Flavor Mixings in Intersecting D-Brane Models

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Collaboration with Bhaskar Dutta

(based on PLB**633**, 761 (2006) and hep-ph/0604126)

1. Motivation of Intersecting D-branes
2. Almost Rank 1 Yukawa Matrix
3. Mixings of quarks and leptons
4. Summary

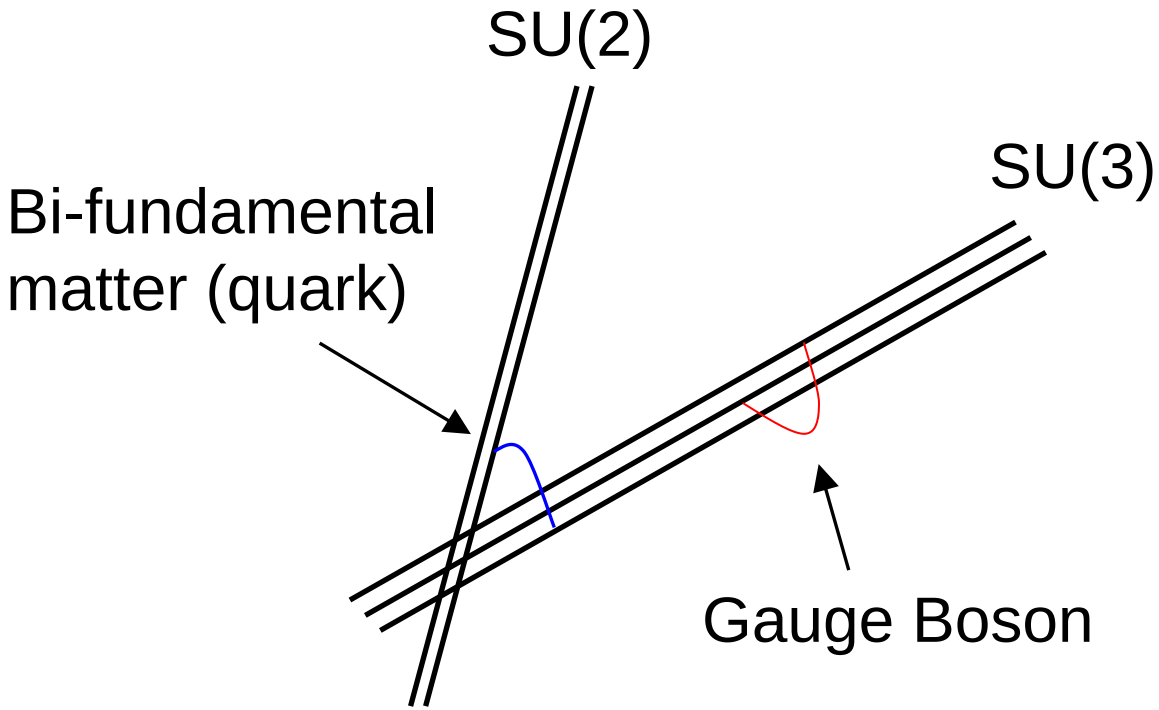
Motivation of Intersecting D-Brane Models

- Standard-like gauge symmetry
- Bi-fundamental representation for matters
- Replication of generations

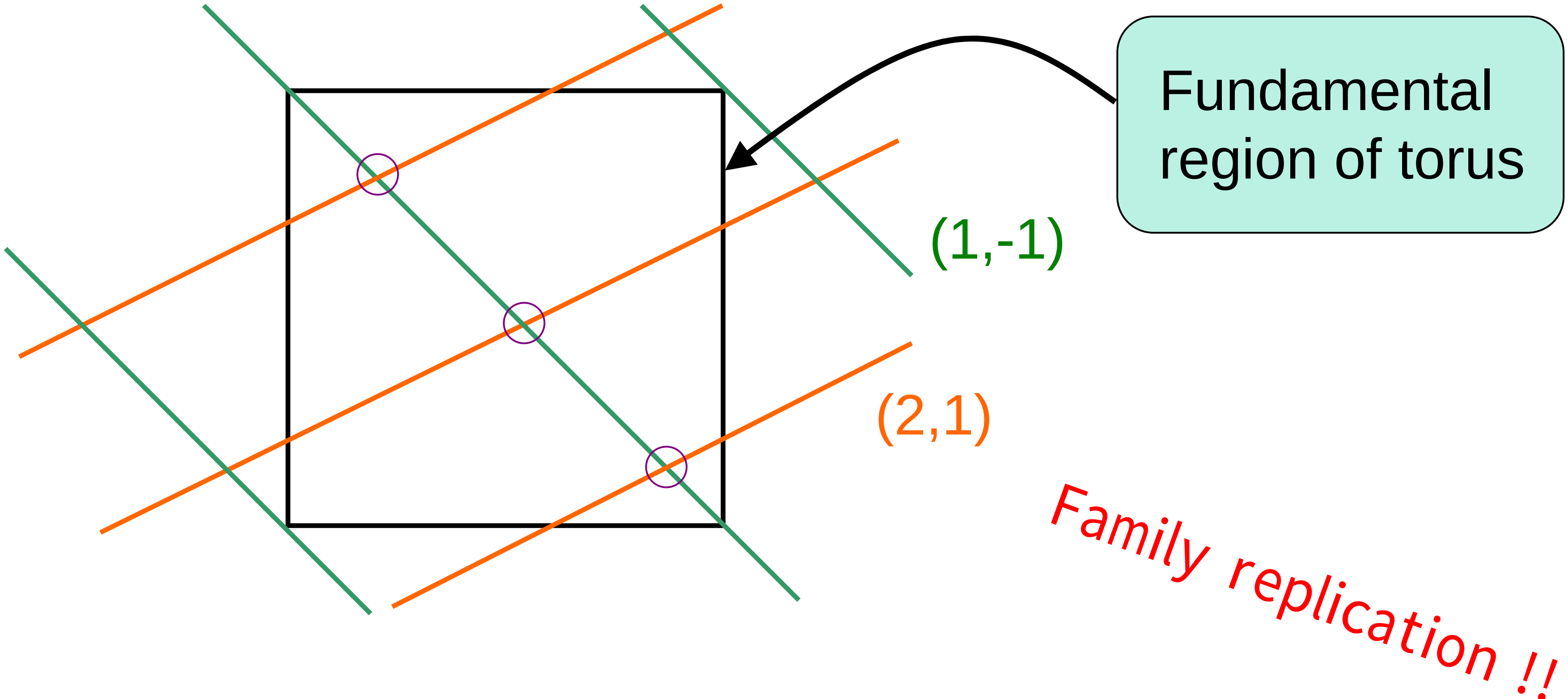
Easily realized !

