N=4 SYM on R x S^3 and Theories with 16 Supercharges

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N=4 SYM on R x S^3

Nicolai-Sezgin-Tanii
Okuyama

- R x S^3 is the boundary of AdS_5 in the global coordinates

$$ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_3^2)$$

boundary $(\rho \to \infty) \sim R \times S^3$

- action

$$S = \frac{1}{g_{YM}^2}\int d^4x\sqrt{-g}\Big(-\frac{1}{4}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho} - \frac{1}{2}g^{\mu\nu}\partial_\mu X_m\partial_\nu X_m - \frac{1}{12}R_3 X_m X_m + [X_m, X_n]^2$$
$$-\frac{i}{2}e_a^\mu\bar{\lambda}\Gamma^a D_\mu\lambda - \frac{1}{2}\bar{\lambda}\Gamma_m[X_m,\lambda]\Big)$$

$m = 4 \sim 9$ $\quad R_3 = 6$ $\quad D_\mu\psi = \partial_\mu\psi + \frac{i}{2}\omega_\mu^{ab}\Sigma_{ab}\psi + i[A_\mu,\psi]$

- superconformal symmetry $SU(2,2|4) \supset SO(2,4)\times SO(6)$

32 real supercharges ⇦ conformal Killing spinors on R x S^3

⇧ Killing spinors on AdS_5

- N=4 SYM :on R x S^3 ~ on R^4 at conformal point

But...

A family of theories with 16 supercharges

Start with $\mathcal{N}=4$ SYM on $R\times S^3$ — Lin-Maldacena

$$SU(2,2|4)\supset SO(2,4)\times SO(6)\supset R\times SO(4)\times SO(6)$$

$SO(4)=SU(2)\times \tilde{SU}(2)$ isometry of S^3

dividing by various subgroups of $\tilde{SU}(2)$

$\Longrightarrow$ theories with SU(2|4) symmetry

consistent truncations

$Z_k \Longrightarrow$ $\mathcal{N}=4$ SYM on $R\times S^3/Z_k$

$U(1) \Longrightarrow$ $2+1$ SYM on $R\times S^2$

Maldacena-Sheikh-Jabbari-Raamsdonk

$\tilde{SU}(2) \Longrightarrow$ plane wave matrix model — BMN

Kim-Plefka-Klose

$k\to\infty$

remove fuzzyness of fuzzy sphere vacuum

Truncated theories have many vacua !

extension of bubbling AdS

Lin-Maldacena

- General solutions of IIA sugra that preserve SU(2|4)

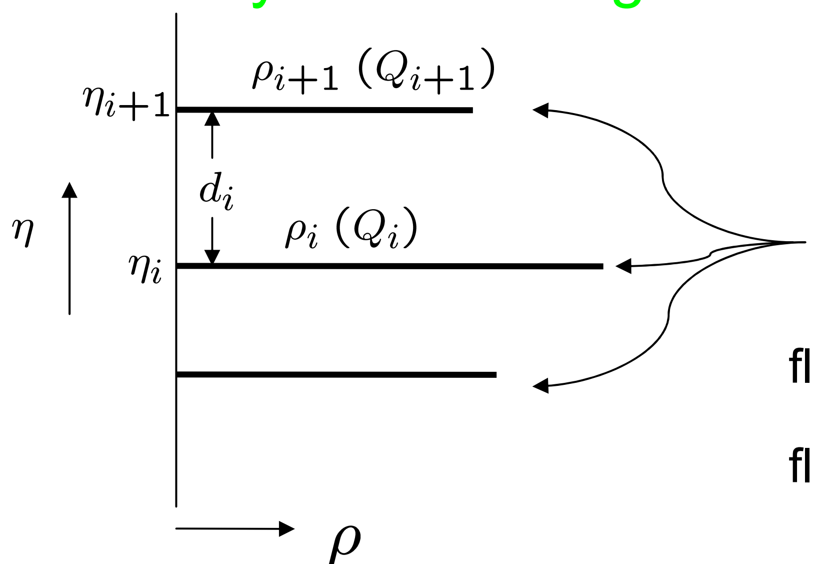
$$ds_{10}^2 = \left(\frac{\ddot{V} - 2\dot{V}}{-V''}\right)\left\{-4\frac{\ddot{V}}{\ddot{V} - 2\dot{V}}dt^2 + \frac{-2V''}{\dot{V}}(d\rho^2 + d\eta^2) + 4d\Omega_5^2 + 2\frac{V''\dot{V}}{\Delta}d\Omega_2^2\right\}$$

