkowski) (KKLT model)

Kachru, Kallosh, Linde and Trivedi, Phys.Rev.D68 (2003) 046005

$$V_{\rm lift} = D(T+\bar{T})^{n_P-2} \qquad \longrightarrow \quad V + V_{\rm lift} \;=\; V_{\rm dS} \;\geq\; 0$$

At the linear order of δ^t, $\quad t = t_{SUSY}(1+\delta^t), \quad F^T|_{t=t_{SUSY}} = 0$

Choi, Falkowski, Nilles, Olechowski, Nucl.Phys.B718 (2005) 113

$$\delta^t \;\simeq\; \frac{1-n_P}{2(at)^2} \qquad : \; (n_P = 0 \text{ for } \bar{D}3)$$

SUSY breaking order parameters

flux+GC : $\quad \frac{F^C}{C_0} \;\simeq\; m_{3/2} \;\approx\; w_0, \qquad \frac{F^T}{T+\bar{T}} \;\simeq\; \frac{2-n_P}{2}\frac{1}{at}m_{3/2}$

Racetrack : $\quad \frac{F^C}{C_0} \;\simeq\; m_{3/2} \;\approx\; Ae^{-at}, \qquad \frac{F^T}{T+\bar{T}} \;\simeq\; \frac{3(2-n_P)}{4}\frac{1}{at\,bt}m_{3/2}$

$$\alpha \;\equiv\; \frac{1}{4\pi^2}\frac{F^C}{C_0}\frac{T+\bar{T}}{F^T} \;\approx\; \begin{cases} \dfrac{at}{\ln(M_{Pl}/m_{3/2})}\dfrac{2}{2-n_P} \;\sim\; 1 & : \text{ flux+GC} \\ \dfrac{at\,bt}{\ln(M_{Pl}/m_{3/2})}\dfrac{4}{3(2-n_P)} \;\sim\; 4\pi^2 & : \text{ racetrack} \end{cases}$$

Ⅲ. Moduli-mixing racetrack model

Moduli-mixing gauge coupling $\mathrm{Re}\, f_a = 1/g_a^2, \quad S = s + i\sigma_s, \quad T = t + i\sigma_t$

Type IIB $O3/O7$, magnetized D-branes:

Lüst, Mayr, Richter, Stieberger, Nucl.Phys. B696 (2004) 205

$$\begin{aligned} f_{mD7} &= |m_{D7}|S + |w_{D7}|T \\ f_{mD9} &= m_{D9}S - w_{D9}T \end{aligned} \qquad \left\{ \begin{aligned} m_{D7,D9} &: \text{ magnetic flux number} \\ w_{D7,D9} &: \text{ winding number} \end{aligned} \right.$$

Heterotic (M-)theory:

Choi, Kim, Phys.Lett.B165 (1985) 71, ...

Lukas, Ovrut, Waldram, Nucl.Phys. B532 (1998) 43

$$f_H = S - \beta T + f_{M5}, \qquad \beta \sim \frac{1}{16\pi^4}\int_{CY} J \wedge \left(\mathrm{tr}\, F_2^2 - \frac{1}{2}\mathrm{tr}\, R^2 \right)$$

$SU(N_a)$ gaugino condensation:

$$W_{GC} = N_a e^{-8\pi^2 f_a / N_a} \qquad \longrightarrow \qquad \textbf{moduli-mixing superpotential}$$

Single light modulus

H.A., Higaki, Kobayashi, Phys.Rev.D73 (2006) 046005

Assuming $\langle \partial_S \partial_{U^\alpha} W_{flux} \rangle \neq 0$, $\langle W_{flux} \rangle = 0$ (S is stabilized at high scale)

$$K = -\ln\langle S + \bar{S} \rangle - 3\ln(T + \bar{T})$$

$$W = Ae^{-a(m_a\langle S\rangle \pm w_a T)} \pm Be^{-b(m_b\langle S\rangle \pm w_b T)}$$

Typical models: $A' = Ae^{-am_a\langle S\rangle}$, $B' = Be^{-bm_b\langle S\rangle}$

