Spontaneous Partial Breaking of $\mathcal{N}=2$ Supersymmetry and the $U(N)$ Gauge Model

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- original construction
- properties
- manifestly $\mathcal{N}=2$ SUSY formulation
- inclusion of $\mathcal{N}=2$ matter

c.f.
with K.Maruyoshi
· [hep-th/0603180]

I). Introduction

• basic mechanism: $\left\{\bar{Q}^j_{\dot{\alpha}}, \mathcal{S}^m_{\alpha i}(x)\right\} = 2(\sigma^n)_{\alpha\dot{\alpha}}\delta_i{}^j T^m_n(x) + (\sigma^m)_{\alpha\dot{\alpha}} C_i{}^j$

· $C_i{}^j$: not a VEV but follows simply from the algebra.

· permitted by the Jacobi id. constraints.

· not modify the SUSY algebra on the elementary fields.

The model:

• $\mathcal{N}=2$ $U(N)$ gauge group. (cf. $U(1)$ case Antoniadis-Partouche-Taylor in 1995)

• In $\mathcal{N}=1$ language $\Phi^a = (A^a, \psi^a, F^a),\ V^a = (v^a_m, \lambda^a, D^a)$

• noncanonical kinetic term from the Kähler ptl and gauged .

• three parameter $e,\ m$ and ξ

The model predicts:

• $C_i{}^j = 4m\xi\tau_1 \xrightarrow{90^{\rm o}{\rm rot.}} 4m\xi\tau_3$ The scalar ptl VEV $\langle\langle\mathcal{V}\rangle\rangle = \mp 2m\xi = 2|m\xi|$

• $\therefore$ Half of the supercharges annihilates the vacuum while the remaining half takes $\infty \sim |m\xi| \int d^4x$ matrix elements.

$\therefore$ Partial Breaking of Extended SUSY is a Reality.

II). Original $\mathcal{N}=2$ Strategy and Our Model

$$\boldsymbol{\lambda}_i^a = \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} \rightarrow \boldsymbol{\lambda}^{ia} = \begin{pmatrix} \psi^a \\ -\lambda^a \end{pmatrix} = R\boldsymbol{\lambda}_i^a R^{-1}$$

$R\delta^{(1,\xi)}_{\eta_1=\theta}R^{-1} \equiv \delta^{(2,-\xi)}_{\eta_2=\theta}$ so that $0=\delta^{(2,\xi)}_{\eta_2=\theta}S(\xi)$ follows from $R\delta^{(1,\xi)}_{\eta_1=\theta}S(\xi)R^{-1}=0$

- Kähler: kinetic term for A

$$\mathcal{L}_K = \int d^2\theta d^2\bar{\theta}\ K(\Phi^a,\bar{\Phi}^a), \qquad K=\frac{i}{2}(\Phi^a\bar{\mathcal{F}}_a - \bar{\Phi}^a\mathcal{F}_a),$$

- U(N) gauging $\mathcal{L}_\Gamma = \int d^2\theta d^2\bar{\theta}\Gamma, \quad \Gamma = \left[\int_0^1 d\alpha e^{\frac{i}{2}\alpha v^a(k_a-k_a^*)}v^c\mathfrak{D}_c\right]_{v^a\to V^a}$
- gauge kinetic action $\mathcal{L}_{\mathcal{W}^2} = -\frac{i}{4}\int d^2\theta\tau_{ab}\mathcal{W}^a\mathcal{W}^b + c.c.\ , \quad \mathcal{W}_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$
- superpotential $\mathcal{L}_W = \int d^2\theta\ W(\Phi) + c.c.$ Fayet-Iliopoulos D-term $\mathcal{L}_D = \xi\int d^2\theta d^2\bar{\theta}V^0 = \sqrt{}$

The total action $\mathcal{L} = \mathcal{L}_K + \mathcal{L}_\Gamma + \mathcal{L}_{\mathcal{W}^2} + \mathcal{L}_W + \mathcal{L}_D$

- Impose R invariance. $-\frac{i}{4}g^{cd^*}\partial_c\tau_{ab}\partial_{d^*}\overline{W} = \frac{1}{2}g^{cd^*}\partial_c W g_{ad^*,b} - \frac{1}{2}\partial_a\partial_b W,$

The solution $W = eA^0 + m\mathcal{F}_0, \quad \tau_{ab} = \mathcal{F}_{ab}$

III). Transformation Laws and Analysis of Vacua

Doublet of fermions $\boldsymbol{\lambda}_I{}^a \equiv \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix}$ & parameters $\boldsymbol{\eta}_I \equiv \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$.