$$V = V(\rho,\eta) \qquad \Delta = (\ddot{V} - 2\dot{V})V'' - (\dot{V}')^2, \quad \dot{} = \rho\partial_\rho, \quad ' = \partial_\eta$$

$$\frac{1}{\rho}\partial_\rho(\rho\partial_\rho V) + \partial_\eta^2 V = 0$$

Interpret V as electric potential for a 3d axially symmetric system

boundary conditi ~ b.g. field and config. of conducting disks



conducting disks

flux of $*\tilde{F}_4$ $\qquad N_2 = \frac{8Q_i}{\pi^2}$

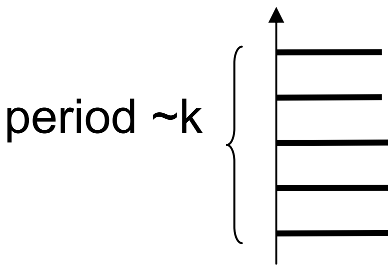
flux of H_3 $\qquad N_5 = \frac{2d_i}{\pi}$

$$V = V_b + V_c$$

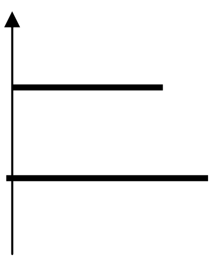
background electric field V_b $\longleftrightarrow$ the theory we consider

configuration of disks V_c $\longleftrightarrow$ a vacuum we select

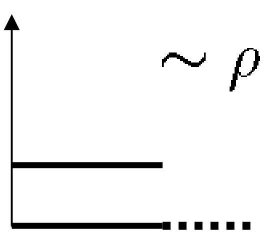
Examples



$\frac{1}{g_s k}(\rho^2 - 2\eta^2)$ $\Longleftrightarrow$ a vacuum of $\mathcal{N}=4$ SYM on $R \times S^3/Z_k$



$\sim \rho^2 - 2\eta^2$ $\Longleftrightarrow$ a vacuum of $2+1$ SYM on $R \times S^2$



$\sim \rho^2 \eta - \frac{2}{3}\eta^3$ $\Longleftrightarrow$ a vacuum of plane wave matrix model