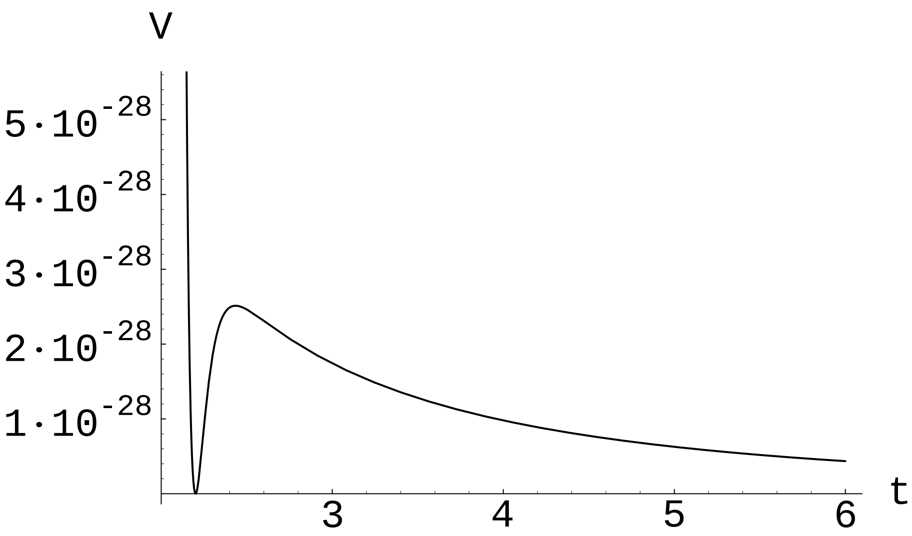
$$\begin{aligned}
W_{D3+mD7} &= A' - B'e^{-bw_bT} &:&\quad 0 < \alpha \lesssim 1 \\
W_{D3+mD9} &= A' + B'e^{+bw_bT} &:&\quad -1 \lesssim \alpha < 0 \\
W_{mD7+mD7'} &= A'e^{-aw_aT} - B'e^{-bw_bT} &:&\quad 1 < \alpha \lesssim 4\pi^2 \\
W_{mD7+mD9} &= A'e^{-aw_aT} + B'e^{+bw_bT} &:&\quad -4\pi^2 \lesssim \alpha < -1
\end{aligned}$$

$$\alpha = \frac{1}{4\pi^2}\frac{F^C}{C_0}\frac{T+\bar{T}}{F^T}$$

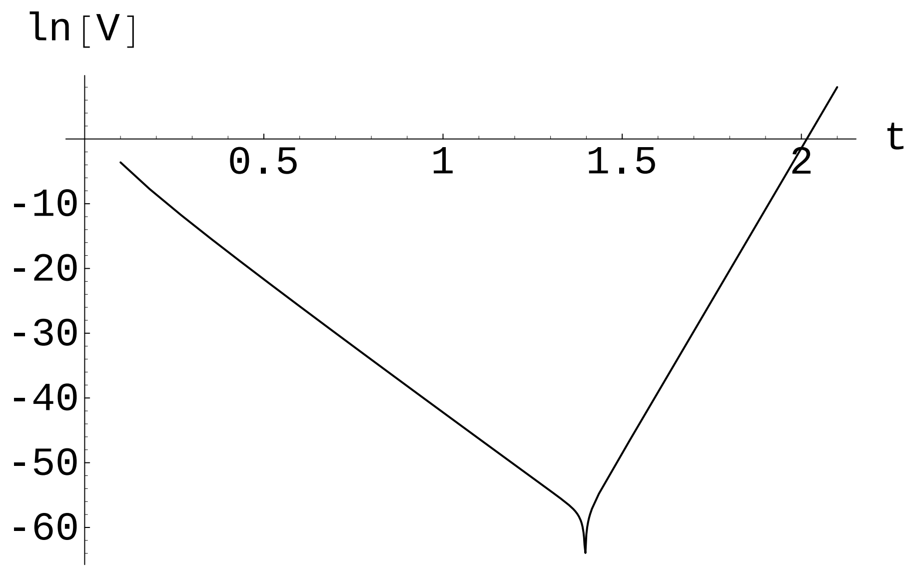
Implications on SUSY phenomenology/cosmology

- Fine-tuning $w_0 \ll 1$ is not necessary if $\langle W_{flux} \rangle = 0$ and Ae^{-aS} exists in W_{GC}.