Standard Model has lots of parameters, especially in flavor sector.

⇨ Can we explain the variety of the mixings for fermions?



Quark & lepton fields are zero modes of open strings stretched at the intersection.



When compactified, there are multiple intersections.

Intersection number : $I_{ab} = \prod_{r=1}^{3} (n_a^r m_b^r - m_a^r n_b^r)$

$[(n^1, m^1), (n^2, m^2), (n^3, m^3)]$: wrapping numbers

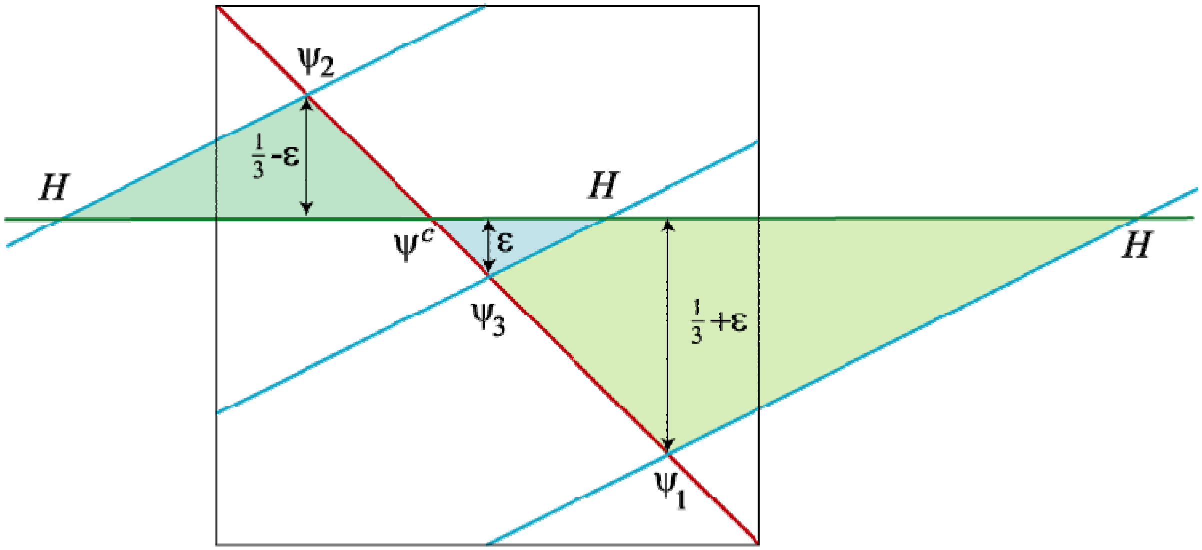
$(T^6 = T^2 \times T^2 \times T^2)$

Low energy action can be calculated by using brane configuration parameters.

Yukawa couplings $y \propto \prod_{r=1}^{3} \vartheta \left[\begin{array}{c} \delta^{(r)} \\ \phi^{(r)} \end{array} \right] (\kappa^{(r)})$

$$\vartheta \left[\begin{array}{c} \delta \\ \phi \end{array} \right] (\kappa) = \sum_{\ell \in Z} e^{-\pi\kappa(\delta+\ell)^2} e^{2\pi i(\delta+\ell)\phi}$$

(Cremades-Ibanez-Marchesano)



Naively,

$$y \sim e^{-\kappa A}$$

A : Area of the triangle

$$y \propto \prod_{r=1}^{3} \vartheta \left[\begin{array}{c} \delta^{(r)} \\ \phi^{(r)} \end{array} \right] (\kappa^{(r)})$$