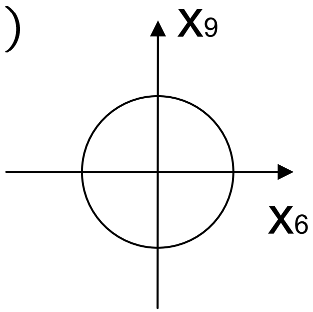
Time-dependent BPS solution

In the original and truncated theories

$$Z = \begin{pmatrix} \xi \, e^{it} & \\ & 0 \end{pmatrix}$$

$$Z = \frac{1}{\sqrt{2}}(X_6 + iX_9)$$

s-wave



In original N=4 SYM on $R \times S^3$, AdS giant graviton

Hashimoto-Hirano-Itzaki

classically

$\Longleftrightarrow$ a vacuum with Higgs

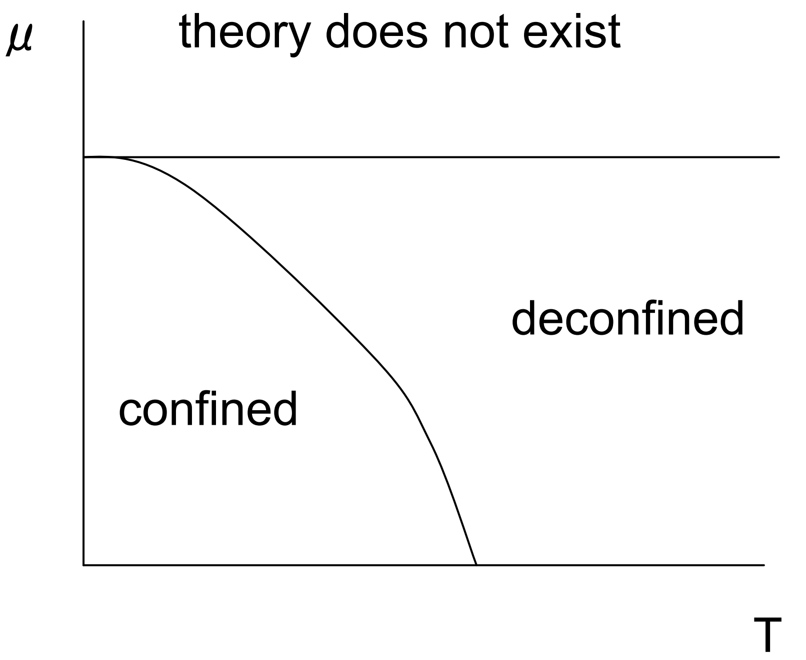
$\not\Longleftrightarrow$ vev of N=4 SYM on R^4

quantum mechanically

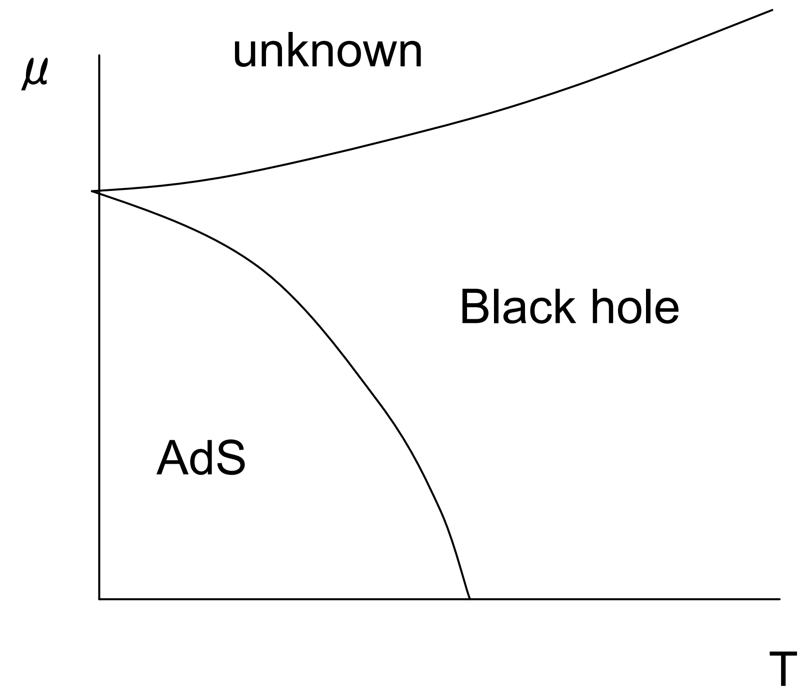
In PWMM, a spherical membrane solution

finite temperature N=4 SYM on $S^1 \times S^3$ and blackhole thermodynamics

Witten, Sundborg, Aharony et al, Yamada-Yaffe, Cvetic-Gubser



zero coupling limit (gauge side)



strong coupling (gravity side)

