2T-physics and the Standard Model of Particles and Forces

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- Success of 2T-physics for particles on worldlines.
- Field theory version of 2T-physics.
- Standard Model in 4+2 dimensions.
- Fundamental SM_{4+2} gives emergent SM_{3+1}, New features:
  - Avoid strong CP violation (no $U(1)_{\text{Peccei-Quinn}}$, no elusive axion)
  - New concepts on source of mass [1) dilaton, 2) higher dim.]
  - New methods of investigation: duality, holography, hidden symm., emergent 1T spacetimes and dynamics.

Sp(2,R) gauge symmetry
(X^M, P^M) indistinguishable at any instant

Generalizes τ reparametrization

$$\partial_\tau x^\mu p_\mu - \frac{1}{2} e p_\mu p_\nu \eta^{\mu\nu}$$

i=1,2; symmetric matrix 2x2

$$\mathcal{L}_{2T} = \partial_\tau X^M P_M - \frac{1}{2} A^{ij} Q_{ij}(X,P)$$

1) Quantum commutation rules

2) Any Lagrangian L=X'.P – …

Spinless particle in any

background

Sp(2,R) Lie algebra required

$$[J_\mu, J_\nu] = i\varepsilon_{\mu\nu\lambda}\eta^{\lambda\sigma}J_\sigma,\ \varepsilon_{012} = 1$$

μ=0,1,2 , η=diag(-1,1,1)
SO(1,2)=Sp(2,R)

$$[Q_{ij}, Q_{kl}] = \frac{i}{2}(\varepsilon_{ik}Q_{jl} + \varepsilon_{jk}Q_{il} + \varepsilon_{il}Q_{jk} + \varepsilon_{jl}Q_{ik})$$

$$J_2 = Q_{12}$$
$$J_0 - J_1 = Q_{11}$$
$$J_0 + J_1 = Q_{22}$$

3 local symmetry parameters of Sp(2,R)

$$\delta X^M = \omega^{ij}(\tau)\{Q_{ij}, X^M\} = \omega^{ij}(\tau)\,\partial Q_{ij}/\partial P_M$$

$$\delta P^M = \omega^{ij}(\tau)\{Q_{ij}, P^M\} = -\omega^{ij}(\tau)\,\partial Q_{ij}/\partial X^M$$

Transformation law of (X,P) depends on form of Q_{ij}(X,P)

$$\delta A_i^{\ j} = \partial_\tau \omega_i^{\ j} + [A, \omega]_i^{\ j}$$

3 gauge fields of Sp(2,R).
2 more compared to τ reparametrization

Example: flat background

$$Q_{11} = \frac{X \cdot X}{2}$$

$$Q_{22} = \frac{P \cdot P}{2}$$

$$Q_{12} = \frac{X \cdot P + P \cdot X}{4}$$

Sp(2,R) doublet: $\begin{pmatrix} X^M(\tau) \\ P^M(\tau) \end{pmatrix}$

$X_i^M(\tau)$, i=1,2

Physical sector, gauge invariant →

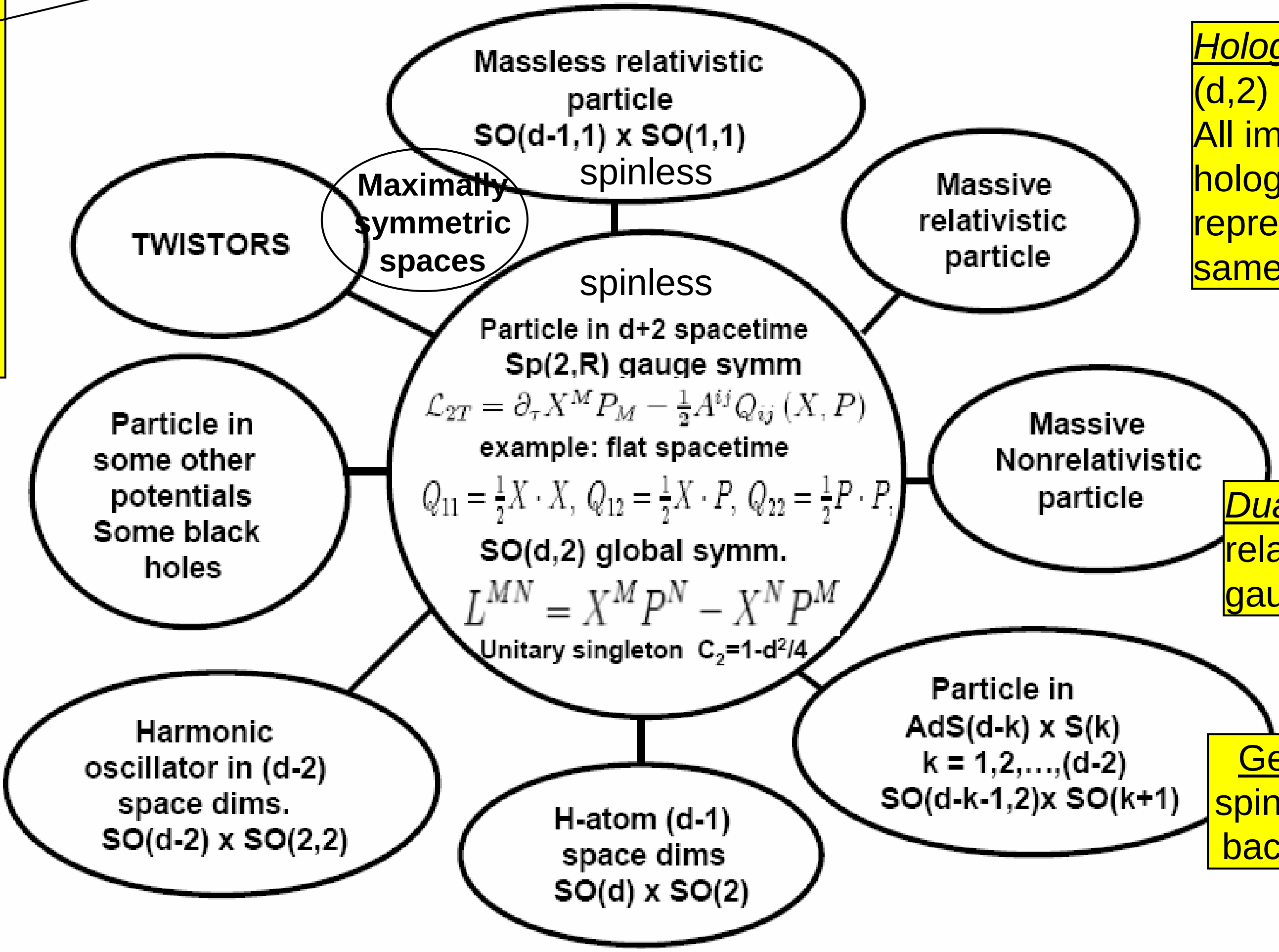
Q_{ij} (X,P)= 0 has nontrivial solutions only if signature is (d,2) : (- - ++++…+)