⇓

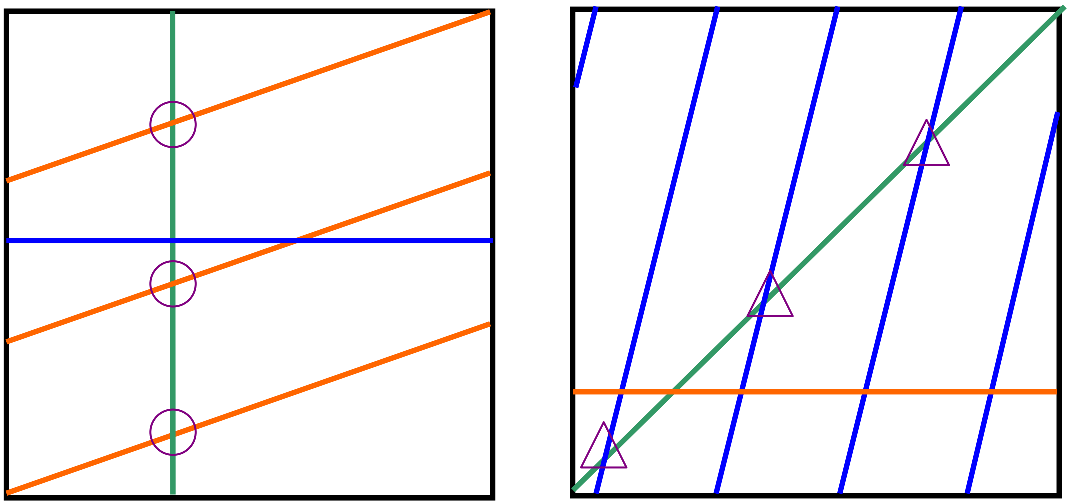
When left- and right-handed fermions
are replicated at different tori,
Yukawa matrix is given as a multiplicative form.

$$y_{ij} = y_i^L \, y_j^R$$

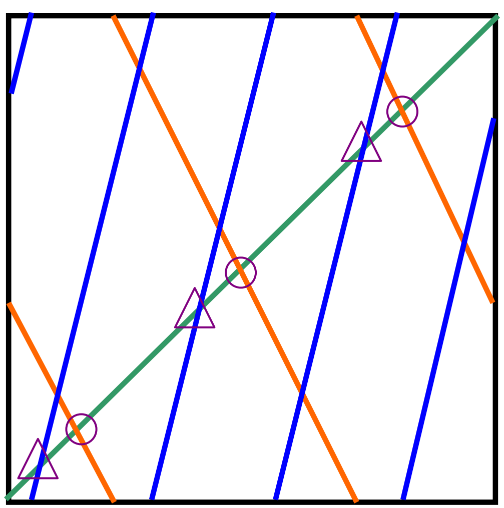
⇓

Yukawa matrix is Rank 1.

Left-hand　　　　Right-hand



Rank 1



Rank 3

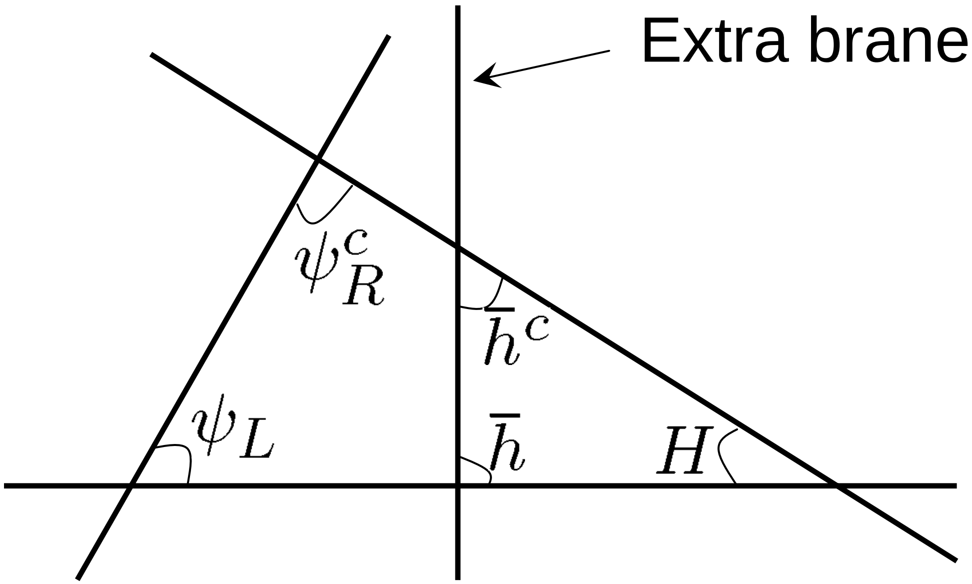
But it is difficult to fit fermion masses & mixings.

(Chamoun-Khalil-Lashin)

Abel-Lebedev-Santiago
Kitazawa-Kobayashi-Maru-Okada, Noguchi
Cvetic-Li-Liu, Chen-Li-Nanopoulos

If Yukawa matrix is
rank 1 + small correction,
realistic fermion mixings can be reproduced naturally.

Candidate of small correction :
Higher order multi-point function



$\psi_L \psi_R^c \bar{h}^{c\dagger} \bar{h}^\dagger$ can arise in Kähler potential.

