Higher derivative terms in type IIB and M-theory

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1 Introduction

At low energies, string theory can be reduced to an effective field theory of the massless modes. All string theories have a massless graviton, and to leading order, the action for this field is the Einstein-Hilbert action.

$$S_2 = \int d^{10}x \ R \tag{1.1}$$

Supersymmetric type II strings have many fields in addition to the graviton, and the complete two-derivative action for these fields is the $N = 2$ supergravity action in ten dimensions. The action for type II theories is completely determined by SUSY.

$$S_2 = \int d^{10}x(R + (\partial\phi)^2 + H^2 + ..) \tag{1.2}$$

H is the 3-form field strength, ϕ is the dilaton.

The effective action also contains an infinite series of higher derivative terms, suppressed by powers of the string scale α', and the complete action has the form

$$S = S_2 + (\alpha')^4 S_8 + (\alpha')^5 S_{10} + \dots \tag{1.3}$$

where S_n contains terms with n derivatives. S_2 is the supergravity action.

The leading correction in type II theories is the eight-derivative action, which contains the famous R^4 term (Gross, Witten)

$$S_{8;R^4} = \int d^{10}x t^8 t^8 R^4 \tag{1.4}$$

This term occurs in all string theories and in the eleven-dimensional theory.

There are also several other terms at the eight derivative level, which involve the other fields of the theory. These terms are believed to be related to the R^4 term by supersymmetry.

How can we determine these terms?

2 Why do we care?

There are several reasons that one wishes to know the full action at the eight-derivative level.

At the basic level, knowledge of these terms will tell us a lot more about actions with maximal supersymmetry, which may lead to fundamental understandings like the off-shell nature of the theory.

From a phenomenological viewpoint, there has been a lot of interest in flux compactifications, where fluxes are turned on in the internal Calabi-Yau manifold. The potential for moduli in this background can be efficiently computed in the low energy effective theory, and can be used to gain information about stable compactifications at large radius. However, one needs to know the full action including the Ramond-Ramond (RR) field strengths.

The full action will also be needed to consider the stabilization of the brane moduli.

Another place where the full effective action is required is for computing α' corrections in Anti-de-Sitter (AdS) backgrounds, for applications to the AdS/CFT correspondence. These can be applied to find corrections to black hole entropy, or to correlation functions.

Recently there has been a lot of interest in calculating corrections to black hole entropy due to higher derivative terms (Dabholkar, Sen,..). To do this correctly, we need all the terms in the effective action.

3 Approaches

It has not been possible so far to determine the complete eight derivative action. Several different approaches have been tried.

The action can be computed by evaluating all the relevant string diagrams, and extracting the low energy action (Gross, Sloan; Sakai, Tanii; Grisaru, van de Ven, Zanon; K. Peeters, P. Vanhove and A. Westerberg).

Problem: String diagrams contain much more information than just the eight-derivative terms. One needs an effective way of extracting the low energy limit without doing the entire computation. Furthermore, once we get to five-point amplitudes and beyond, we have to worry about extracting contributions to the amplitude involving the exchange of massless fields, for example those coming from a combination of the four-point eight derivative amplitude and a tree level three-point interaction. Furthermore, the plethora of fields in the supergravities means that many amplitudes need to be computed.

It is believed that the eight-derivative action is completely determined by supersymmetry alone. One can therefore attempt to construct the action by using the Noether method to generate terms step by step until supersymmetry is satisfied. This has been attempted for the heterotic string action (Romans, Warner; Bergshoeff, de Roo; de Roo, Suelmann, Wiedemann).

Problem: The vast number of fields and the plethora of possible terms make it impractical to use this method directly in ten dimensional supergravity.

Most promising: If the complete superfield can be found, then the action can be written as an integral over one-half of superspace. We will now discuss this approach.

4 Type IIB

The symmetry of type IIB is $SL(2)/U(1)$. In the standard formulation, the fields transform under this U(1). There are two scalars u,v; gauge fixing the U(1) kills one of these scalars.

The field content of Type IIB supergravity consists then of:

the vielbein $e_\mu{}^a$ charge 0

a complex two-form field $a_{\mu\nu}$ charge 1

a real four form field $a_{\mu\nu\rho\sigma}$ charge 0

two complex scalars u^*, v charge 2

gravitino ψ_μ charge 1/2

dilatino λ charge 3/2 .