Emergent spacetimes & dynamics, hidden symmetries from gauge fixing the simplest model of 2T theory

$(d,2)-(1,1)$ signature of extra gauge parameters $=(d-1,\ 1)$ emergent space-time

Emergent spacetime: Sp(2,R) gauge choices. Some combination of X^M, P^M is fixed as t,H. Can fix 3 gauges, but fix 2 or 3

Holography: from (d,2) to (d-1,1). All images holographically represent the same 2T system



spinless

Particle in d+2 spacetime

Sp(2,R) gauge symm

$$\mathcal{L}_{2T}=\partial_\tau X^M P_M-\tfrac{1}{2}A^{ij}Q_{ij}\,(X,P)$$

example: flat spacetime

$$Q_{11}=\tfrac{1}{2}X\cdot X,\ Q_{12}=\tfrac{1}{2}X\cdot P,\ Q_{22}=\tfrac{1}{2}P\cdot P,$$

SO(d,2) global symm.

$$L^{MN}=X^M P^N-X^N P^M$$

Unitary singleton C_2=1-d²/4

Massless relativistic particle
SO(d-1,1) x SO(1,1)
spinless

Maximally symmetric spaces

TWISTORS

Massive relativistic particle

Massive Nonrelativistic particle

Particle in some other potentials
Some black holes

Harmonic oscillator in (d-2) space dims.
SO(d-2) x SO(2,2)

H-atom (d-1) space dims
SO(d) x SO(2)

Particle in AdS(d-k) x S(k)
k = 1,2,…,(d-2)
SO(d-k-1,2)x SO(k+1)

Hidden symmetry: All images have hidden SO(d,2) symmetry, for the example.

Duality: Sp(2,R) relates one fixed gauge to another

Generalizations
spin, susy, strings, background fields

Unification: 2T-physics unifies diverse forms of 1T-physics into a single theory.

2T-physics

- Fundamental concept is Sp(2,R) gauge symmetry
  (X^M,P^M) are indistinguishable at any instant. (1998)

1) Quantum commutation rules
2) Any Lagrangian L=X'.P – …

Symmetry requires target space signature (d,2); 1 extra time, 1 extra space.

Gauge symmetry effectively reduces (d,2) to (d-1,1) [**NOT** Kaluza-Klein]

Nontrivial because of many ways of embedding (d-1,1) in (d,2)
different components of (X^M,P^M) become time & Hamiltonian in (d-1,1)
same system in (d,2) looks very different 1T-dynamics depending on which (d-1,1)

- Advantages/features: Notice structures in 1T-physics that were missed before
  Holography, Duality, Hidden global symmetries, Unification

- 2T-physics works. Correct description of Nature!!
  Tested and verified in simple "everyday" 1T systems, classical & quantum.
  Standard Model of Particles and Forces
  4+2 theory gives 3+1 theory, and explains more … and new tools…

Field equations in 2T-physics

Derived from Sp(2,R) in hep-th/0003100; also Dirac 1936 other approach

$$X^2|\Phi\rangle = 0,\ \ P^2|\Phi\rangle = 0,\ \ (X\cdot P + P\cdot X)\,|\Phi\rangle = 0.$$

Constraints = 0 on physical states
i.e. Sp(2,R) gauge invariant

$$\hat{\Phi}\,(X)\ =\ \langle X|\Phi\rangle$$

Probability
amplitude
is the field

$$X^2\hat{\Phi}\,(X) = 0,\ \partial_M\partial^M\hat{\Phi}\,(X) = 0,\ X^M\partial_M\hat{\Phi}\,(X) + \partial_M\left(X^M\hat{\Phi}\,(X)\right) = 0.$$

kinematic #1

$$\hat{\Phi}\,(X) = \delta\left(X^2\right)\Phi\,(X)$$

kinematic #2

$$\left(X\cdot\partial\Phi + \frac{d-2}{2}\Phi\right)_{X^2=0} = 0$$

Kinematic eom´s say how to embed d dims in d+2 dims.

dynamical eq. extended with interaction

$$\left[\partial^2\Phi - V'\,(\Phi)\right]_{X^2=0} = 0$$

3 eqs. in d+2
= KG in d

$$\frac{\partial}{\partial X^M}\delta\left(X^2\right) = 2X_M\delta'\left(X^2\right),\ X\cdot\frac{\partial}{\partial X}\delta\left(X^2\right) = 2X^2\delta'\left(X^2\right) = -2\delta\left(X^2\right),$$
$$\partial^2\delta\left(X^2\right) = 2\,(d+2)\,\delta'\left(X^2\right) + 4X^2\delta''\left(X^2\right) = 2\,(d-2)\,\delta'\left(X^2\right).$$

Subtleties of derivatives of delta function

$$\delta_\Lambda\Phi = X^2\Lambda\,(X)$$

gauge
symmetry

$$\Phi\,(X) = \Phi_0\,(X) + X^2\tilde{\Phi}\,(X)$$

Physical
part of field

remainder

$$\Phi_0 \equiv \left[\Phi\,(X)\right]_{X^2=0}$$

Action for scalar field in 2T-physics

Obtain 3 equations not just one : 2 kinematic and 1 dynamic.

$$S(\Phi)=\int d^{d+2}X\ \left\{B(X)\,\partial^2\left[\Phi\,\delta\left(X^2\right)\right]-\delta\left(X^2\right)\left[B(X)\,V'(\Phi)+U(\Phi)\right]\right\}$$