$$\boldsymbol{\delta\lambda}_J{}^a = (\sigma^{mn}\boldsymbol{\eta}_J)v^a_{mn} + \sqrt{2}i(\sigma^m\bar{\boldsymbol{\eta}}_J)\mathcal{D}_m A^a + i(\boldsymbol{\tau}\cdot\boldsymbol{D}^a)_J{}^K\boldsymbol{\eta}_K - \frac{1}{2}\boldsymbol{\eta}_J f^a_{\ bc}A^{*b}A^c \; .$$

$$\boldsymbol{D}^a = \hat{\boldsymbol{D}}^a - \sqrt{2}g^{ab^*}\partial_{b^*}\left(\boldsymbol{\mathcal{E}}A^{*0} + \boldsymbol{\mathcal{M}}\mathcal{F}^*_0\right) .$$

fermion bilinears

$$\boldsymbol{\mathcal{E}} = (0,\ -e,\ \xi)\ , \quad \boldsymbol{\mathcal{M}} = (0,\ -m,\ 0),$$

The scalar potential of the model, $\mathcal{V} = -\mathcal{L}_{\text{pot}}$ is

$$\mathcal{V} = g^{ab}\left(\frac{1}{8}\mathfrak{D}_a\mathfrak{D}_b + \xi^2\delta^0_a\delta^0_b + \partial_a W\partial_{b^*}W^*\right) = \frac{1}{8}g_{bc}\mathfrak{D}^b\mathfrak{D}^c + \frac{1}{2}g^{bc}\tilde{\boldsymbol{D}}^*_b\cdot\tilde{\boldsymbol{D}}_c$$

$$\therefore \quad \langle\langle \mathcal{F}_{\underline{jj}}\rangle\rangle = -2(\frac{e}{m} \mp i\frac{\xi}{m}) = -2\zeta \ ; \text{the vac. condition}$$

$$\langle\langle g_{\underline{jj}}\rangle\rangle = \mp 2\frac{\xi}{m},$$

$$\langle\langle \boldsymbol{D}^{\underline{j}}\rangle\rangle = \frac{m}{\sqrt{N}}\begin{pmatrix} 0 \\ -i \\ \pm 1 \end{pmatrix}$$

IV). Partially Broken Supersymmetry and Gauge Symmetry Breaking

$$\begin{aligned} \langle\!\langle \delta \left(\frac{\lambda^{\underline{i}} + \psi^{\underline{i}}}{\sqrt{2}} \right) \rangle\!\rangle &= im\sqrt{\frac{2}{N}}(\eta_1 - \eta_2) \ , \\ \langle\!\langle \delta \left(\frac{\lambda^{\underline{i}} - \psi^{\underline{i}}}{\sqrt{2}} \right) \rangle\!\rangle &= 0 \ . \end{aligned}$$

• $\mathcal{N} = 2$ supersymmetry is spontaneously broken to $\mathcal{N} = 1$ and we obtain the Nambu-Goldstone fermion $\frac{1}{\sqrt{2}}(\lambda^0 + \psi^0)$ associated with the overall $U(1)$ part.

• The diagonal entries $\lambda^{\underline{i}}$ of $\langle\!\langle A \rangle\!\rangle$ belong to $k-2$ complex roots of the vac. cond. These define a grouping of N eigenvalues into $k-2$ sets and hence determine a breaking pattern of $U(N)$ gauge symmetry into a product gauge group $\prod_{i=1}^{k-2} U(N_i)$ with $\sum_{i=1}^{k-2} N_i = N$.

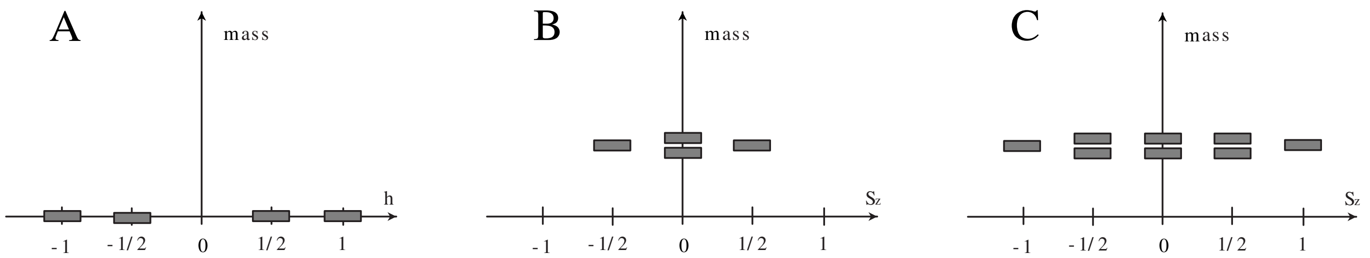
V). Mass Spectrum

index labelling $a, b, \cdots = \begin{cases} \alpha, \beta, \cdots \text{ for unbroken generators} \\ \mu, \nu, \cdots \text{ for broken generators} \end{cases}$