On flat spce, only deconfined phase

Hawking-Page transition

Harmonic expansion on S^3

Salam-Strathdee

$S^3 = G/H = SO(4)/SO(3)$

$G = SO(4) = SU(2) \times \tilde{SU}(2)$

$J_i \qquad \tilde{J}_i \qquad i = 1,2,3$

$H = SU(2) \quad L_i = J_i + \tilde{J}_i$ ~identified with 3d local `Lorentz' group

basis of $(J,\tilde{J})$ rep. of G $\quad |Jm\rangle|\tilde{J}\tilde{m}\rangle$

basis of spin L rep. of H $\quad |Ln;J\tilde{J}\rangle\rangle = \sum_{m\tilde{m}} C^{Ln}_{Jm\ \tilde{J}\tilde{m}}|Jm\rangle|\tilde{J}\tilde{m}\rangle$

representative of G/H $\quad \Upsilon(\Omega) = e^{-i\psi L_1}e^{-i\varphi L_3}e^{-i\theta K_1}$

$K_i = J_i - \tilde{J}_i \qquad \Omega = (\theta,\varphi,\psi)$

spin L spherical harmonics $\quad \mathcal{Y}^{Ln}_{Jm,\tilde{J}\tilde{m}}(\Omega) = N^{L}_{J\tilde{J}}\langle\langle Ln;J\tilde{J}|\Upsilon^{-1}(\Omega)|Jm\rangle|\tilde{J}\tilde{m}\rangle$

scalar $L=0, \quad (J,\tilde{J}) = (J,J)$ $\qquad$ spinor $L=\frac{1}{2}, \quad (J,\tilde{J}) = (J+\frac{1}{2},J)$ or $(J,J+\frac{1}{2})$

transverse vector $\quad L=1, \quad (J,\tilde{J}) = (J+1,J)$ or $(J,J+1)$

integral of product of three harmonics

new result !

$$\int d\Omega \sum_{n_1 n_2 n_3} (\mathcal{Y}^{L_1 n_1}_{J_1 m_1, \tilde{J}_1 \tilde{m}_1})^* \, \mathcal{Y}^{L_2 n_2}_{J_2 m_2, \tilde{J}_2 \tilde{m}_2} \, \mathcal{Y}^{L_3 n_3}_{J_3 m_3, \tilde{J}_3 \tilde{m}_3} \, C^{L_1 n_1}_{L_2 n_2 \; L_3 n_3}$$

$$= \sqrt{(2L_1+1)(2J_2+1)(2\tilde{J}_2+1)(2J_3+1)(2\tilde{J}_3+1)} \left\{ \begin{array}{ccc} J_1 & \tilde{J}_1 & L_1 \\ J_2 & \tilde{J}_2 & L_2 \\ J_3 & \tilde{J}_3 & L_3 \end{array} \right\} C^{J_1 m_1}_{J_2 m_2 \; J_3 m_3} C^{\tilde{J}_1 \tilde{m}_1}_{\tilde{J}_2 \tilde{m}_2 \; \tilde{J}_3 \tilde{m}_3}$$

free part of action

$$S_{free} = \int dt \Bigg[\sum_{JM} (-1)^{m-\tilde{m}} \frac{1}{2} \mathrm{Tr}(\dot{X}^{J\,-M}_m \dot{X}^{JM}_m - \omega^{X\,2}_J X^{J\,-M}_m X^{JM}_m)$$

$$+ \sum_{\rho JM} (-1)^{m-\tilde{m}+1} \frac{1}{2} \mathrm{Tr}(\dot{A}_{J-M\rho} \dot{A}_{JM\rho} - \omega^{A\,2}_J A_{J-M\rho} A_{JM\rho} + \cdots) \Bigg]$$

all fields are massive $\omega^X_J = 2J+1, \quad \omega^A_J = 2J+2, \quad \omega^\psi_J = 2J + \frac{3}{2}$

interaction terms (example)