BRST approach for Sp(2,R)
Like string field theory
I.B.+Kuo hep-th/0605267

Gauge symmetries Λ and b

$$\delta_\Lambda\Phi=X^2\Lambda(X)$$

Works only for unique V(Φ)→ $\Phi^{\frac{2d}{d-2}}$

$$\delta_b B=\left(X\cdot\partial+\frac{d-2}{2}\right)b-\frac{1}{4}X^2\left(\partial^2 b-bV''(\Phi)\right),\ \text{any } b(X).$$

Gauge fixed version is more familiar looking

$$S(\Phi)=2\gamma\int d^{d+2}X\ \delta\left(X^2\right)\left[\frac{1}{2}\Phi\partial^2\Phi-\lambda\frac{d-2}{2d}\Phi^{\frac{2d}{d-2}}\right]$$

$$\Phi(X)=\Phi_0(X)+X^2\tilde{\Phi}(X)$$

$$\left(X\cdot\partial+\frac{d+2}{2}\right)\tilde{\Phi}=0$$

Gauge fixed to homogeneous remainder, but general Φ_0

There is remaining gauge freedom and remaining gauge symmetry that is sufficient to still uniquely determine V(Φ)

Minimizing the action gives two equations, so get all 3 Sp(2,R) constraints from the action

$$\delta S(\Phi)=2\gamma\int d^{d+2}X\ \delta\Phi\left\{\begin{array}{c}\delta\left(X^2\right)\left[\partial^2\Phi-V'(\Phi)\right]\\+2\delta'\left(X^2\right)\left[X\cdot\partial\Phi+\frac{d-2}{2}\Phi\right]\end{array}\right\}$$

kinematic #1,2 dynamical eq.

Gauge symmetries for the Standard Model in 4+2 dimensions

Guiding principles : 2Tgauge symmetry, SU(3)xSU(2)xU(1) YM gauge symmetry, renormalizability

2Tgauge-symmetry given by

There is a separate 2Tgauge parameter for every field, so remainder of every field is gauge freedom.

$$\Phi(X)=\Phi_0(X)+\cancel{X^2\tilde{\Phi}(X)}$$

$$A_M(X)=A_M^0(X)+\cancel{X^2\tilde{A}_M(X)}$$

$$\Psi^{L,R}(X)=\Psi_0^{L,R}(X)+\cancel{X^2\Psi_1^{L,R}(X)}$$

remainders proportional to X^2

Dilaton

Higgs

$$\left.\begin{array}{l}\delta_\Lambda\Phi=X^2\Lambda,\ \delta_\Lambda H^i=X^2\Lambda^i,\\ \delta_b B_\Phi,\delta_b B_{H^i}\ \text{as in Eq.(2.11)}\end{array}\right\}\ i\ \text{spans all other scalar fields,}$$

$$\left.\begin{array}{l}\delta_\zeta\Psi^{L_\alpha}=X^2\zeta_1^{L_\alpha}+\not{X}\zeta_2^{R_\alpha},\\ \delta_\zeta\Psi^{R_\beta}=X^2\zeta_1^{R_\beta}+\overline{\not{X}}\zeta_2^{L_\beta},\end{array}\right\}\ \alpha,\beta\ \text{span all fermions,}$$

3 families of quarks and leptons but all are left/right quartet spinors of SU(2,2)=SO(6,2)

$$\left.\begin{array}{l}\delta_a A_M^r=X^2 a_M^r\Phi^{-\frac{2(d-4)}{d-2}},\\ \delta_b B_{A_M^r}\ \text{similar}^{14}\ \text{to Eq.(2.11)}\end{array}\right\}\ r\ \text{spans all gauge bosons.}$$

SU(3)xSU(2)xU(1) gauge bosons, but all are SO(6,2) vectors

Action of the Standard Model in 4+2 dimensions

$$S\left(A,\Psi^{L,R},H,\Phi\right)=Z\int\left(d^6X\right)\ \delta\left(X^2\right)\ L\left(A,\Psi^{L,R},H,\Phi\right)$$

$$L\left(A,\Psi^{L,R},H,\Phi\right)=L\left(A\right)+L\left(A,\Psi^{L,R}\right)+L\left(\Psi^{L,R},H\right)+L\left(A,\Phi,H\right)$$

Gauge fields

SU(3) SU(2) U(1)

$$L\left(A\right)=-\frac{1}{4}Tr_3\left(G_{MN}G^{MN}\right)-\frac{1}{4}Tr_2\left(W_{MN}W^{MN}\right)-\frac{1}{4}B_{MN}B^{MN}.$$

quarks & leptons 3 families

$$\begin{aligned}L\left(A,\Psi^{L,R}\right)=&\frac{1}{2}\left(\bar{Q}^{L_i}\ \not{X}\overline{\not{D}}Q^{L_i}+\bar{Q}^{L_i}\overleftarrow{\overline{\not{D}}}\overline{\not{X}}Q^{L_i}\right)+\frac{1}{2}\left(\bar{L}^{L_i}\ \not{X}\overline{\not{D}}L^{L_i}+\bar{L}^{L_i}\overleftarrow{\overline{\not{D}}}\overline{\not{X}}L^{L_i}\right)\\&+\frac{1}{2}\left(\bar{d}^{R_j}\overline{\not{X}}\ \not{D}d^{R_j}+\bar{d}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}d^{R_j}\right)+\frac{1}{2}\left(\bar{e}^{R_j}\overline{\not{X}}\ \not{D}e^{R_j}+\bar{e}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}e^{R_j}\right)\\&+\frac{1}{2}\left(\bar{u}^{R_j}\overline{\not{X}}\ \not{D}u^{R_j}+\bar{u}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}u^{R_j}\right)+\frac{1}{2}\left(\bar{\nu}^{R_j}\overline{\not{X}}\ \not{D}\nu^{R_j}+\bar{\nu}^{R_j}\overleftarrow{\overline{\not{D}}}\ \not{X}\nu^{R_j}\right)\end{aligned}$$