Define a chiral superfield V satisfying (Howe, West)

$$D^*_\alpha V = 0 \tag{4.5}$$

The linearized superfield is

$$V| = v \tag{4.6}$$

$$D_\alpha V| = -2\lambda_\alpha \tag{4.7}$$

$$D_{[\alpha}D_{\beta]}V| = \frac{i}{12}\gamma^{abc}_{\alpha\beta} f_{abc} \tag{4.8}$$

$$D_{[\gamma}D_\beta D_{\alpha]}V| = -\frac{1}{4}\gamma^{abc}_{\beta\alpha}(\gamma_a)_{\gamma\epsilon}\psi^\epsilon_{bc} \tag{4.9}$$

while the bosonic part of the fourth component is

$$D_{[\delta}D_\gamma D_\beta D_{\alpha]}V| = \frac{1}{16}\gamma^{abc}_{[\beta\alpha}\gamma^{def}_{\gamma\delta]}(g_{ad}R_{bcef} - \frac{i}{6}D_b g_{acdef}) \tag{4.10}$$

To get the R^4 action, we integrate V^4 over one half of superspace

$$\int d^{16}\theta V^4 = (t^{abcdefgh}t_{ijklmnop} + \epsilon^{IJabcdefgh}\epsilon_{IJijklmnop})R_{ab}{}^{ij}R_{cd}{}^{kl}R_{ef}{}^{mn}R_{gh}{}^{op} + \dots \tag{4.11}$$

This is in agreement with the string calculation, which is an indication that we may be on the right track.

5 The nonlinear case and a failure

When we try to go beyond the quartic action, we need a supersymmetric measure; the supersymmetric analogue to the $\sqrt{g}$ factor. The suggested form of the eight-derivative action is then

$$S_8 = \int d^{10}x \int d^{16}\theta \Delta W(V) = \int d^{10}x \sum_{n=0}^{16} \frac{1}{n!} D_{\alpha_1}..D_{\alpha_n}\Delta|\, D^{16-n,\alpha_1..\alpha_n} W| \qquad (5.12)$$

where Δ is by definition a superfield whose lowest component is

$$\Delta|_{\theta=0} = \sqrt{g} \qquad (5.13)$$

Invariance of the action under supersymmetry requires

$$\delta S = \int d^{10}x \sum_{n=0}^{16} \frac{1}{n!} \left(\delta D_{\alpha_1}..D_{\alpha_n}\Delta|D^{16-n,\alpha_1..\alpha_n} W| + D_{\alpha_1}..D_{\alpha_n}\Delta|\delta D^{16-n,\alpha_1..\alpha_n} W|\right) = 0. \qquad (5.14)$$

and Δ is to be constructed order by order by requiring that the action be supersymmetric.

Requiring the cancellation of variations containing $D^{16}W$ yields

$$D_\alpha \Delta|_{\theta=0} = -ie\gamma^c_{\alpha\beta}\psi^{*\beta} \qquad (5.15)$$

Similarly, the cancellation of terms proportional to $D^{15}W$ determines

$$[D_\alpha, D_\beta]\Delta|_{\theta=0} = \frac{1}{12} ie\gamma^{abc}{}_{\alpha\beta} f^*_{abc} + O[\psi^*\psi^*, \lambda^*\psi] \qquad (5.16)$$

At the same time, the terms proportional to ζ^* need to cancel as well, and this has to happen automatically for this construction to work. As it turns out, the terms containing $D^{16}W$ and ζ^* do cancel, but at next order the cancellation does not work. The uncancelled term is (de Haro, Sinkowicz, Skenderis)

$$\delta S = \frac{1}{2} \int d^{10}x e \zeta^{*\alpha} T^{\gamma}_{\bar{\alpha}\bar{\delta}} T^{\bar{\delta}}_{\gamma\beta} D^{\beta,15} W| \qquad (5.17)$$

6 Towards a subset of the action

We give up the idea of reproducing the complete action from this integral. Instead we try to use the superfield to reproduce a *subset* of the terms in the action.

Under the $U(1)$ symmetry, the curvature and five-form field strength g_5 are both uncharged. Look for bosonic terms containing only these e.g R^4, g_5^8 etc. Call this set of terms $S_{8;0;R,g_5}$.

Under a supersymmetry variation, these terms produce a huge set of variations that need to be cancelled. We will consider the subset of variations which have at most one fermion field, and where we set $\partial\tau^* = \partial\tau = \lambda = a_2 = 0$. The remaining variations are of the schematic form

$$\delta S_{8;0;R,g_5} = \int d^{10}x(R^3\psi^*\epsilon + (g_5)^7\psi^*\epsilon + \ldots) \tag{6.18}$$

These can only be cancelled by variations coming from terms containing fermion bilinears which are of the schematic form

$$S_{8;0;R,g_5,\psi^*,\psi} = \int d^{10}x(R^2\psi^*\psi + (g_5)^7\psi^*\psi + \ldots) \tag{6.19}$$

As long as we only look for the cancellation of the subset of the variations $\delta S_{8;0;R,g_5}$ discussed above, we can restrict ourselves to the subset of terms $S_{8;0;R,g_5}$ and $S_{8;0;R,g_5,\psi^*,\psi}$. The superfield offers a quick way to reproduce these terms in the action.

