Scherk-Schwarz SUSY breaking from the viewpoint of 5D conformal supergravity

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**Introduction**

**SUSY**: a solution of hierarchy problem

- SUSY breaking is necessary
- **Scherk-Schwarz SUSY breaking**:
  - Basic theory is 5D with U(1) sym.
  - different b.c. on bosons and fermions
- In 5D conformal SUGRA, various properties of SS mechanism can be easily understood.
SS breaking in 5D SUGRA

5D SUGRA on $S^1/Z_2$ with the radius $R$

It has $SU(2)_R$ symmetry.

- Orbifold b.c.: $\Phi(x, -y) = \pm Z \Phi(x, y)$
  
  We choose as $Z = \sigma_3$

- Twisted b.c. for $U(1) \subset SU(2)_R$
  
  $\Phi(x, y + 2\pi R) = T \Phi(x, y)$,
  
  $T = e^{-2\pi i \vec{\omega} \cdot \vec{\sigma}}$
Consistency condition:

\[ TZ = ZT^{-1} \]

This requires \( \bar{\omega} = (\omega_1, \omega_2, 0) \).
5D conformal supergravity

- **Symmetries:**
  \[ (P_m, M_{mn}, D, U_{ij}, K_m, Q_i, S_i) \]
  [Kugo & Ohashi]

- **Field content:**
  - Weyl multiplet: \( (e^m_\mu, \psi^i_\mu, V^{ij}_\mu, \cdots) \)
  - Hypermultiplet: \( \mathcal{H}^\alpha = (A^\alpha_i, \zeta^\alpha, F^\alpha_i) \)
    \( (\alpha = 1, 2, 3, 4, \cdots, 2(n_H + 1)) \)  
    - **compensator**
      - **Physical fields**
  - Vector multiplet: \( \nu^I = (M^I, \Omega^{II}_i, W^I_\mu, Y^{Iij}) \)
    \( (I = 0, 1, 2, \cdots, n_\nu) \)
Superconformal gauge fixing

- Extra symmetries: \((D, U, S, K)\)
- Ordinary gauge fixing conditions

\[ U\text{-fixing} : \ A^a_i \propto \delta^a_i \quad (a = 1, 2) \]

\[ SU(2)_U \times SU(2)_C \rightarrow SU(2)_R \]

rotate \(i\) \quad rotate \(a\)
Combining $D$- and $U$- gauge fixings,

\[ A^a_i = \delta^a_i \sqrt{1 + A^\alpha_j A_{\bar{\alpha}}^j} \]

We will modify this as

\[ A^a_i = \left( e^{i\bar{\omega} \cdot \bar{\sigma} f(y)} \right)^a_i \sqrt{1 + A^\alpha_j A_{\bar{\alpha}}^j}, \]

\[ (f(y + 2\pi R) = f(y) + 2\pi) \]

while the physical fields remain periodic.
$SU(2)_U$-gauge field on-shell is given by

$$V^{ij}_\mu = -\mathcal{A}^{a(i} \partial_{\mu} \mathcal{A}_{a)j} + \mathcal{A}^{\alpha(i} \partial_{\mu} \mathcal{A}_{\alpha)j} + \cdots$$

$V^{ij}_y$ is shifted and

$$\int_0^{2\pi R} dy \ V^{i,j}_y = 2\pi i \ (\vec{\omega} \cdot \vec{\sigma})^{i,j} + \cdots$$

This corresponds to the nonvanishing F-term of the radion superfield in the superfield formalism. (G.von Gersdorff, M.Quiros, …)
SUSY breaking terms:

\[ e^{-1} \mathcal{L}_\omega = f'(y) (i \bar{\omega} \cdot \sigma)_{ij} \left( 2i \bar{\psi}_m \gamma^m \gamma^4 \psi_n \psi_j + a_{IJ} \Omega^I \gamma_4 \Omega^J \psi_j \right) \]

\[ -2 \left( f'(y) |\bar{\omega}| \right)^2 A_{\alpha}^i A_{\bar{\alpha}}^i + \cdots \]

All SUSY terms are proportional to \( f'(y) \).
Singular gauge fixing

We take a gauge fixing parameter as

\[ f(y) = \frac{\pi}{2} \sum_n (\text{sgn}(y - n\pi R) - \text{sgn}(-n\pi R)) \]
Namely,

\[ f'(y) = \pi \sum_n \delta(y - n\pi R) \]

Thus \( \mathcal{L}_\omega \) becomes boundary terms.

\textbf{SS twist}

\[ \leftrightarrow \text{Boundary Constant superpotential} \]

for \( \mathbb{Z}_2 \)-even part of the action

provided that \( W = \pi(\omega_2 + i\omega_1) \)
**SS twist and warped geometry**

*SS twist yields an inconsistency on the warped geometry.*  (Hall, Nomura, Okui, Oliver (2004))

- Gauging of $U(1) \subset SU(2)_R$ by graviphoton is necessary to realize $AdS_5$ geometry.
  
  \[ \mathcal{D}_\mu A^a_i = \partial_\mu A^a_i - A^a_j V_{\mu ij} - g_R W^R_{\mu} (\bar{q} \cdot i\bar{\sigma})^a b A^b_i + \cdots \]

- From \( \mathcal{D}^\mu A^a_i \mathcal{D}_\mu A^a_i \), a graviphoton mass term appears, which is proportional to \( [(\bar{q} \cdot \bar{\sigma}), (\bar{\omega} \cdot \bar{\sigma})] \).
Consistency condition: 
\[ [\tilde{q} \cdot \tilde{\sigma}, (\tilde{\omega} \cdot \tilde{\sigma})] = 0 \quad (\tilde{\omega} = (\omega_1, \omega_2, 0)) \]

- **Models of** (Gherghetta, Pomarol; Falkowski, Lalak, Pokorski) gauging $U(1)_R$ with $\mathbb{Z}_2$-odd coupling
  i.e., $\tilde{q} = (0, 0, q_3) \iff \tilde{\omega} = 0$

- **Model of** (Altendorfer, Bagger, Nemeschansky) gauging $U(1)_R$ with $\mathbb{Z}_2$-even coupling
  i.e., $\tilde{q} = (q_1, q_2, 0) \iff \tilde{\omega} \propto \tilde{q}$
  SS twisting seems possible.
  But this model is not derived from known off-shell SUGRA.
Summary

From the viewpoint of 5D conformal SUGRA,

- SS twisting for $SU(2)_R$ is understood as twisted $SU(2)_U$-gauge fixing.

- SS twist $\Leftrightarrow$ Boundary constant superpotential for $Z_2$-even part of the Lagrangian

- SS twist in AdS$_5$ leads to massive graviphoton.
Future works

- The case that both SS twist and boundary constant superpotentials exist (model of Altendorfer, Bagger, Nemeschansky?)

- Extension to two-compensator case (e.g. effective theory of heterotic M theory)