$$\bar{Q}^{L_i}\ \not{X}\overline{\not{D}}Q^{L_i}$$

Yukawa couplings to Higgs

$$L\left(\Psi^{L,R},H\right)=-i\ \left(\begin{array}{c}\left(g_u\right)_{ij}\bar{Q}^{L_i}\ \not{X}u^{R_j}H^c-\left(g_u^\dagger\right)_{ji}\bar{H}^c\bar{u}^{R_j}\overline{\not{X}}Q^{L_i}\\+\left(g_d\right)_{ij}\bar{Q}^{L_i}\ \not{X}d^{R_j}H-\left(g_u^\dagger\right)_{ji}\bar{H}\bar{d}^{R_j}\overline{\not{X}}Q^{L_i}\\+\left(g_\nu\right)_{ij}L^{L_i}\ \not{X}\nu^{R_j}H^c-\left(g_\nu^\dagger\right)_{ji}\bar{H}^c\bar{\nu}^{R_j}\overline{\not{X}}L^{L_i}\\+\left(g_e\right)_{ij}L^{L_i}\ \not{X}e^{R_j}H-\left(g_e^\dagger\right)_{ji}\bar{H}\bar{e}^{R_j}\overline{\not{X}}L^{L_i}\end{array}\right)$$

$$\bar{Q}^{L_i}\ \not{X}d^{R_j}H$$

Higgs and dilaton

$$L\left(A,\Phi,H\right)=\frac{1}{2}\Phi\partial^2\Phi+\frac{1}{2}\left(H^\dagger D^2H+\left(D^2H\right)^\dagger H\right)-V\left(\Phi,H\right)$$

$$V\left(\Phi,H\right)=\frac{\lambda}{4}\left(H^\dagger H-\alpha^2\Phi^2\right)^2+V\left(\Phi\right)$$

quadratic mass terms not allowed

No F*F terms

Emergent scalars in 3+1 dimensions

$$\text{lightcone type basis in } 4+2 \text{ dimensions}$$

$$ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

$$X^{\pm'} = \frac{1}{\sqrt{2}}\left(X^{0'} \pm X^{1'}\right)$$

$$X^{+'} = \kappa,\ X^{-'} = \kappa\lambda,\ X^\mu = \kappa x^\mu \longleftarrow$$

Embedding of 3+1 in 4+2 defines emergent spacetime x^μ. This is analog of Sp(2,R) gauge fixing

$$\kappa = X^{+'},\ \ \lambda = \frac{X^{-'}}{X^{+'}},\ x^\mu = \frac{X^\mu}{X^{+'}}$$

x^μ and λ are homogeneous coordinates

$X^2=0$

$$\left(d^6X\right)\ \delta\left(X^2\right) = \kappa^5 d\kappa\ d^4x\ d\lambda\ \delta\left(\kappa^2\left(2\lambda - x^2\right)\right)$$

Solve kinematic equations in extra dimensions

$$\left(X\cdot\partial + \tfrac{d-2}{2}\right)\Phi = \left(\kappa\frac{\partial}{\partial\kappa} + 1\right)\Phi = 0$$

$$\Phi_0 + X^2\tilde{\Phi}$$

$$\Phi\left(X\right) = \Phi\left(\kappa, \lambda, x^\mu\right) = \kappa^{-1}\underline{\Phi}\left(x, \lambda\right) = \kappa^{-1}\left[\phi\left(x\right) + \left(\lambda - \frac{x^2}{2}\right)\cancel{\tilde{\phi}\left(x, \lambda\right)}\right]$$

Remainder is gauge freedom, remove it by fixing the 2Tgauge-symmetry at any λ,κ,x

Result of gauge fixing and solving kinematic eoms is fields only in 3+1

$$\Phi\left(X\right) = \kappa^{-1}\phi\left(x\right)$$

Dynamics only in 3+1

$$\partial^M\partial_M\Phi\left(X\right) = \frac{1}{\kappa^3}\frac{\partial^2\phi\left(x\right)}{\partial x^\mu \partial x_\mu}$$

Emergent fermions in 3+1 dimensions

$$\Psi^{L,R}(X) = \Psi_0^{L,R}(X) + X^2 \cancel{\tilde{\Psi}^{L,R}}(X)$$

choose $X^2\xi_1$
2Tgauge symm.

$$\left(X\cdot\partial+\frac{d}{2}\right)\Psi^{L,R} = \left(\kappa\frac{\partial}{\partial\kappa}+2\right)\Psi^{L,R} = 0$$

Impose kinematical eom in extra dimension

$$\Psi^{L,R}(X) = \kappa^{-2}\chi^{L,R}(x)$$

4 component SU(2,2) chiral fermions

choose ξ_2
2Tgauge symm.

$$\Gamma^{+'}\Psi^{L,R} = 0 \qquad \Psi^{L,R}(X) = \frac{1}{2^{1/4}\kappa^2}\begin{pmatrix}\psi^{L,R}(x)\\ 0\end{pmatrix}$$

2 component SL(2,C) chiral fermions

$$\overline{\not{D}}\Psi^L = \frac{1}{2^{1/4}\kappa}\begin{pmatrix}\bar{\sigma}^\mu D_\mu & -i\sqrt{2}\left(\kappa D_\kappa-\lambda\partial_\lambda-x^\mu D_\mu\right)\\ -i\sqrt{2}\partial_\lambda & -\sigma^\mu D_\mu\end{pmatrix}\begin{pmatrix}\frac{1}{\kappa^2}\psi^L(x)\\ 0\end{pmatrix} = \frac{1}{2^{1/4}\kappa^3}\begin{pmatrix}\bar{\sigma}^\mu D_\mu\psi^L(x)\\ 0\end{pmatrix}$$

4+2 Lagrangian descends to 3+1 standard Lagrangian. No explicit X.

$$\bar{\Psi}^L\ \not{X}\,\overline{\not{D}}\Psi^L = \frac{i}{\kappa^4}\bar{\psi}^L\bar{\sigma}^\mu D_\mu\psi^L\ ,\quad -i\,g\bar{\Psi}^L\ \not{X}\Psi^R H = \frac{g}{\kappa^4}\bar{\psi}^L\psi^R h$$

standard 3+1 kinetic term

standard 3+1 Yukawa term

Translation invariance in 3+1 comes from rotation invariance in 4+2

Emergent gauge bosons in 3+1 dimensions

start with YM axial gauge

$$X \cdot A = 0$$

kinematic equation simplifies → homogeneous

$$X^N F_{NM} = \left(X \cdot \partial + 1\right) A_M = \left(\kappa \partial_\kappa + 1\right) A_M = 0$$

