Holographic quantum statistics
&
dual thermodynamics

- Black hole mechanics & thermodynamics
- Energy-entropy duality
- Holographic quantum statistics

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hep-th/0605190

hep-th/0608nnn

Black hole mechanics

0th law

$$\kappa = \mathrm{Const}(R)$$

surface gravity is the same everywhere on the horizon

1st law

$$dM = \frac{\kappa}{8\pi} dA$$

2nd law

$$dA \geq 0$$

3rd law

$$\kappa \neq 0$$

it's impossible to reduce κ to zero by a finite sequence of operations

Thermodynamics

0^{th} law

$$T_A = T_C \,\&\, T_B = T_C \Rightarrow T_A = T_B$$

A and B in thermal eq. with C ⇒ A in thermal eq. with B

1^{st} law

$$dE = T\,dS$$

2^{nd} law

$$dS \geq 0$$

3^{rd} law

$$T \neq 0$$

it's impossible to reduce T to zero by a finite sequence of operations

The 3rd law

Bekenstein-Hawking

$$S = \pi R^2$$

Hawking

$$T = \frac{1}{4\pi R}$$

Black hole entropy

$$S = \frac{1}{16\pi T^2}$$

Planck-Nernst

$$\lim_{T \to 0} S(T) = \mathrm{Const}(T)$$

Gravity →← quantum physics

hard to construct quantum models of black holes

Energy-Entropy duality

$$S = \frac{1}{16\pi T^2} \quad \Rightarrow \quad \lim_{T_D \to 0} S(T_D) = 0$$

$$T_D = \frac{1}{T}$$

$$dE = \frac{1}{T_D} dS$$

$$E_D = S, \quad S_D = E$$

$$\boxed{dS_D = \frac{1}{T_D} dE_D}$$

Dual thermodynamics

1st law

$$dE = T\,dS - p\,dV + \mu\,dN$$

$$dS_D = \frac{1}{T_D}\,dE_D + \frac{p_D}{T_D}\,dV_D - \frac{\mu_D}{T_D}\,dN_D$$

2nd law

$$dS_D = dE = T\,dS - p\,dV + \mu\,dN \geq 0$$

$$\text{if} \qquad -p\,dV + \mu\,dN \geq 0$$

3rd law

$$\lim_{T_D \to 0} S_D(T_D) = \lim_{\frac{1}{T} \to 0} E\left(\frac{1}{T}\right)$$

$$\text{Black hole: } E = \frac{R}{2} = \frac{1}{8\pi T} \xrightarrow[\frac{1}{T} \to 0]{} 0$$

What does Energy-Entropy (E-)duality mean?

Electric-magnetic duality

$$\mathcal{E} \to \mathcal{H} \qquad \mathcal{H} \to -\mathcal{E} \qquad e \to g = \frac{1}{2e} \qquad g \to -e = -\frac{1}{2g}$$

T-duality

$$R_C \leftrightarrow \frac{1}{R_C}$$

$$T_D = \frac{1}{T} = 4\,\pi\,\mathrm{R} \quad \Leftrightarrow \quad R_D = \frac{1}{R} = 4\,\pi\,\mathrm{T}$$

$$R \leftrightarrow \frac{1}{R}$$

AdS/CFT

4D black hole $\leftrightarrow$ 1D quantum gas

Wheeler: "It from bit"

$$\mathrm{E} = S_D, \quad E_D = S$$

Holographic quantum statistics

One dimensional quantum gas

$$E_D = p_D V_D = \frac{V_D}{2\pi} \int_0^\infty f(\epsilon, T_D, \mu_D)\, \epsilon\, d\epsilon$$
$$N_D = \frac{V_D}{2\pi} \int_0^\infty f(\epsilon, T_D, \mu_D)\, d\epsilon$$

$$f(\epsilon, T_D, \mu_D) = \left(e^{(\epsilon - \mu_D)/T_D} \pm 1\right)^{-1}$$

$$S_D = \frac{1}{T_D} E_D + \frac{p_D}{T_D} V_D - \frac{\mu_D}{T_D} N_D$$

Quantum effects small if

$$\left| \frac{\mu_D}{T_D} \right| \ll 1$$

Holographic quantum statistics

One dimensional Bose gas

$$E_D = p_D V_D = \frac{4\pi^2 V_D}{3}\left(\frac{1}{16\pi T^2} + \frac{3\log(\mu)}{8\pi^3 T^2}\mu + O\left(\frac{\mu}{T}\right)\right)$$

$$S_D = \frac{4\pi^2 V_D}{3}\left(\frac{1}{8\pi T} - \frac{3\log(\mu)}{8\pi^3 T}\mu + O\left(\frac{\mu}{T}\right)\right)$$

$$N_D = \frac{4\pi^2 V_D}{3}\left(\frac{3\log(\mu)}{8\pi^3 T} + \frac{3\mu}{16\pi^3 T}\mu + O(\mu^2)\right)$$

Quantum effects small if

$$\mu \ll 1$$

Holographic quantum model

Application for black holes

$$E = S_D = \frac{R}{2} + O\left(\frac{1}{R}\right)$$

$$\text{if} \quad V_D = \frac{3}{4\pi^2}$$

$$S = E_D = \pi R^2 - \frac{3}{2}\log(\pi R^2) + O\left(\frac{1}{R}\right)$$

$$\text{if} \quad \mu = \frac{\pi}{16 M^2}$$

$$N = N_D = \frac{6\log(2)}{\pi^2} M$$

2^{nd} law of dual thermodynamics holds

$$dV = dV_D = 0 \quad \text{and} \quad \mu\, dN \geq 0$$

Holographic quantum model

Reproduced black hole properties

$$E \sim R \sim \frac{1}{T}, \quad dE = T\,dS, \quad E = 2\,T\,S \quad \text{for}\ \mu = 0$$

$$S = \pi R^2 - \frac{3}{2}\log(\pi R^2) + O(\frac{1}{R})$$

$$N = \frac{6\log(2)}{\pi^2}\,M/M_P$$

$$\dim(V_D) = 1 \quad \text{and} \quad V_D = \frac{3}{4\pi^2}\,L_P$$

entropy cutoff $\leftrightarrow$ dual Fermi/Bose distribution

energy cutoff $\leftrightarrow$ dual Fermi/Bose distribution

Conclusions

Black hole entropy conflicts Planck-Nernst theorem →
it's hard to count quantum degrees of freedom in BHs

Energy-entropy duality bypasses conflict and maps
black holes to weakly interacting quantum systems

The statistical mechanics of the simplest dual quantum
systems reproduce classical black hole properties

Quantum corrections to black hole entropy and energy
can also be easily calculated