Properties of Almost Rank 1 Yukawa matrices

$$(Y_u)_{ij} = \underline{y_i^q} y_j^u + (\delta Y_u)_{ij}$$

$$(Y_d)_{ij} = \underline{y_i^q} y_j^d + (\delta Y_d)_{ij}$$

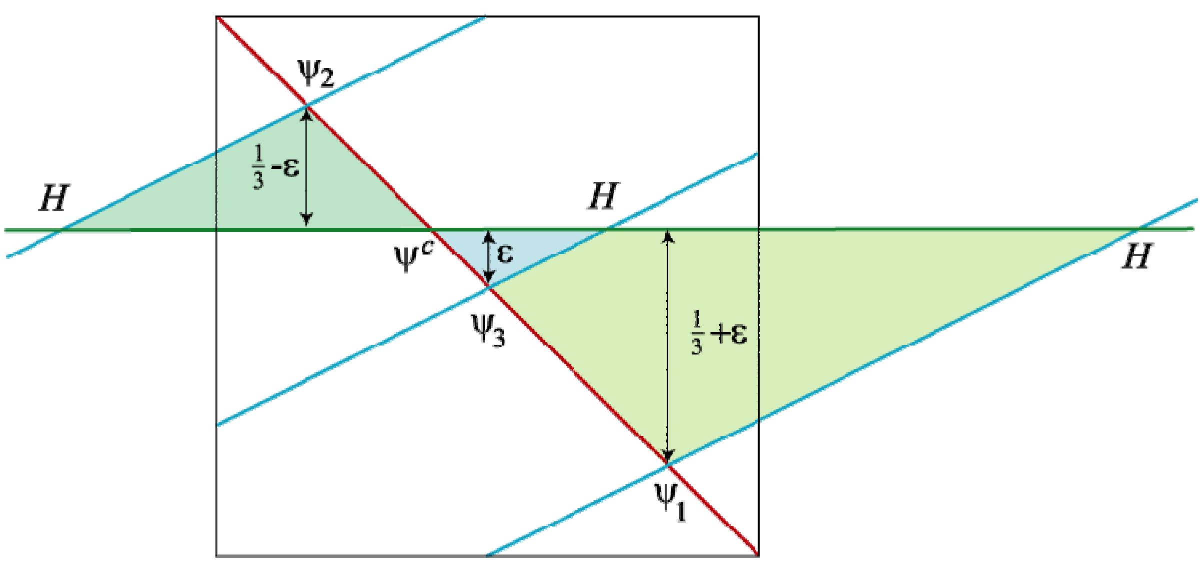
$$(Y_e)_{ij} = \underline{y_i^\ell} y_j^e + (\delta Y_e)_{ij}$$

$$(Y_\nu)_{ij} = \underline{y_i^\ell} y_j^\nu + (\delta Y_e)_{ij}$$

y_i's are determined
by the triangle areas.

$$y_i \sim e^{-kA_i}$$

Left-handed y_i : common



$$Y_0 = \begin{pmatrix} c \\ b \\ a \end{pmatrix} \begin{pmatrix} c & b & a \end{pmatrix} = \begin{pmatrix} c^2 & bc & ac \\ bc & b^2 & ab \\ ac & ab & a^2 \end{pmatrix} \qquad (a, b, c \sim O(1))$$

$$U_0 Y_0 U_0^T = \begin{pmatrix} 0 & & \\ & 0 & \\ & & y \end{pmatrix} \qquad (y = a^2 + b^2 + c^2)$$

U(2) symmetry remains.

We parameterize

$$U_0 = \begin{pmatrix} \cos\theta_s & -\sin\theta_s & 0 \\ \sin\theta_s \cos\theta_a & \cos\theta_s \cos\theta_a & -\sin\theta_a \\ \sin\theta_s \sin\theta_a & \cos\theta_s \sin\theta_a & \cos\theta_a \end{pmatrix}$$

$$\tan\theta_s = \frac{c}{b}, \qquad \tan\theta_a = \frac{\sqrt{b^2 + c^2}}{a}$$

Suppose $\delta Y = \begin{pmatrix} 0 & & \\ & 0 & \\ & & \epsilon \end{pmatrix}$, $Y = Y_0 + \delta Y$ is rank 2.

$$\begin{pmatrix} c^2 & bc & ac \\ bc & b^2 & ab \\ ac & ab & a^2 + \epsilon \end{pmatrix} \begin{pmatrix} b \\ -c \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Eigenvector for zero eigenvalue is $\propto (b, -c, 0)$.

$$U = \begin{pmatrix} \cos\theta_s & -\sin\theta_s & 0 \\ \sim \sin\theta_s \cos\theta_a & \sim \cos\theta_s \cos\theta_a & \sim -\sin\theta_a \\ \sim \sin\theta_s \sin\theta_a & \sim \cos\theta_s \sin\theta_a & \sim \cos\theta_a \end{pmatrix}$$

where $U(Y_0 + \delta Y)U^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sim \sin^2\theta_a\, \epsilon & 0 \\ 0 & 0 & \sim y \end{pmatrix}$

Successful scenario :

If $\delta Y_e \simeq \begin{pmatrix} 0 & & \\ & 0 & \\ & & \epsilon \end{pmatrix}$, in the basis where light neutrino mass matrix is diagonal

$$\sin\theta_{13} \simeq 0, \qquad \tan\theta_{\rm sol} \simeq \frac{c}{b}, \qquad \tan\theta_{\rm atm} \simeq \frac{\sqrt{b^2+c^2}}{a}$$

More precisely,

$$\sin\theta_{13} \simeq \sin\theta_{\rm atm}\sin\theta^e_{12},$$

$$\theta_{\rm sol} \simeq \theta_s \pm \theta_{13}\cot\theta_{\rm atm}\cos\delta_{\rm MNSP},$$

$$\tan\theta_{\rm atm} \simeq \tan\theta_a\cos\theta^e_{12},$$

Quark-lepton unif.