There is leftover YM gauge symm.

$$X \cdot \delta_\Lambda A = 0 \rightarrow X \cdot \partial \Lambda = 0$$

homogeneous Λ enough to gauge fix $A^{+'}=0$

$$A_{-'} = -\eta_{-'+'} A^{+'} = 0$$

Solution of X.A=0

$$A^{-'} = -A_{+'} = \frac{1}{\kappa} x^\mu \underline{A}_\mu$$

Only independent

$$\rightarrow A^\mu\left(X\right) = \frac{1}{\kappa} \underline{A}^\mu\left(x^\mu, \lambda\right)$$

Use 2Tgauge symmetry to eliminate V_μ gauge freedom proportional to X^2

$$A_\mu\left(X\right) = \frac{1}{\kappa}\left[A_\mu\left(x\right) + \left(\lambda - \frac{x^2}{2}\right) \cancel{V_\mu\left(x, \lambda\right)}\right] = \frac{1}{\kappa} A^\mu\left(x\right)$$

$$F_{\mu\nu}\left(X\right) = \kappa^{-2} F_{\mu\nu}\left(x\right), \text{ with } F_{\mu\nu}\left(x\right) = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$$

$$F_{+'\mu}\left(X\right) = \kappa^{-2} x^\nu F_{\mu\nu}\left(x\right), \quad F_{-'\mu}\left(X\right) = 0, \quad F_{+'-'}\left(X\right) = 0.$$

F_{MN} is YM gauge invariant but 2Tgauge dependent

result is standard 3+1 YM Lagrangian

$$L\left(A\left(X\right)\right) = -\frac{1}{4} Tr\left(F_{MN} F^{MN}\right)\left(X\right) = -\frac{1}{4\kappa^4} Tr\left(F_{\mu\nu} F^{\mu\nu}\right)\left(x\right)$$

Emergent Standard Model in 3+1 dimensions

Every term in the 4+2 action is

- proportional to κ^{-4} after solving kinematic eoms
- and is independent of λ after 2Tgauge fixing,

remainders proportional to X^2 eliminated by 2Tgauge

$$\Phi(X) = \Phi_0(X) + \cancel{X^2\tilde{\Phi}(X)}$$

$$A_M(X) = A_M^0(X) + \cancel{X^2\tilde{A}_M(X)}$$

$$\Psi^{L,R}(X) = \Psi_0^{L,R}(X) + \cancel{X^2\Psi_1^{L,R}(X)}$$

$$S = Z\int |\kappa|^5 d\kappa\, d^4x\, d\lambda\, \delta\left(\kappa^2\left(2\lambda - x^2\right)\right) \times \frac{1}{\kappa^4} L\left(A_\mu(x), \phi(x), h(x), \psi^{L,R}(x)\right)$$

$$= \left\lfloor Z \int d\kappa du\, \delta\left(2|\kappa| u\right)\right\rfloor \int d^4x L\left(A_\mu(x), \phi(x), h(x, \psi^{L,R}(x)\right)$$

↑ Normalize to 1

Emergent Standard Model in 3+1 has dilaton in addition to usual matter

What is new in 3+1 ?

1. Resolution of the strong CP violation problem of QCD
2. Mass generation: a) new mechanisms, b) dilaton (perhaps observable phenomenology)

Resolution of the strong CP problem

strong CP problem in QCD

(instantons)

$$\frac{\theta}{4!}\int dx^4 \varepsilon_{\mu\nu\lambda\sigma} Tr\left(G^{\mu\nu}G^{\lambda\sigma}\right) \text{ can be added to the QCD action in } 3+1$$

There is no observed CP violation in the strong interactions, so why is θ zero or so small?

θ can be made zero if there is an extra $U(1)_{PQ}$ suggested by Peccei & Quinn, but electroweak spontaneous breaking generates a Goldstone boson = the axion. It does not seem to exist !! So there is an outstanding fundamental problem.

The 4+2 Standard Model solves the strong CP violation problem of QCD

There is no term in 4+2 that can descend to the troublesome F*F terms in 3+1
No need for the Peccei-Quinn symmetry, and no elusive axion.

$J_{M_1M_2}$ renormalizable term, homogeneous of degree 0, does not exist

$$\int \left(d^6X\right)\ \delta\left(X^2\right) \boxed{X_{M_1}\partial_{M_2}} Tr\left(F_{M_3M_4}F_{M_5M_6}\right)\varepsilon^{M_1M_2M_3M_4M_5M_6} \longrightarrow 0$$

Non-renormalizable J_{MN} made from composite fields OK. Good for pion-decay, etc.

$$\int \left(d^6X\right) B_{M_1M_2} Tr\left(G_{M_3M_4}G_{M_5M_6}\right)\varepsilon^{M_1M_2M_3M_4M_5M_6} \longrightarrow 0$$

topological term vanishes: $F_{+'-'}(X) = 0 \quad F_{-'\mu}(X) = 0,$

Mass generation via Higgs & dilaton

The 4+2 Standard Model has 2Tgauge symmetry which forbids quadratic mass terms in the scalar potential. Only quartic interactions are permitted. → Scale invariance
Quantum effects break scale inv. But give insufficient mass to the Higgs (10 GeV).

$$V\left(\Phi,H\right)=\frac{\lambda}{4}\left(H^{\dagger}H-\alpha^{2}\Phi^{2}\right)^{2}+V\left(\Phi\right)$$

$$\partial^{2}H=\lambda H\left(H^{\dagger}H-\alpha^{2}\Phi^{2}\right)$$

$$\partial^{2}\Phi=-2\alpha^{2}\Phi\left(H^{\dagger}H-\alpha^{2}\Phi^{2}\right)+V'\left(\Phi\right)$$

$$\langle H\left(\kappa,\lambda,x^{\mu}\right)\rangle=\frac{v}{\kappa}\begin{pmatrix}0\\1\end{pmatrix}\qquad \langle\Phi\left(X\right)\rangle=\pm\frac{v}{\kappa\alpha}\qquad V\left(\Phi\right)=\frac{\lambda'}{4}\Phi^{4}\quad=0$$