The terms we are looking for all occur in the superfield only in the third, fourth and fifth components i.e. with a factor of θ^3, θ^4 or θ^5. In particular the bosonic terms are all found in the θ^4 component.

We should now consider the action

$$S_8 = \int d^{10}x \int d^{16}\theta \Delta V^4 \tag{6.20}$$

7 Checking the variations

We are setting $\partial\tau = \lambda = a_2 = 0$. Furthermore, we are considering variations with at most one fermion field. In this case, we can set $D^n W| = 0$ in the supersymmetry variations for all $n \leq 14$. We then only need to cancel the variations proportional to $D^{16}W$ and $D^{15}W$.

The first two components of Δ have already been computed

$$\Delta|_{\theta=0} = \sqrt{g} \tag{7.21}$$

$$D_\alpha \Delta|_{\theta=0} = -ie\gamma^c_{\alpha\beta}\psi^{*\beta} \tag{7.22}$$

Furthermore, it was shown that the variations proportional to $D^{16}W$ and $D^{15}W$ do cancel up to the obstruction shown in equation (5.17). However, the torsion factor is

$$T^{\bar{\delta}}_{\gamma\beta} = (\gamma^a)_{\gamma\beta}(\gamma_a)^{\delta\theta}\lambda^*_\theta - 2\delta^\delta_{(\gamma}\lambda^*_{\beta)} \tag{7.23}$$

which vanishes if we set $\partial\tau = \lambda = a_2 = 0$ in the variations. This means that the obstruction vanishes, and the variations $\delta S_{8;0;r,g_5}$ indeed cancel in the action (**??**).

8 Type IIB: Summary

We have found a superfield expression for the action involving, R, g_5

$$S_8 = \int d^{10}x \int d^{16}\theta \Delta V^4 \tag{8.24}$$

The bosonic part of the action can be immediately written down; up to an overall constant

$$S_{8;0;R,g_5} = \int d^{10}x f^{(0,0)}(\tau, \tau^*) \times \\ \epsilon^{\alpha_1..\alpha_{16}} (\gamma^{a_1a_2a_3}_{\alpha_1\alpha_2} \gamma^{b_1b_2b_3}_{\alpha_3\alpha_4})(\gamma^{c_1c_2c_3}_{\alpha_5\alpha_6} \gamma^{d_1d_2d_3}_{\alpha_7\alpha_8}) \ldots (\gamma^{g_1g_2g_3}_{\alpha_{13}\alpha_{14}} \gamma^{h_1h_2h_3}_{\alpha_{15}\alpha_{16}}) \times \\ \mathcal{R}_{a_1a_2a_3b_1b_2b_3} \mathcal{R}_{c_1c_2c_3d_1d_2d_3} \ldots \mathcal{R}_{g_1g_2g_3h_1h_2h_3} \tag{8.25}$$

where

$$\mathcal{R}_{abcdef} = \frac{1}{16}(g_{ad}R_{bcef} - \frac{i}{6}D_b g_{acdef}) - \frac{1}{1536}(3g_{bafmn}g_{ced}{}^{mn} - g_{abcmn}g_{def}{}^{mn}) \tag{8.26}$$

The fermionic terms can be written down similarly.

This supersymmetrizes the type IIB R^4 action with the inclusion of g_5.

9 Eleven dimensions

In IIA theory, the R^4 term occurs only at tree-level and one-loop. The R^4 action in M-theory can be obtained by taking the strong coupling limit of IIA theory (Green, Gutperle, Vanhove). Again, we would like to know the supersymmetric completion. Here, we shall show that the superfield approach can be modified to obtain the complete eight-derivative effective action in M-theory.

The lowest component of the superfield must be a scalar, so a superfield cannot be constructed in eleven dimensions. We then need to try and perform the construction in a lower dimension. That is, instead of trying to find the M-theory action directly, we will try to find the dimensional reduction of the action. The dimensionally reduced action will be constructed by the superfield.

In fact, we do not even need to find the full action in the lower dimensional theory. For example, the eleven dimensional term of the form $R^2F_4^2$ will yield, after dimensional reduction, terms like $R^2F_4^2$ as well as $R^2H_3^2$. If we can establish the exact form of either of these terms, we can dimensionally oxidize to reproduce the eleven dimensional term.