↓

$$\theta^e_{12} \sim \theta_{\rm Cabibbo}$$

In quark sector, the large mixings cancel since left-handed y_i^q is common.

(Type II seesaw is favored to avoid the cancellation.)

$LL\Delta$ (Δ : $\mathrm{SU}(2)_L$ triplet)

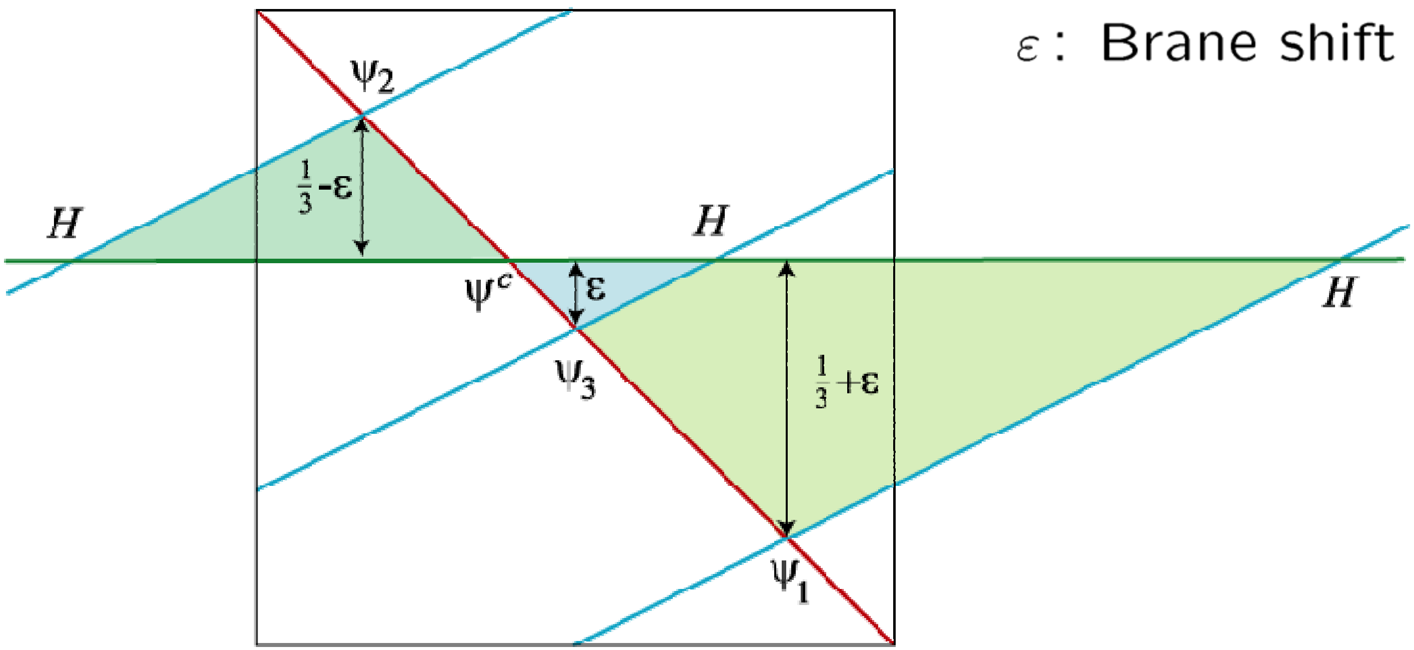
Relations :

$$V_{cb} \simeq \cot\theta_a \frac{m_s}{m_b}$$

$$V_{ub}V_{cb} \simeq V_{us}\left(\frac{m_s}{m_b}\right)^2$$

Empirical relation $V_{us} \simeq \sqrt{\frac{m_d}{m_s}}$ is assumed.