Electroweak vev is probe of extra dimension

All space filled with vev. Makes sense to have dilaton & gravity & strings involved

$$\text{small fluctuations }V\left(\Phi,H\right)=\frac{1}{\kappa^{4}}V\left(h,\phi\right)=\frac{\lambda}{4\kappa^{4}}\left(h-\alpha\phi\right)^{2}\left(h+\alpha\phi+2v\right)^{2}$$

Goldstone boson due to spontaneous breaking of scale invariance

$$h=\frac{\tilde{h}+\alpha\tilde{\phi}}{\sqrt{1+\alpha^{2}}},\quad \phi=\frac{-\alpha\tilde{h}+\tilde{\phi}}{\sqrt{1+\alpha^{2}}}\qquad V\left(\tilde{h},\tilde{\phi}\right)=\frac{\lambda}{4}\tilde{h}^{2}\left(\left(1-\alpha^{2}\right)\tilde{h}+2\alpha\tilde{\phi}+\sqrt{1+\alpha^{2}}2v\right)^{2}$$

$\alpha m_f/v$

Goldstone boson couples to everything the Higgs couples to, but with reduced strength factor α. It is not expected to remain massless because of quantum anomalies that break scale symmetry. Can we see it ? LHC? Dark Matter?

Conclusions

- Local Sp(2,R) (X,P indistinguishable) is a fundamental principle that agrees with everything we know about Nature as embodied by the Standard Model → 2T-physics works!

- The Standard Model in 4+2 dimensions provides new guidance:
  -resolves the strong CP violation problem of QCD.
  -dilaton driven electroweak spontaneous breakdown.
  Conceptually more appealing source for vev - choice of vacuum in string theory.
  Weakly coupled dilaton, possibly not very massive; LHC ? Dark Matter ?

- Beyond the Standard Model

  GUTS, SUSY, gravity; all can be elevated to 2T-physics in d+2 dimensions.
  Strings, branes; tensionless, and twistor superstring, 2T OK. Tensionful incomplete.
  M-theory*;* expect 11+2 dimensions → OSp(1|64) global SUSY.

- Advantages of formulating 1T physics from the vantage point of d+2 dims: new tools – emergent spacetimes and dynamics, unification, holography, duality, hidden symmetries.

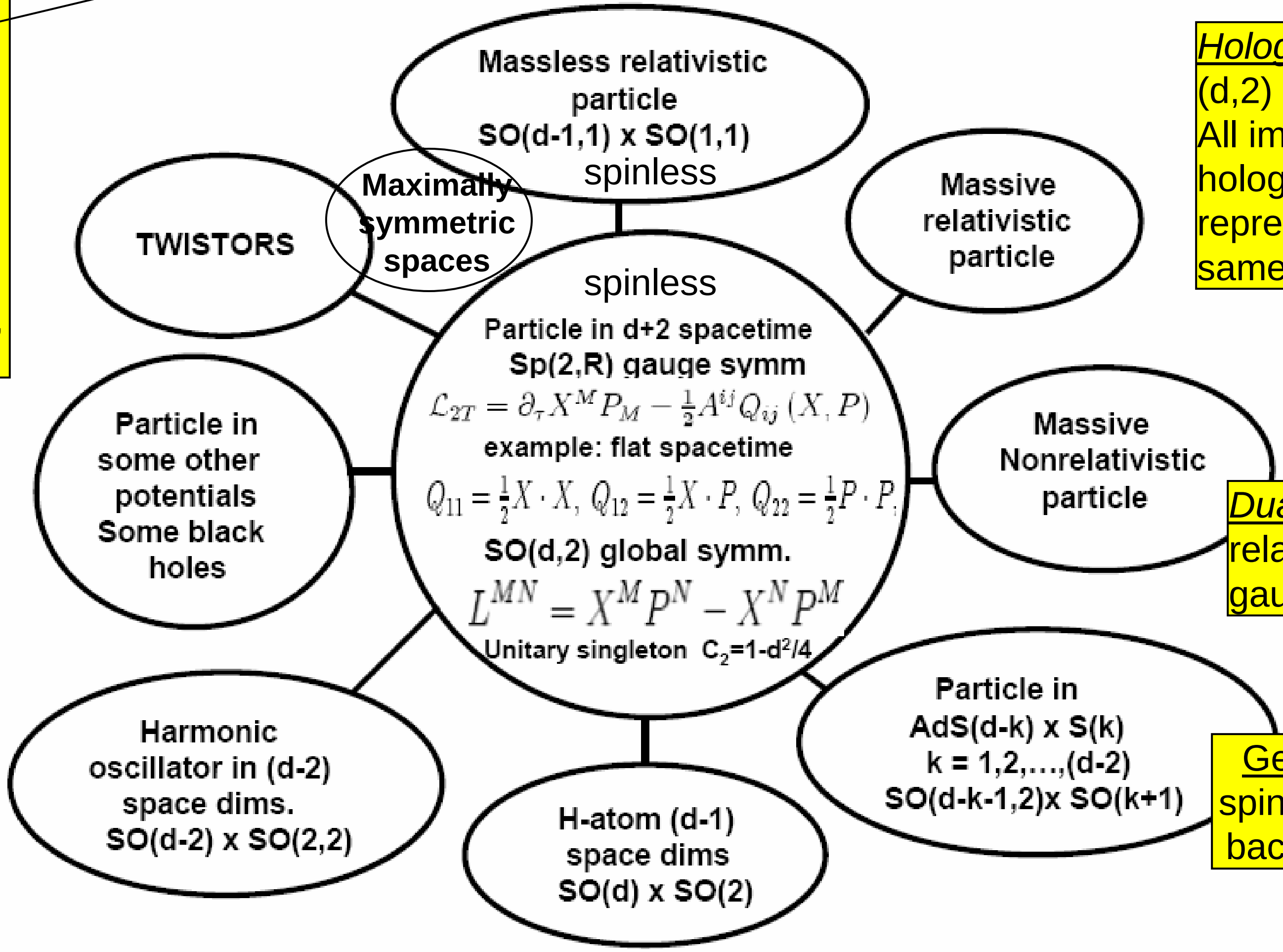
  Hopes for nonperturbative analysis of field theory, including QCD?