Now the requirement of having a chiral superfield forces us to reduce on a three-torus. This dimensional reduction produces $N = 2, D = 8$ supergravity (Bergshoeff et al.). Crucially, the symmetry of the theory is $SL(3,R)/SO(3,R) \times SL(2,R)/U(1)$, and we can use the last factor to build actions analogous to type IIB.

10 N=2, D=8 supergravity

N=2 D=8 supergravity can be obtained as a direct dimensional reduction of D=11 N=1 supergravity to eight dimensions.

The reduction of the vierbein produces an eight-dimensional vierbein, 6 scalars, 3 gauge fields.

$$g_{\mu\nu} \to g_{\mu\nu}, A_{\mu i}, e^{\varphi}, g_{ij} \tag{10.27}$$

From the three-form field in eleven dimensions, we get a self-dual three-form, three 2-forms, three gauge fields, 1 scalar

$$\hat{C}_{\mu\nu\rho} \to C_{\mu\nu\rho}, C_{\mu\nu i}, C_{\mu ij}, C_{ijk} \sim l \tag{10.28}$$

The symmetry of this theory is $SL(3)/SO(2) \times SL(2)/U(1)$. To represent the SL(2, R) symmetry linearly, we introduce an extra scalar. The U(1) is now a gauge transformation that can kill this scalar.

Under the local U(1) symmetry, the scalars u, v have charge 1. The gauge fields and the three-form have charge 1, the two-forms and the metric have zero charge.

Define a chiral superfield by $\bar{D}\Phi = 0$. The lowest component is the scalar u. We find for the first few components at the linearized level

$$\Phi|_{\theta=0} = u \tag{10.29}$$

$$D_\alpha \Phi|_{\theta=0} = \frac{1}{2iv}(\Gamma^0 \lambda)_\alpha \tag{10.30}$$

$$D_{[\alpha} D_{\beta]} \Phi|_{\theta=0} = -\frac{1}{2iv}\left(\frac{i}{4}\Gamma^i L_i{}^m \Gamma^{\mu\nu}\Gamma^0 f_{\mu\nu m} + \frac{1}{96}\Gamma^{\mu\nu\rho\sigma}\Gamma^0 F_{\mu\nu\rho\sigma}\right)_{\beta\alpha} \tag{10.31}$$

$$D_{[\alpha} D_\beta D_{\gamma]} \Phi|_{\theta=0} = \frac{1}{8v}\left[(\Gamma^i \Gamma^{\mu\nu}\Gamma^0)_{\gamma\beta}\left((\Gamma^0\Gamma^i)_{\alpha\delta}\tilde{\psi}^\delta_{\mu\nu} - 2(\Gamma^0\Gamma_{[\mu})_{\alpha\delta}\partial_{\nu]}\chi^{i\delta}\right)\right. \tag{10.32}$$

$$\left. +\frac{1}{4}\left(\Gamma^{\mu\nu\rho\lambda}\Gamma^0\right)_{\gamma\beta}(\Gamma^0\Gamma_{[\mu\nu})_{\alpha\delta}\tilde{\psi}^\delta_{\lambda\rho]}\right] \tag{10.33}$$

$$D_{[\alpha} D_\beta D_\gamma D_{\delta]} \Phi|_{\theta=0} = -\frac{1}{16v}\left[(\Gamma^i \Gamma^{\lambda\rho}\Gamma^0)_{\delta\gamma}(\Gamma^0\Gamma^i\Gamma_{\sigma\tau})_{\beta\alpha}\right. \tag{10.34}$$

$$\left. +\frac{1}{4}\left(\Gamma^{\mu\nu\rho\lambda}\Gamma^0\right)_{\delta\gamma}(\Gamma^0\Gamma_{\mu\nu\sigma\tau})_{\beta\alpha}\right] R_{\lambda\rho}{}^{\sigma\tau} + \ldots \tag{10.35}$$

If we integrate Φ^4 over half of superspace, we will produce an eight derivative action

$$S_8 = \int d^{10}x d^{16}\theta \Phi^4 \tag{10.36}$$

which will have linearized supersymmetry. the R^4 obtained in this way agrees with string calculations.