- Destabilization/overshooting problem may be avoided in some case.

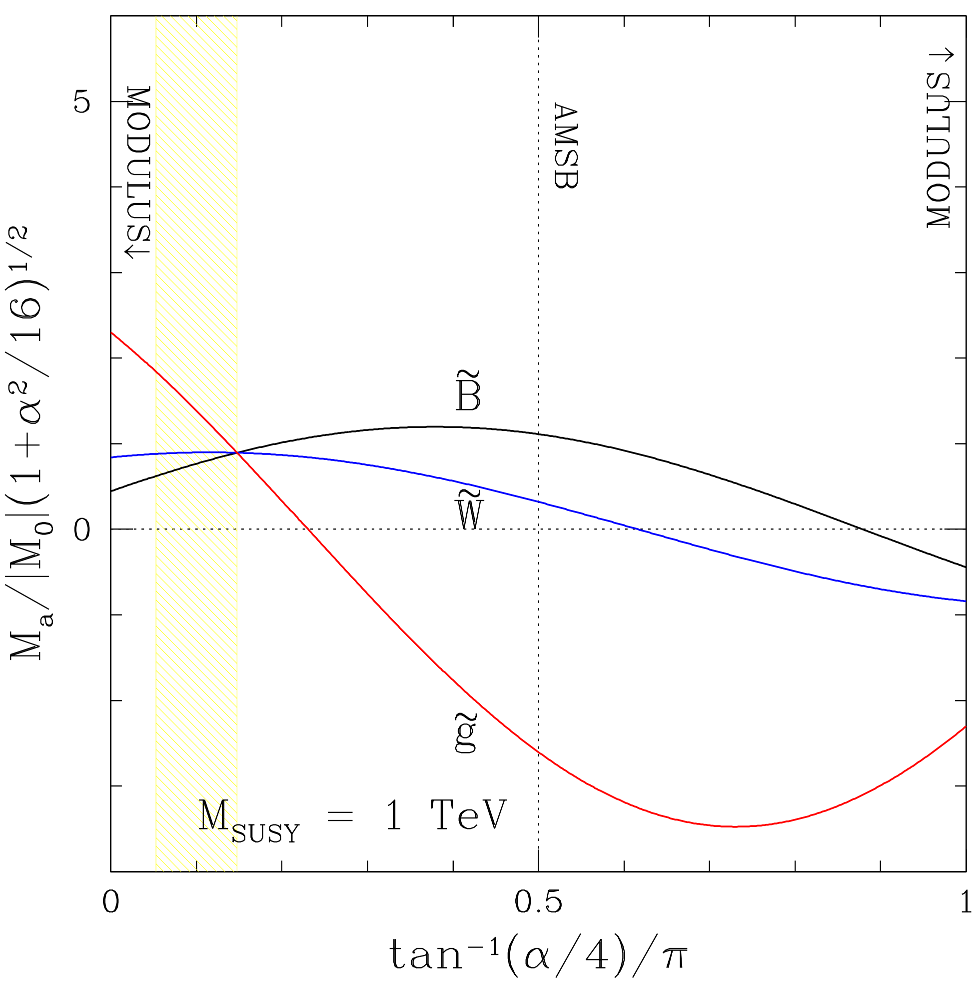


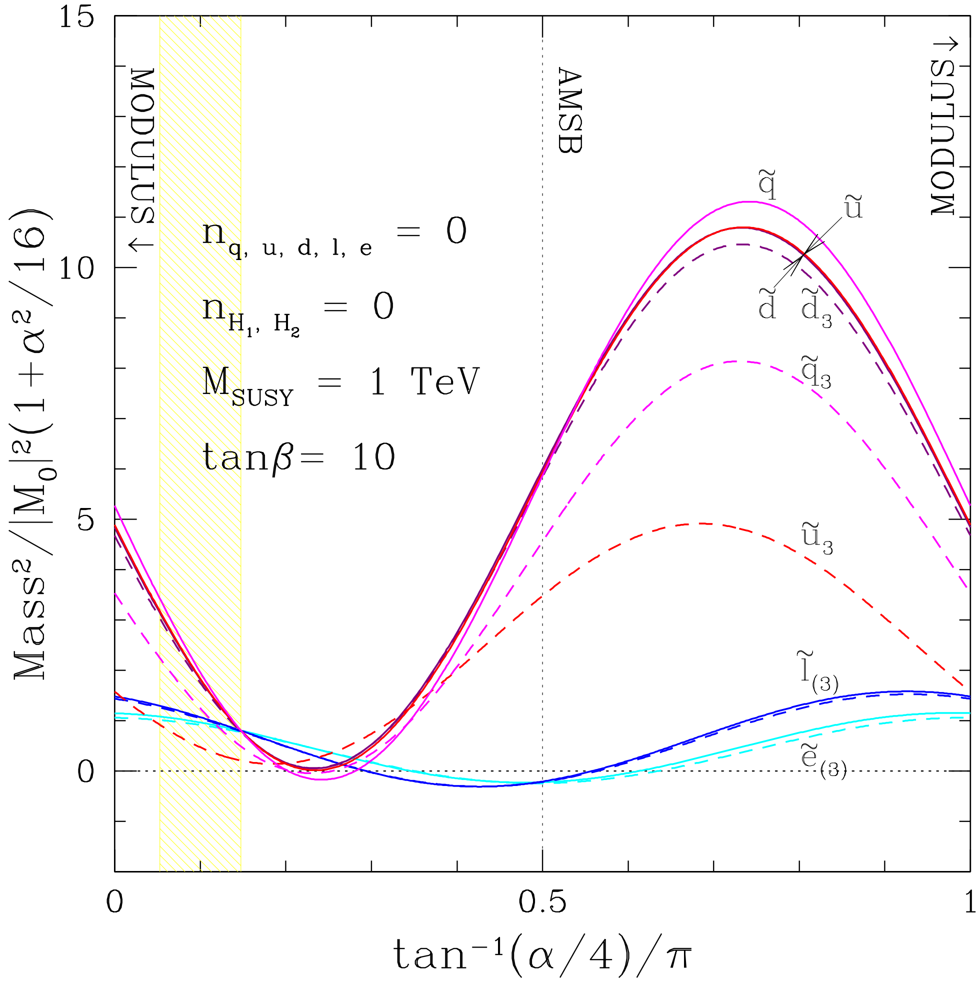
$\Longrightarrow$



- Modulus/anomaly ratio of SUSY breaking mediation, α can take various value depending on m, w and $\langle S \rangle = -\langle \partial_U f^{RR}(U) / \partial_U f^{NS}(U) \rangle$.

Gaugino and sfermion masses at TeV scale





Choi, Jeong, Okumura, JHEP09 (2005) 039

Two light moduli

H.A., Higaki, Kobayashi, Nucl.Phys.B742 (2006) 187

Assuming $\langle \partial_S \partial_{U^\alpha} W_{flux} \rangle = 0$, $\langle W_{flux} \rangle = 0$ (S also remains light)

$$K = -n_S \ln(S+\bar{S}) - n_T \ln(T+\bar{T})$$

$$W = A e^{-a(S-\alpha T)} - B e^{-b(S+\beta T)}$$

a, b, α, β, A, B, n_S, n_T: parameters of effective SUGRA

SUSY stationary point: F^S, $F^T = 0$ for $|s_{SUSY}| \gg n_S/2a,\ n_S/2b$

$$s_{SUSY} \sim \frac{1}{b-a} \ln \frac{bB}{aA}, \qquad t_{SUSY} \simeq \frac{n_T(b-a)}{2ab(\alpha+\beta)}, \qquad (W_{SUSY} \neq 0)$$

Negative vacuum energy: $V_{SUSY} = -3(m_{3/2}^{SUSY})^2 < 0$

$$m_{3/2}^{SUSY} \sim \frac{b-a}{b}\left(\frac{aA}{bB}\right)^{\frac{a}{b-a}} \frac{A}{(2s_{SUSY})^{n_S/2}(2t_{SUSY})^{n_T/2}}$$