Emergent spacetimes & dynamics, hidden symmetries from gauge fixing the simplest model of 2T theory

$(d,2)-(1,1)$ signature of extra gauge parameters $=(d-1,\ 1)$ emergent space-time

Emergent spacetime: Sp(2,R) gauge choices. Some combination of X^M, P^M is fixed as t,H. Can fix 3 gauges, but fix 2 or 3

Holography: from (d,2) to (d-1,1). All images holographically represent the same 2T system



Massless relativistic particle
SO(d-1,1) x SO(1,1)
spinless

Maximally symmetric spaces

TWISTORS

Massive relativistic particle

spinless
Particle in d+2 spacetime
Sp(2,R) gauge symm

$$\mathcal{L}_{2T}=\partial_\tau X^M P_M-\tfrac{1}{2}A^{ij}Q_{ij}\,(X,P)$$

example: flat spacetime

$$Q_{11}=\tfrac{1}{2}X\cdot X,\ Q_{12}=\tfrac{1}{2}X\cdot P,\ Q_{22}=\tfrac{1}{2}P\cdot P,$$

SO(d,2) global symm.

$$L^{MN}=X^M P^N-X^N P^M$$

Unitary singleton C_2=1-d^2/4

Particle in some other potentials Some black holes

Massive Nonrelativistic particle

Harmonic oscillator in (d-2) space dims.
SO(d-2) x SO(2,2)

H-atom (d-1) space dims
SO(d) x SO(2)

Particle in AdS(d-k) x S(k)
k = 1,2,…,(d-2)
SO(d-k-1,2)x SO(k+1)

Hidden symmetry: All images have hidden SO(d,2) symmetry, for the example.

Duality: Sp(2,R) relates one fixed gauge to another

Generalizations
spin, susy, strings, background fields

Unification: 2T-physics unifies diverse forms of 1T-physics into a single theory.

1) *Massless particle*: gauge fix for *all* τ :

$$\mathcal{L}_{2T} = \partial_\tau X^M P_M - \frac{1}{2}A^{ij}Q_{ij}(X,P)$$

$X^{+'}(\tau) = 1,\ P^{+'}(\tau) = 0$

$$ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'}dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

emergent space-time

$$X^M = (\overset{+'}{1},\ \overset{-'}{\frac{x^2}{2}},\ \overset{\mu}{x^\mu}) \qquad X\cdot X = -2X^{+'}X^{-'} + X^\mu X^\nu \eta_{\mu\nu} = 0$$

$$P^M = (0,\ x\cdot p,\ p^\mu) \qquad X\cdot P = -X^{+'}P^{-'} - X^{-'}P^{+'} + X^\mu P^\nu \eta_{\mu\nu} = 0$$

$$P\cdot P = -2P^{+'}P^{-'} + P^\mu P^\nu \eta_{\mu\nu} = p^2 = \text{wait}$$

$$\dot X^M = (0,\ \dot x\cdot x,\ \dot x^\mu) \qquad \dot X\cdot P = -0 + \dot x\cdot p$$

2 gauge choices made.
τ reparametrization remains.

Gauge invariants: Action S, global SO(d,2) $L^{MN} = \varepsilon^{ij}X_i^M X_j^N$ $\qquad \partial_\tau L^{MN} = 0$

emergent dynamics

gauge fixed $S = \int d\tau\left(\dot x\cdot p - \frac{1}{2}A^{22}p^2\right)$

gauge fixed $L^{MN} = \varepsilon^{ij}X_i^M X_j^N = X^M P^N - X^N P^M$ becomes conformal SO(d,2)

L^{MN} is the only Sp(2,R) gauge invariant

$$L^{+'\mu} = p^\mu \qquad L^{+'-'} = x\cdot p,$$

$$L^{\mu\nu} = x^{[\mu}p^{\nu]}, \qquad L^{-'\mu} = \frac{x^2}{2}p^\mu - x\cdot p\, x^\mu$$

After quantum ordering : $C_2 = \frac{1}{2}L^{MN}L_{MN} = 1 - \frac{d^2}{4}$

same as **covariant** quantization in Sp(2,R) invariant space 9803188

Massive relativistic particle gauge

$$X^M = \left(\frac{\overset{+'}{1+a}}{2a},\ \frac{\overset{-'}{x^2 a}}{1+a},\ \overset{\mu}{x^\mu} \right),\ a \equiv \sqrt{1+\frac{m^2 x^2}{(x\cdot p)^2}}$$

$$P^M = \left(\frac{-m^2}{2(x\cdot p)a},\ (x\cdot p)\,a\ ,\ p^\mu \right),\ P^2 = p^2 + m^2 = 0.$$

$$S = \int d\tau \left(\dot{X}^M P^N - \frac{1}{2} A^{ij} X_i^M X_j^N \right) \eta_{MN} = \int d\tau \left(\dot{x}^\mu p_\mu - \frac{1}{2} A^{22} \left(p^2 + m^2\right) \right)$$

$$L^{MN} = X^M P^N - X^N P^M$$

$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu, \quad L^{+'-'} = (x\cdot p)\,a,$$

$$L^{+'\mu} = \frac{1+a}{2a} p^\mu + \frac{m^2}{2\,(x\cdot p)\,a} x^\mu$$

$$L^{-'\mu} = \frac{x^2 a}{1+a} p^\mu - (x\cdot p)\,a x^\mu$$

conformal group warped by mass

Field equations for fermions in 2T-physics

Worldline gauge symmetry OSp(1|2)

$$X^2,\ P^2,\ (X\cdot P+P\cdot X),\ X\cdot\psi,\ P\cdot\psi$$

$$X\cdot\psi|\Psi\rangle = P\cdot\psi|\Psi\rangle = 0$$

Vanishing constraints on physical states

ψ^M act like SO(d,2) gamma matrices $\Gamma^M,\ \bar{\Gamma}^M$ on the two SO(d,2) Weyl spinors $\hat{\Psi}^R_{\dot{\alpha}}(X)$ $\hat{\Psi}^L_{\alpha}(X)$

$$\left(\not{X}\hat{\Psi}\right)_\alpha = 0,\ \left(\not{\partial}\hat{\Psi}\right)_\alpha = 0,\ \hat{\Psi}_\alpha(X) = \text{chiral spinor of SO}(d,2)$$

kinematic #1

$$\hat{\Psi}(X) = \delta\left(X^2\right)\overline{\not{X}}\Psi(X)$$

used $\not{X}\overline{\not{X}} = X^2$ and $X^2\delta\left(X^2\right) = 0$

Notation: $\not{X}\equiv\Gamma^M X_M$ $\overline{\not{\partial}}\equiv\bar{\Gamma}^M\partial_M$

$$\left[\left(X\cdot\partial+\frac{d}{2}\right)\Psi_{\dot{\alpha}}\right]_{X^2=0} = 0,$$

kinematic #2 (homogeneous)

$$\left[\not{X}\,\overline{\not{\partial}}\Psi\right]_{X^2=0} = 0.$$

Dynamic eq. of motion

Action for fermion field in 2T-physics

Obtain 3 equations not just one : 2 kinematic and 1 dynamic.