Calculation of neutrino mixing



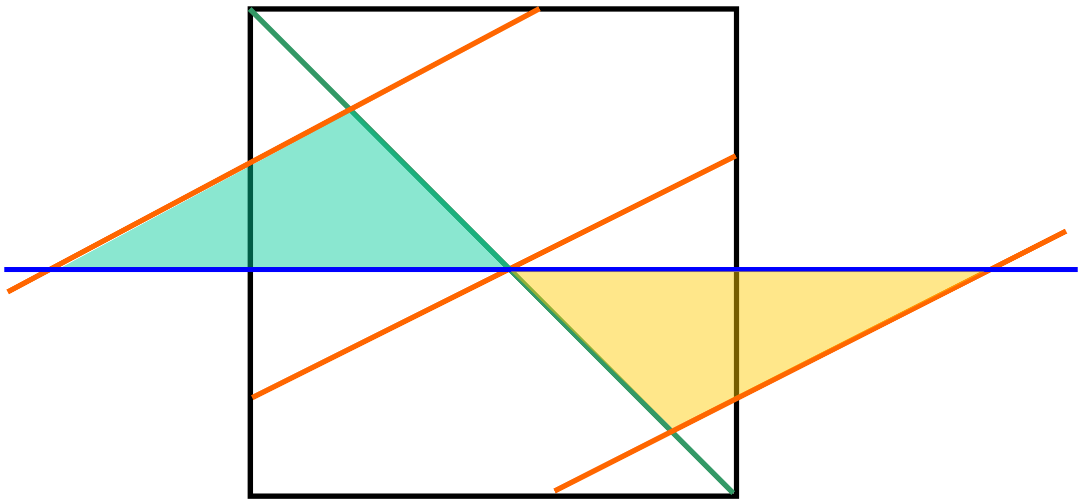
ε : Brane shift parameter

$$a : b : c = \vartheta \begin{bmatrix} \varepsilon \\ 0 \end{bmatrix} (t) : \vartheta \begin{bmatrix} -\frac{1}{3} + \varepsilon \\ 0 \end{bmatrix} (t) : \vartheta \begin{bmatrix} \frac{1}{3} + \varepsilon \\ 0 \end{bmatrix} (t)$$

$$\sim e^{-\pi\varepsilon^2 t} : e^{-\pi(\frac{1}{3}-\varepsilon)^2 t} : e^{-\pi(\frac{1}{3}+\varepsilon)^2 t} \quad (\text{when } t \gtrsim 1)$$

$$a \geq b \geq c \quad \longleftrightarrow \quad 0 \leq \varepsilon \leq \frac{1}{6}$$