11 The Nonlinear case

When we try to go beyond the quartic action, we need a supersymmetric measure; the supersymmetric analogue to the $\sqrt{g}$ factor. The suggested form of the eight-derivative action is then

$$S_8 = \int d^{10}x \int d^{16}\theta \Delta \Phi^4$$
$$= \int d^{10}x\, \epsilon^{\alpha_1..\alpha_{16}} \sum_{n=0}^{16} \frac{1}{n!(16-n)!} D_{\alpha_1}..D_{\alpha_n}\Delta|\, D_{\alpha_{n+1}}...D_{\alpha_{16}} W| \tag{11.37}$$

where $W = \Phi^4$, and Δ is by definition a superfield whose lowest component is

$$\Delta|_{\theta=0} = \sqrt{g} \tag{11.38}$$

Δ is to be constructed order by order by requiring that the action be supersymmetric. As in the very similar situation of type IIB supergravity, it can be shown that such a measure does not exist.

As for type IIB, we can look for a subset of the terms in the action.

We restrict attention to the bosonic terms which involve only the field strengths which are uncharged under the U(1), viz. the curvature R, the three-form field strengths $H_{\mu\nu\rho m}$, and the scalars L_m^i. Examples of such terms are R^4, R^2H^4 etc. Call this set of terms $S_{8;0;R,H}$.

Under a supersymmetry variation, these terms produce a huge set of variations that need to be cancelled. We will consider the subset of variations which have at most one fermion field; for instance, the variation of the R^4 term will produce variations of the generic form $R^3\bar{\zeta}D^2\psi$. We will denote this set of variations as $\delta S_{8;0;R,H}$.

The only way to cancel these terms is by the variation of terms bilinear in fermions, for example, a term of the form $R^2 D\psi D^2\psi$. These terms must be of a particular form: they involve the uncharged fields R, $H_{\mu\nu\rho m}$, and L_m^i, and in addition they have two fermions, one of which carries a (1/2) charge under the U(1) (i.e. ψ_μ or χ_a), and one with a -(1/2) charge under the U(1) (i.e. ψ_μ^* or χ_a^*). Call these terms $S_{8;0;R,H,\psi^*,\psi}$.

As long as we only look for the cancellation of the subset of the variations $\delta S_{8;0;R,g_5}$ discussed above, we can restrict ourselves to the subset of terms $S_{8;0;R,g_5}$ and $S_{8;0;R,g_5,\psi^*,\psi}$. The superfield offers a quick way to reproduce these terms in the action.

We should now consider the action

$$S_8 = \int d^{10}x \int d^{16}\theta \Delta \Phi^4 \tag{11.39}$$

To construct the action, we need the first two components of Δ, i.e. $\Delta|_{\theta=0} \equiv \sqrt{g}$ and $D_\alpha \Delta|_{\theta=0}$. We may truncate the action to

$$S = \int d^8x \, \frac{1}{16!} \epsilon^{\alpha_1..\alpha_{16}} \left(\sqrt{g}\, D_{\alpha_1}...D_{\alpha_{16}} W| + 16 D_{\alpha_1}\Delta|\, D_{\alpha_2}...D_{\alpha_{16}} W|\right) \tag{11.40}$$

We now need to show that this action is supersymmetric, and we can do this exactly as we did for type IIB. We take the variations explicitly, and cancel all the terms linear in fermions. At the end of this, we find the action

$$S_8 = \int d^8x \, \frac{1}{16!} \sqrt{g} \epsilon^{\alpha_1..\alpha_{16}} \left(D_{\alpha_1}...D_{\alpha_{16}} W| - 8i(\Gamma^0\Gamma^a)_{\alpha_1\bar{\alpha}} \psi_a^{*\bar{\alpha}} D_{\alpha_2}...D_{\alpha_{16}} W| \right) \tag{11.41}$$

$$W = \left(\bar{\theta}\Gamma^{\mu\nu\rho\lambda}\theta\bar{\theta}\Gamma_{\mu\nu}\tilde{\Psi}_{\lambda\rho} + \bar{\theta}\Gamma^{ij\rho\lambda}\theta\bar{\theta}\Gamma_{ij}\tilde{\Psi}_{\lambda\rho} + 4\bar{\theta}\Gamma^{i\nu j\lambda}\theta\bar{\theta}\Gamma_{i\nu}\tilde{\Psi}_{j\lambda} + \bar{\theta}\Gamma^{\mu\nu ij}\theta\bar{\theta}\Gamma_{\mu\nu}\tilde{\Psi}_{ij} \right)^4 \tag{11.42}$$

This action is the supersymmetrization of the R^4 term in 8 dimensions. It includes all the two-form fields.

12 Summary and Conclusion

We can now construct this action, and uplift to eleven dimensions.

This yields the supersymmetrization of the R^4 terms in eleven dimensions. We get the *complete* set of bosonic terms in the M-theory action, since we found the full action in 8 dimensions with R, H.

The fermions can be done similarly.

Future work: Do explicitly, work out implications for dimensional reductions of M-theory.