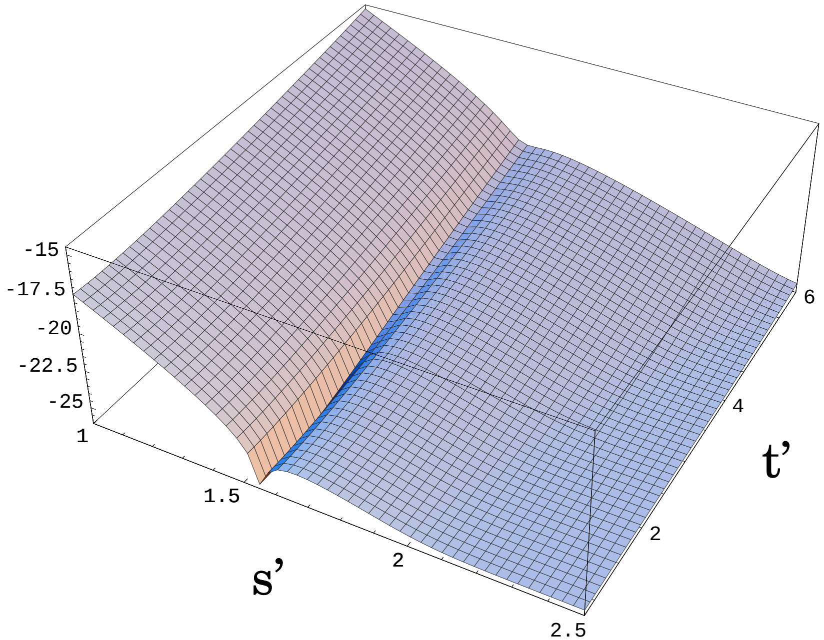
For $s_{SUSY}, t_{SUSY} > 1$ with a, $b \gg 1$ and A, $B \sim \mathcal{O}(1)$

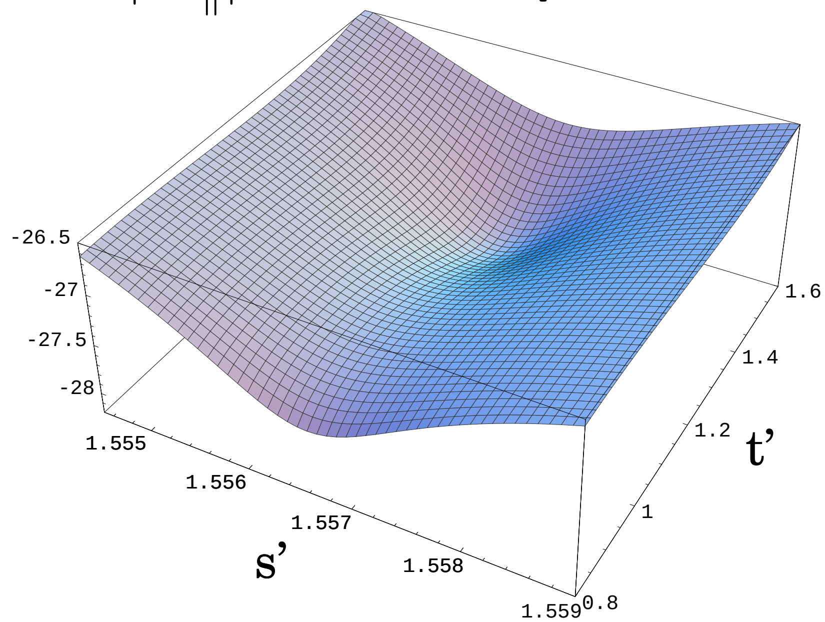
$$b - a \sim \mathcal{O}(1), \qquad B/A > a/b, \qquad \alpha, \beta \sim \mathcal{O}(1/ab),$$