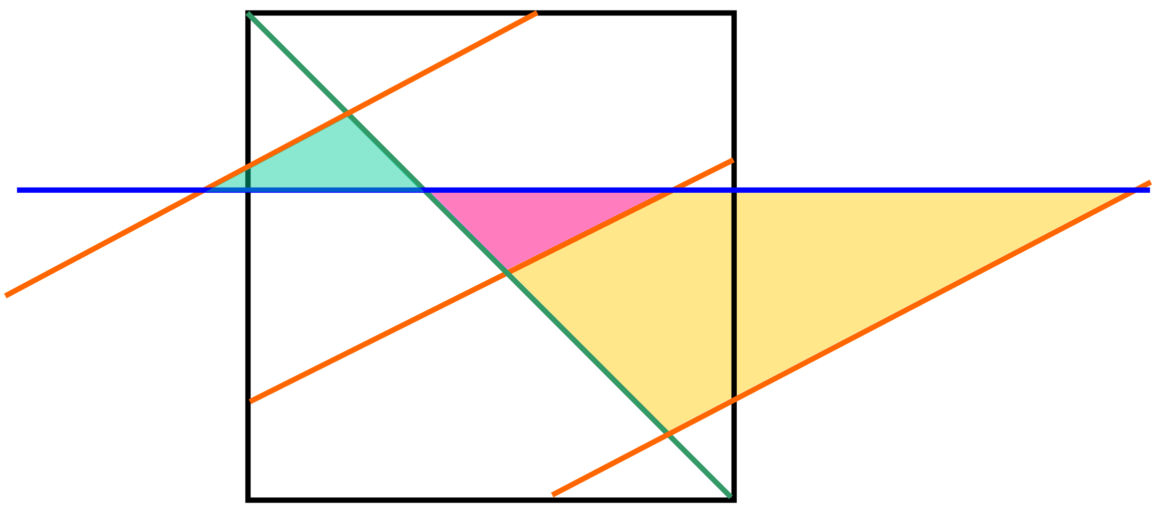
Special points



$$\varepsilon = 0$$

$$b = c$$

$$\tan\theta_s = 1$$



$$\varepsilon = \frac{1}{6}$$

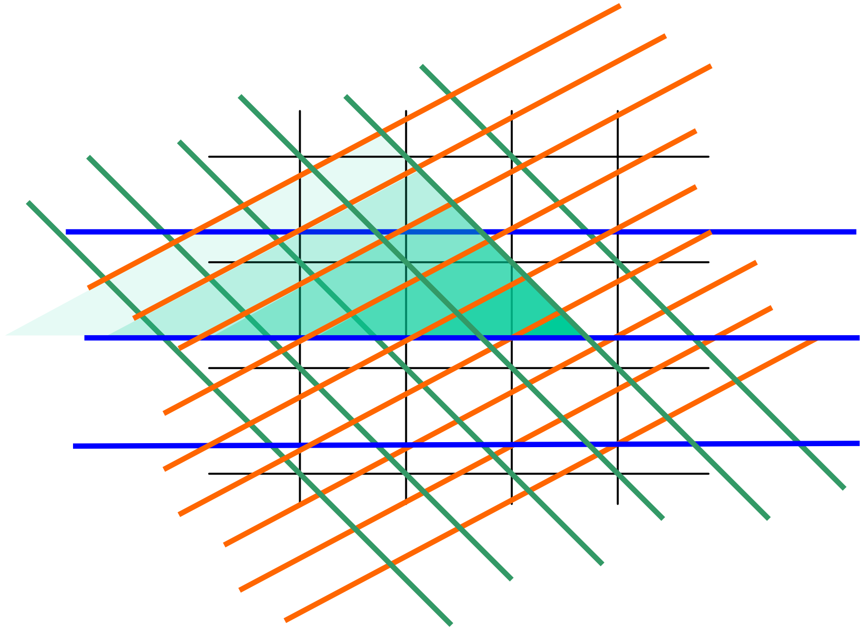
$$a = b$$

$$\tan\theta_a \to 1$$

$$(t \to \infty)$$

↑

weak coupling limit



Strong coupling limit

$(t \to 0)$

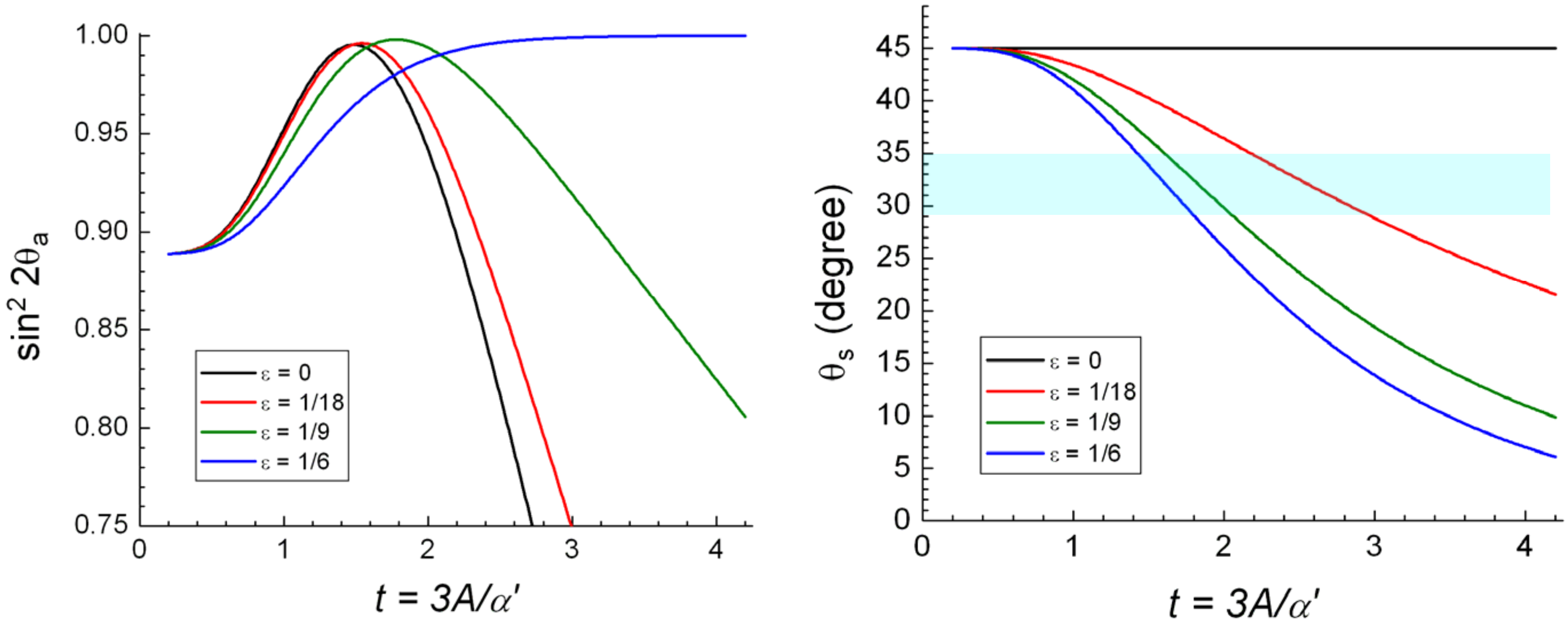
$$a = b = c \to \infty$$

$$\tan\theta_s \to 1$$

$$\tan\theta_a \to \sqrt{2}$$

$$\left(\sin^2 2\theta_a \to \frac{8}{9}\right)$$

$$\left(\tan\theta_s = \frac{c}{b}, \quad \tan\theta_a = \frac{\sqrt{b^2+c^2}}{a}\right)$$



When θ_a is maximal, θ_s separates from maximal mixing.

Experimental data : $\sin^2 2\theta_{\text{atm}} = 1$ (Best fit value)

$$\theta_{\text{sol}} = (32 \pm 3)^\circ$$

SUSY breaking

When generation is replicated simply $(I_{ab} = I_{ca} = 3)$,
Kähler potential doesn’t depend on the generation index.