$$S_0(\Psi) = \frac{1}{2}\int (d^{d+2}X)\,\delta(X^2)\left(\bar{\Psi}\ \not{X}\,\overline{\not{\partial}}\Psi + \bar{\Psi}\,\overleftarrow{\not{\partial}}\,\overline{\not{X}}\Psi\right)$$

$$\delta S_0(\Psi) = \int (d^{d+2}X)\ \delta(\underset{\uparrow}{X^2})\ \delta\bar{\Psi}\left[\not{X}\,\overline{\underset{\uparrow}{\not{\partial}}}\Psi - \left(X\cdot\underset{\uparrow}{\partial} + \frac{d}{2}\right)\Psi\right] + h.c.$$

kinematic #1 dynamical eq. kinematic #2

Although it looks like one equation one can show that each term vanishes separately due to $X^2=0$.

$$\delta_\zeta\bar{\Psi} = X^2\underset{\uparrow}{\bar{\zeta}_1} + \underset{\uparrow}{\bar{\zeta}_2}\,\overline{\not{X}}$$

fermionic 2Tgauge-symmetry

$$\left(X\cdot\partial\bar{\zeta}_2 + \frac{d+2}{2}\bar{\zeta}_2\right)_{X^2=0} = 0$$

Any general spinor. Eliminates all spinor components proportional to $X^2=0$.

Homogeneous spinor. Eliminates half of the leftover spinor to remain with spinor in d dimensions rather than spinor in d+2

Minimizing the action gives two equations, so get all OSp(1|2) constraints as eom´s from the action

These kinematic + dynamical equations for left/right spinors in d+2 dimensions descend to Dirac equations for left/right spinors in d dimensions.
Extra components are eliminated because of kappa type fermionic symmetry.

Yukawa interactions in 2T-physics

$$S_{int}\left(\psi, scalars\right) = \int \left(d^{d+2}X\right)\ \delta\left(X^2\right)\ \left[\bar{\Psi}^L\ \not{X}\Psi^R \times \left(scalars\right) + h.c.\right]$$

Ψ^L is a 4 of SU$(2,2)$ Ψ^R is a 4^* $\bar{\Psi}^L$ is a 4^*

$4^* \times 4^*$ antisymmetrized is
the SO$(6,2)$ vector $\bar{\Psi}^L\Gamma_M\Psi^R$

d=4 SO(4,2) group theory explains why there should be X^M

fermionic gauge transformation

$$\delta_\zeta\left(\bar{\Psi}^L\ \not{X}\Psi^R\right) = \left(\bar{\zeta}_1^L X^2 + \bar{\zeta}_2^R\overline{\not{X}}\right)\ \not{X}\Psi^R + \bar{\Psi}^L\ \not{X}\left(X^2\zeta_1^R + \overline{\not{X}}\zeta_2^L\right)$$

$$= X^2\left[\left(\bar{\zeta}_1^L\ \not{X}\Psi^R + \bar{\Psi}^L\ \not{X}\zeta_1^R\right) + \left(\bar{\zeta}_2^R\Psi^R + \bar{\Psi}^L\zeta_2^L\right)\right]$$

vanishes against
delta function

2Tgauge symmetry also explains why there should be X^M

$$\left(scalars\right) = g_H\ H\,\Phi^{-\frac{d-4}{d-2}}$$

must include dilaton factor Φ if d is not 4
due to 2Tgauge symmetry, or homogeneity

Equations for gauge fields in 2T-physics

two approaches give the same kinematic equations for A_M

1) $\mathrm{OSp}(2|2)$ superquartet $\left(\psi_1^M, \psi_2^M, X^M, P^M\right)$ worldline gauge symmetry for spin 1

2) Spinless particle in gauge field background, and subject to Sp(2,R) gauge symmetry. Then the gauge field background must be kinematically constrained.

$$X^M F_{MN} = 0, \text{ where } F_{MN} = \partial_M A_N - \partial_N A_M - ig_A \left[A_M, A_N\right]$$

$$X \cdot A = 0 : \; X^M F_{MN} = \left(X \cdot \partial + 1\right) A_N = 0$$

In the fixed ‘axial’ gauge it amounts to homogeneity

The dynamical equation follows from the $\mathrm{OSp}(2|2)$ approach

$$\left(D_M \left(\Phi^{\frac{2(d-4)}{d-2}} F^{MN}\right)\right)_{X^2=0} = sources.$$

must include dilaton factor Φ if d is not 4 due to 2Tgauge symmetry, or homogeneity

Action for gauge fields in 2T-physics

Obtain kinematic and dynamic equations from the action

$$S(A) = -\frac{1}{4}\int \left(d^{d+2}X\right)\ \delta\left(X^2\right)\ \Phi^{\frac{2(d-4)}{d-2}}\ Tr\left(F_{MN}F^{MN}\right)$$

dynamical eq.

$$\delta S(A) = \int \left(d^{d+2}X\right) Tr\left\{\delta A_N \begin{bmatrix} \delta\left(X^2\right) D_M\left(\Phi^{\frac{2(d-4)}{d-2}} F^{MN}\right) \\ +2\Phi^{\frac{2(d-4)}{d-2}}\delta'\left(X^2\right) X_M F^{MN} \end{bmatrix}\right\}$$

kinematic #1,2

2Tgauge-symmetry with the transformation

with $\left[(X\cdot D + d - 1)\, a_N - X_N D\cdot a\right]_{X^2=0} = 0,$ and $X\cdot a = 0.$

$$A_M(X) = A_M^0(X) + X^2\tilde{A}_M(X)$$

remainder can be
removed by gauge symm.