- the table

field	mass	label	# of polarization states
v_m^{α}	0	A	$2d_u (d_u \equiv \dim \prod_i U(N_i))$
v_m^{μ}	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$3(N^2 - d_u)$
$\frac{1}{\sqrt{2}}(\lambda^{\alpha} \pm \psi^{\alpha})$	0	A	$2d_u$
$\frac{1}{\sqrt{2}}(\lambda^{\alpha} \mp \psi^{\alpha})$	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\boldsymbol{\lambda}_I^{\mu}$	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$4(N^2 - d_u)$
A^{α}	$\lvert m \langle\langle g^{\alpha\alpha} \rangle\rangle \langle\langle \mathcal{F}_{0\alpha\alpha} \rangle\rangle \rvert$	B	$2d_u$
$\mathcal{P}_{\mu}^{\tilde{\mu}} A^{\mu}$	$\lvert \frac{1}{\sqrt{2}} f^{\nu}_{\mu \underline{i}} \lambda^{\underline{i}} \rvert$	C	$N^2 - d_u$

- $\mathcal{N} = 1$ supermultiplet



VI). Manifestly $\mathcal{N}=2$ Supersymmetric Form in Harmonic Superspace

$\mathcal{N}=2$ harmonic superspace $\mathbb{R}^{4|8}\times$S^2 in the analytic basis.

$$(x_A^m,\ \theta^{\pm},\ \bar{\theta}^{\pm},\ u_I^{\pm}) = (x^m - 2i\theta^I\sigma^m\bar{\theta}^J u^+_{(I}u^-_{J)},\ \theta^I u_I^{\pm},\ \bar{\theta}^I u_I^{\pm},\ u_I^{\pm})$$

S^2 =SU(2)/U(1) is parametrized by harmonic variables $(u_I^+, u_I^-) \in SU(2)$.

- $V^{++}(x_A, \theta^+, \bar{\theta}^+, u^{\pm}) = V^{++a}t_a$; *unconstrained* $\mathcal{N}=2$ vector superfield .
- $\overline{W} = -\frac{1}{4}(D^+)^2 V^{--}$ with $D^+_\alpha = \frac{\partial}{\partial\theta^{-\alpha}}$; the curvature superfield
- $V^{--} = \sum_{n=1}^{\infty}\int dv_1\cdots dv_n \frac{(-i)^{n+1}V^{++}(v_1)\cdots V^{++}(v_n)}{(u^+v_1^+)(v_1^+v_2^+)\cdots(v_n^+u^+)}$ is (in Wess-Zumino gauge)

$$\overline{W} = -i\sqrt{2}\bar{A} - 2\bar{\theta}^I\bar{\lambda}_I + \bar{\theta}^I\bar{\theta}^J D_{IJ} + \bar{\theta}^I\bar{\sigma}^{mn}\bar{\theta}_I v_{mn} + \cdots$$

- $S_V = \frac{i}{4}\int d^4x \left[(\bar{D})^4\mathcal{F}^*(\overline{W}) - (D)^4\mathcal{F}(W)\right]$.

- The electric FI term whose effect is to shift the dual auxiliary field by an imaginary constant, $D_D^{aIJ} \to \boldsymbol{D}_D^{aIJ} \equiv D_D^{aIJ} + 8i\xi^{IJ}\delta_0^a$.

$$S_e = \int du d\zeta^{(-4)}\Xi^{++}V^{++} + c.c. = \int d^4x \xi^{IJ}D^0_{IJ} + c.c.\ , \qquad \Xi^{++} = \xi^{IJ}u_I^+u_J^+\ .$$

- The magnetic FI term S_m^{YM} is obtained by shifting the auxiliary field D^a_{IJ} by an imaginary constant, $D^{aIJ} \to \boldsymbol{D}^{aIJ} \equiv D^{aIJ} + 4i\xi_D^{IJ}\delta_0^a$;

$$S_V + S_m^{\mathrm{YM}} = -\frac{i}{4}\int d^4x (D)^4\mathcal{F}(\hat{W}) + c.c. \quad \text{where} \quad \hat{W} = W|_{D^a_{IJ}\to\boldsymbol{D}^a_{IJ}}\ .$$

• The total action $S_{\rm YM} = S_V + S_e + S_m^{\rm YM}$ is $\mathcal{N} = 2$ supersymmetric.

cf. $\delta_\eta \boldsymbol{D}^I{}_J = \delta_\eta D^I{}_J$

• The vacuum is determined by the scalar potential

$$= \frac{1}{2} g^{ab} \boldsymbol{D}_a^A| \ \bar{\boldsymbol{D}}_b^A| + g_{ab} \mathfrak{D}^a \mathfrak{D}^b + 4i(\xi^A + \bar{\xi}^A)(\xi_D^A - \bar{\xi}_D^A)$$
$$\boldsymbol{D}_a^A| = g_{ab} \boldsymbol{D}^{bA}| = -2 \left[(\xi^A + \bar{\xi}^A)\delta_a^0 + (\xi_D^A + \bar{\xi}_D^A)\bar{\mathcal{F}}_{0a}| \right] \ .$$

• The shift of D induces spontaneous breaking of $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$.