Flavor universality of SUSY breaking scalar mass

Non-proportional term in scalar trilinear coupling only comes from *U* moduli derivative.

$$A_{ij} = A_0(Y_{ij} + c\,\partial_U Y_{ij})$$

U moduli contribution in scalar trilinear coupling

$$A_{ij} = A_0(Y_{ij} + c\,\partial_U Y_{ij})$$

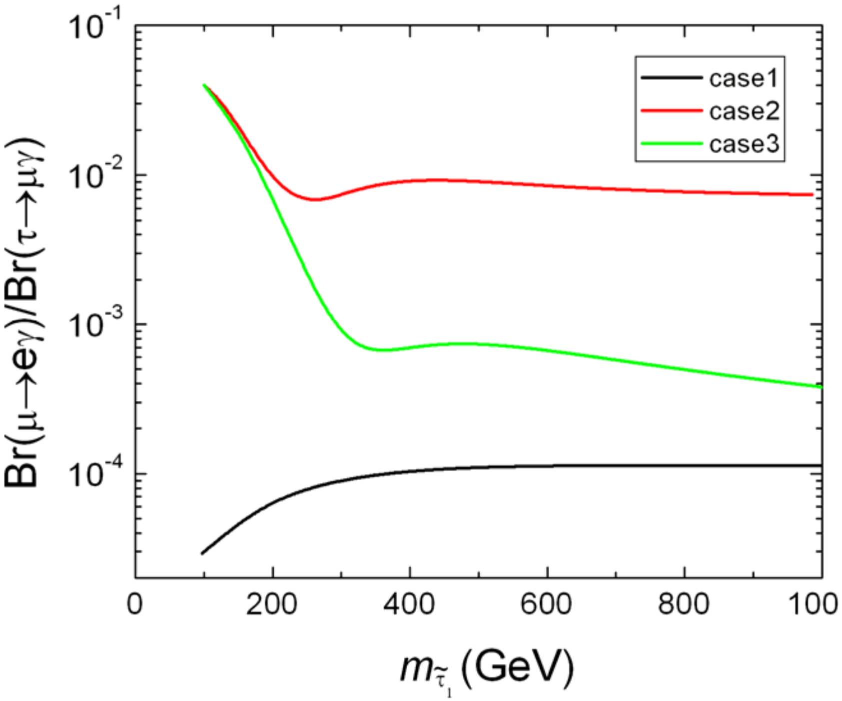
$$\partial_U Y_0 = \begin{pmatrix} \dot{c} \\ \dot{b} \\ \dot{a} \end{pmatrix} \begin{pmatrix} c & b & a \end{pmatrix} + \begin{pmatrix} c \\ b \\ a \end{pmatrix} \begin{pmatrix} \dot{c} & \dot{b} & \dot{a} \end{pmatrix}$$

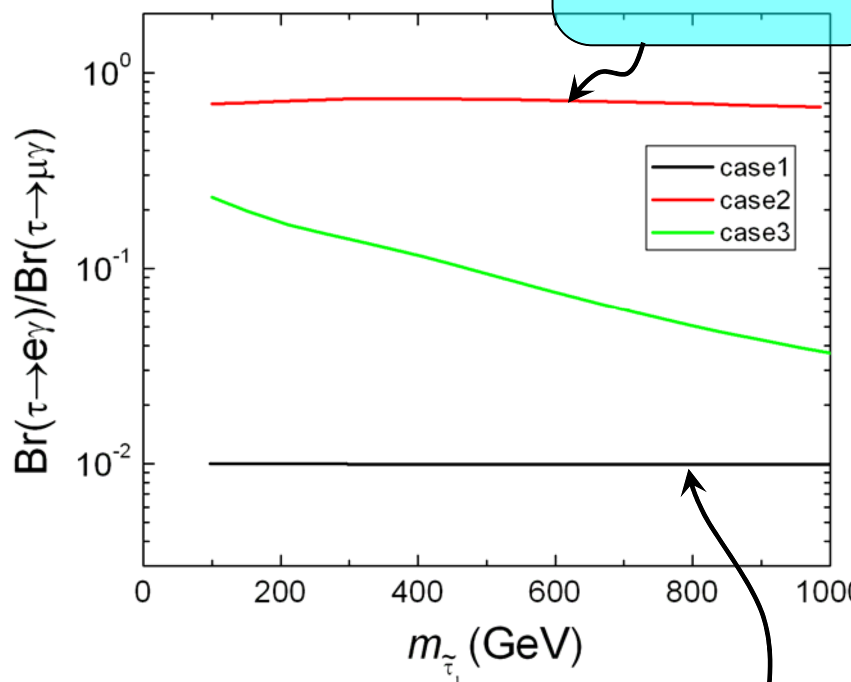
In the basis where Y_0 is diagonal,

$$U_0(\partial_U Y_0)U_0^T = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \begin{pmatrix} 0 & 0 & y \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ y \end{pmatrix} \begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & \beta \\ \alpha & \beta & 2\gamma \end{pmatrix} y$$

Note that A^e_{12} is severely constrained by $\mu \to e\gamma$.





U moduli dominant

Originate from neutrino Dirac Yukawa

$\tau \to e\gamma$ can be comparable to $\tau \to \mu\gamma$.

Summary

- Generations are simply replicated in intersecting D-brane models.
- When left- and right-handed matters are replicated different tori, Yukawa matrices are rank 1.
- Almost rank 1 Yukawa matrices can reproduce realistic flavor mixings for quarks and leptons.
- SUSY breaking can be controlled and contain interesting features when Yukawa matrix is almost rank 1.