Stability of SUSY point: mass eigenvalues $(m_\perp^2, m_\|^2)$

$$\begin{cases} m_\perp^2 \sim \dfrac{128}{n_S^2}\left(\ln \dfrac{bB}{aA}\right)^4 \dfrac{(ab)^2}{(b-a)^4}(m_{3/2}^{SUSY})^2 > 0 \\ m_\|^2 \sim -(m_{3/2}^{SUSY})^2 < 0 \end{cases} \longrightarrow \textbf{saddle point}$$

$$|m_\perp^2| \gg |m_\|^2| \longrightarrow \textbf{sharp racetrack}$$





SUSY breaking local minimum $t_{SB} = t_{SUSY}(1+\delta^t_{SB}), \quad s_{SB} = s_{SUSY}(1+\delta^s_{SB})$

$$\delta^t_{SB} = \frac{\sqrt{n_T+1}-1}{n_T} \lesssim \mathcal{O}(1), \qquad \delta^s_{SB} = -\frac{a\alpha + b\beta}{\ln(bB/aA)}\,\delta^t_{SB}$$

Order parameters

$$m_{3/2} \simeq \frac{e^{-a(\delta^s_{SB}-\alpha\delta^t_{SB})}}{(1+\delta^s_{SB})^{n_S/2}(1+\delta^t_{SB})^{n_T/2}}\, m_{3/2}^{SUSY},$$

$$\frac{F^T}{T+\bar{T}} \simeq -\delta^t_{SB}\, m_{3/2}, \qquad \frac{F^S}{S+\bar{S}} \sim \frac{n_S}{as}\frac{F^T}{T+\bar{T}} \ll \frac{F^T}{T+\bar{T}}$$

Vacuum energy

$$V_{SB} = n_S\left|\frac{F^S}{S+\bar{S}}\right|^2 + n_T\left|\frac{F^T}{T+\bar{T}}\right|^2 - 3m^2_{3/2} < -2m^2_{3/2}$$

AdS minimum with modulus-dominated SUSY breaking

$$\Downarrow \qquad V_{\rm lift} = D\, e^{2K/3}(T+\bar{T})^{n_P}(S+\bar{S})^{m_P}$$

Minkowski minimum with modulus-dominated SUSY breaking $\alpha \sim \frac{1}{4\pi^2} \ll 1$

Numerical results

s_{SUSY}	t_{SUSY}	V_{SUSY}	$(m_\perp^{SUSY}/m_{3/2}^{SUSY})^2$	$\lvert m_\parallel^{SUSY}/m_{3/2}^{SUSY}\rvert^2$
1.41	1.18	-1.79×10^{-26}	1.11×10^{6}	4.00
s_{SB}	t_{SB}	V_{SB}	$(m_\perp^{SB}/m_{3/2})^2$	$(m_\parallel^{SB}/m_{3/2})^2$
1.36	1.57	-1.82×10^{-26}	9.55×10^{5}	7.09
s_{dS}	t_{dS}	V_{dS}	$(m_\perp^{\mathrm{dS}}/m_{3/2}^{\mathrm{dS}})^2$	$(m_\parallel^{\mathrm{dS}}/m_{3/2}^{\mathrm{dS}})^2$
1.30	1.96	$+1.20\times10^{-34}$	8.16×10^{5}	4.15×10

AdS	$F^S/(S+\bar{S})$	$F^T/(T+\bar{T})$	$m_{3/2}$	$m_{3/2}^{SUSY}$
—	4.38×10^{-15}	-2.77×10^{-14}	8.28×10^{-14}	7.72×10^{-14}
dS	$F^S/(S+\bar{S})$	$F^T/(T+\bar{T})$	$m_{3/2}^{\mathrm{dS}}$	D
—	1.34×10^{-14}	-6.49×10^{-14}	9.78×10^{-14}	2.45×10^{-25}

Ⅳ. Summary

Moduli-mixing racetrack model

$$W = Ae^{-af_a} - Be^{-bf_b}, \qquad f_{a,b} = m_{a,b}S + w_{a,b}T$$

Single light modulus $S \to \langle S \rangle$, T

Modulus/anomaly ratio of SUSY breaking mediation α can take various values depending on m, w and $\langle S \rangle$
(c.f. $\alpha = 1$ in KKLT model without moduli mixing)

Two light moduli S, T

SUSY point is saddle point $\longrightarrow$ SUSY breaking AdS local minimum

Modulus-dominated SUSY breaking $\quad \alpha \ll 1$
(c.f. $\alpha \gg 1$ in racetrack model without moduli mixing)