Holographic quantum statistics & dual thermodynamics

- Black hole mechanics & thermodynamics
- Energy-entropy duality
- Holographic quantum statistics

C. Balázs and I. Szapudi  hep-th/0603133
hep-th/0605190
hep-th/0608nnn
Black hole mechanics

0\textsuperscript{th} law

$$\kappa = \text{Const}(R)$$

surface gravity is the same everywhere on the horizon

1\textsuperscript{st} law

$$dM = \frac{\kappa}{8\pi} dA$$

2\textsuperscript{nd} law

$$dA \geq 0$$

3\textsuperscript{rd} law

$$\kappa \neq 0$$

it's impossible to reduce \( \kappa \) to zero by a finite sequence of operations
Thermodynamics

$0^{th}$ law

\[ T_A = T_C \quad \& \quad T_B = T_C \Rightarrow T_A = T_B \]

A and B in thermal eq. with C ⇒ A in thermal eq. with B

$1^{st}$ law

\[ dE = T \, dS \]

$2^{nd}$ law

\[ dS \geq 0 \]

$3^{rd}$ law

\[ T \neq 0 \]

it's impossible to reduce T to zero by a finite sequence of operations
The 3rd law

Bekenstein-Hawking

\[ S = \pi R^2 \]

Hawking

\[ T = \frac{1}{4\pi R} \]

Black hole entropy

\[ S = \frac{1}{16\pi T^2} \]

Planck-Nernst

\[ \lim_{T \to 0} S(T) = \text{Const}(T) \]

Gravity ↔ quantum physics

hard to construct quantum models of black holes
Energy-Entropy duality

\[ S = \frac{1}{16\pi T^2} \Rightarrow \lim_{T_D \to 0} S(T_D) = 0 \]

\[ T_D = \frac{1}{T} \]

\[ dE = \frac{1}{T_D} dS \]

\[ E_D = S, \quad S_D = E \]

\[ dS_D = \frac{1}{T_D} dE_D \]
Dual thermodynamics

1st law

\[ dE = T \, dS - p \, dV + \mu \, dN \]

\[ dS_D = \frac{1}{T_D} \, dE_D + \frac{p_D}{T_D} \, dV_D - \frac{\mu_D}{T_D} \, dN_D \]

2nd law

\[ dS_D = dE = T \, dS - p \, dV + \mu \, dN \geq 0 \]

if \[ -p \, dV + \mu \, dN \geq 0 \]

3rd law

\[ \lim_{T_D \to 0} S_D(T_D) = \lim_{T \to 0} E \left( \frac{1}{T} \right) \]

Black hole: \[ E = \frac{R}{2} = \frac{1}{8 \pi T} \]

\[ \frac{1}{T} \to 0 \]
What does Energy-Entropy (E-)duality mean?

**Electric-magnetic duality**

\[ E \rightarrow H \quad H \rightarrow -E \quad e \rightarrow g = \frac{1}{2}e \quad g \rightarrow -e = -\frac{1}{2}g \]

**T-duality**

\[ R_C \leftrightarrow \frac{1}{R_C} \]

\[ T_D = \frac{1}{T} = 4\pi R \leftrightarrow R_D = \frac{1}{R} = 4\pi T \]

\[ R \leftrightarrow \frac{1}{R} \]

**AdS/CFT**

4D black hole \( \leftrightarrow \) 1D quantum gas

Wheeler: "It from bit"

\[ E = S_D, \quad E_D = S \]
One dimensional quantum gas

\[ E_D = p_D V_D = \frac{V_D}{2\pi} \int_0^\infty f(\epsilon, T_D, \mu_D) \epsilon d\epsilon \]

\[ N_D = \frac{V_D}{2\pi} \int_0^\infty f(\epsilon, T_D, \mu_D) d\epsilon \]

\[ f(\epsilon, T_D, \mu_D) = (e^{(\epsilon-\mu_D)/T_D} \pm 1)^{-1} \]

\[ S_D = \frac{1}{T_D} E_D + \frac{p_D}{T_D} V_D - \frac{\mu_D}{T_D} N_D \]

Quantum effects small if

\[ | \frac{\mu_D}{T_D} | \ll 1 \]
Holographic quantum statistics

One dimensional Bose gas

\[ E_D = p_D V_D = \frac{4 \pi^2 V_D}{3} \left( \frac{1}{16 \pi T^2} + \frac{3 \log(\mu)}{8 \pi^3 T^2} \mu + O\left(\frac{\mu}{T}\right) \right) \]

\[ S_D = \frac{4 \pi^2 V_D}{3} \left( \frac{1}{8 \pi T} - \frac{3 \log(\mu)}{8 \pi^3 T} \mu + O\left(\frac{\mu}{T}\right) \right) \]

\[ N_D = \frac{4 \pi^2 V_D}{3} \left( \frac{3 \log(\mu)}{8 \pi^3 T} + \frac{3 \mu}{16 \pi^3 T} \mu + O(\mu^2) \right) \]

Quantum effects small if

\[ \mu \ll 1 \]
Holographic quantum model

Application for black holes

\[ E = S_D = \frac{R}{2} + O\left(\frac{1}{R}\right) \]
if \[ V_D = \frac{3}{4} \pi^2 \]

\[ S = E_D = \pi R^2 - \frac{3}{2} \log(\pi R^2) + O\left(\frac{1}{R}\right) \]
if \[ \mu = \frac{\pi}{16 M^2} \]

\[ N = N_D = \frac{6 \log(2)}{\pi^2} M \]

2nd law of dual thermodynamics holds

\[ d\mathcal{V} = d\mathcal{V}_D = 0 \quad \text{and} \quad \mu \, dN \geq 0 \]
Holographic quantum model

Reproduced black hole properties

\[ E \sim R \sim \frac{1}{T}, \quad dE = TdS, \quad E = 2TS \quad \text{for} \quad \mu = 0 \]

\[ S = \pi R^2 - \frac{3}{2} \log(\pi R^2) + O\left(\frac{1}{R}\right) \]

\[ N = \frac{6 \log(2)}{\pi^2} \frac{MV}{M_P} \]

\[ \dim(V_D) = 1 \quad \text{and} \quad V_D = \frac{3}{4} \frac{L_P}{\pi^2} \]

entropy cutoff ↔ dual Fermi/Bose distribution
energy cutoff ↔ dual Fermi/Bose distribution
Conclusions

Black hole entropy conflicts Planck-Nernst theorem →
 it's hard to count quantum degrees of freedom in BHs

Energy-entropy duality bypasses conflict and maps
 black holes to weakly interacting quantum systems

The statistical mechanics of the simplest dual quantum
 systems reproduce classical black hole properties

Quantum corrections to black hole entropy and energy
 can also be easily calculated