• By using SU(2), may choose $\xi_D^A + \bar{\xi}_D^A = (0, -m, 0)$, $\xi^A + \bar{\xi}^A = (0, -e, \xi)$ with real constants m, e and ξ. Then the vacuum condition is solved as $\langle \mathcal{F}_{\underline{ii}} \rangle = -2(\frac{e}{m} \pm i\frac{\xi}{m})$.

VII). U(N) Gauge Model with Hypermultiplets

$q^+(x_A, \theta^+, \bar\theta^+, u^\pm)$; $\mathcal{N}=2$ hypermultiplet . (ω hypermultiplets as well in our paper.)
The gauged action is

$$S_q = -\int du d\zeta^{(-4)} \tilde{q}^+_\mu \mathcal{D}^{++} q^{+\mu} \ , \quad \mathcal{D}^{++} q^{+\mu} = D^{++} q^{+\mu} + iV^{++a} (T_a)^\mu{}_\nu q^{+\nu}$$

and $D^{++} = u^{+I}\partial_{u^{-I}} - 2i\theta^+\sigma^m\bar\theta^+\partial_m + \theta^{+\alpha}\partial_{\theta^{-\alpha}} + \bar\theta^{+\dot\alpha}\partial_{\bar\theta^{-\dot\alpha}}$.

- Physical fields and SU(2) doublet complex scalar f^I and SU(2) singlet Weyl spinors ψ and κ.
- The U(N) isometry with the Killing potential $\mathcal{Q}_a^{IJ} = i\bar{f}_\mu^{(I}(T_a)^\mu{}_\nu f^{J)\nu}$ has been gauged.
- The electric FI term is as before.
- $S_V + S_q + S_q + S_m^{\rm QCD} = \left(S_V + S_q + S_q\right)\Big|_{D\to \boldsymbol{D}}$.

That is

$$S_m^{\rm QCD} = S_m^{\rm YM} + \int d^4x \ 2i\hat{\mathcal{Q}}_0^{IJ}(\xi_{DIJ} - \bar\xi_{DIJ}) \ , \quad \hat{\mathcal{Q}}_0^{IJ} = i\bar{f}_u^{(I}(t_0)^u{}_v f^{J)v} \ .$$

The additional f_u-dependent term overcomes the difficulty [Partouche-Pioline'97, Ivanov-Zupnik'97] in coupling fundamental hypermultiplets.

- The total action is $S_{\rm QCD} = S_V + S_q + S_q + S_e + S_m^{\rm QCD}$.

- The scalar potential

$$
\begin{aligned}
&= \frac{1}{4} g_{ab} \boldsymbol{D}^{aIJ}| \; \boldsymbol{D}^b_{IJ}| + g_{ab}\mathfrak{D}^a\mathfrak{D}^b + 2i(\xi+\bar{\xi})^{IJ}(\xi_D - \bar{\xi}_D)_{IJ} \\
&\quad + \overline{f^I}_u(\bar{A}A + A\bar{A})^u{}_v f^{Iv} + \overline{f^I}_a(\bar{A}A + A\bar{A})^a{}_b f^{Ib} \;, \\
\boldsymbol{D}^{aIJ} &= -2g^{ab}\left[(\xi+\bar{\xi})^{IJ}\delta^0_b + (\xi_D + \bar{\xi}_D)^{IJ}\bar{\mathcal{F}}_{0b} + \hat{\mathcal{Q}}^{IJ}_b + \check{\mathcal{Q}}^{IJ}_b\right] \;, \qquad \check{\mathcal{Q}}^{IJ}_b = \mathcal{Q}^{IJ}_b|_{T_a=(t_a)}
\end{aligned}
$$

$$
\text{require } \begin{array}{c} \langle A^r \rangle = 0 \\ \langle A^{\underline{i}} \rangle \neq 0 \end{array} \Rightarrow \text{Coulomb phase}
$$

- $\mathcal{N}=2$ supersymmetry is spontaneously broken to $\mathcal{N}=1$.

VIII). Perspectives

- explore the Higgs phase and connection with $\mathcal{N} = 2$ sugra and the super Higgs effect.
- model with a more general hyperkähler manifold (with U(N) isometry).
- comparison with breaking by the superpotential, namely $e,\ m,\ \xi$ v.s. Λ.
- string theory origin and applications to phenomenology.