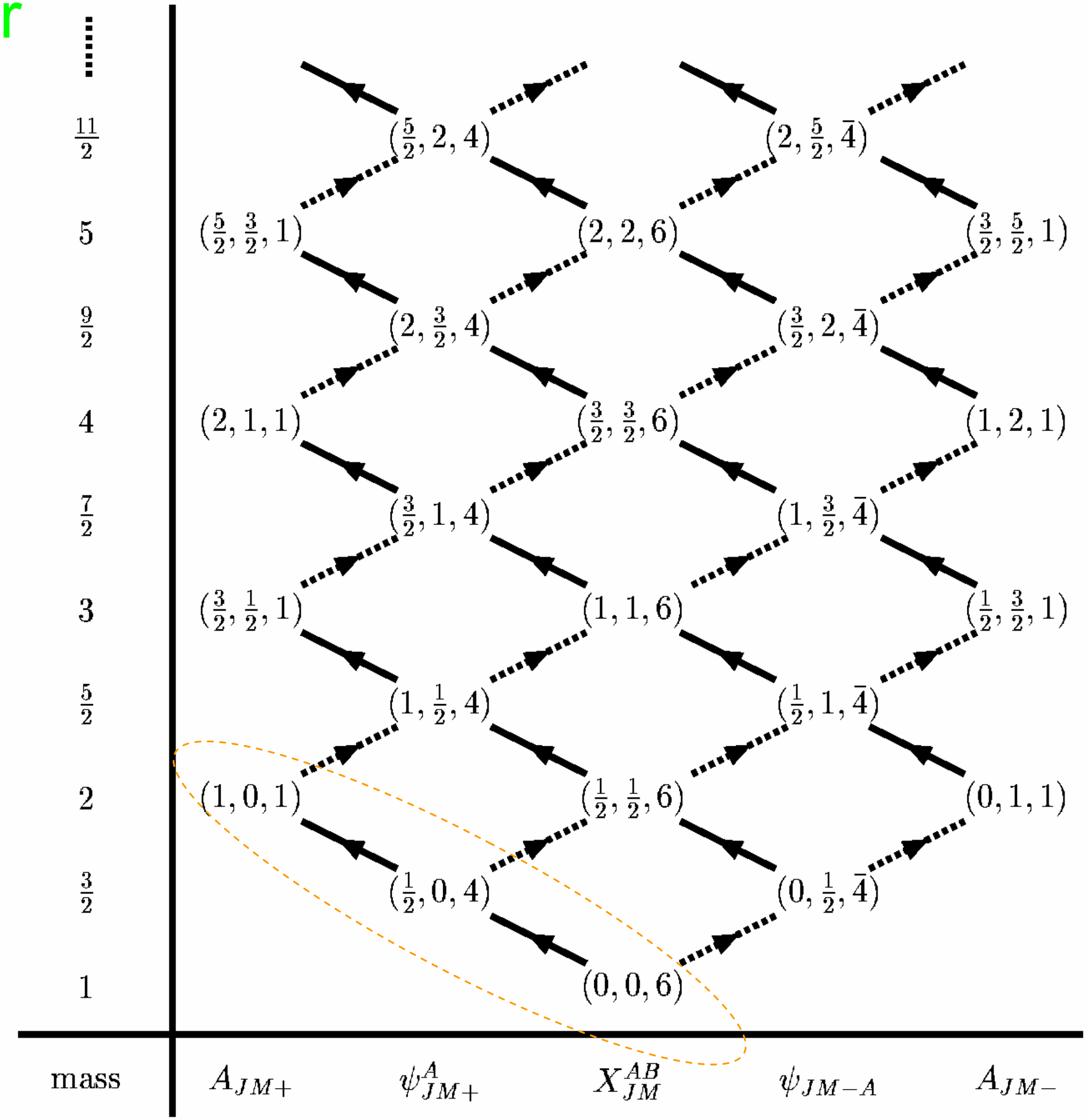
$$g \int dt d\Omega \; \mathrm{Tr}(\psi^\dagger_A \sigma^i [A_i, \psi^A])$$

$$= g G^{J_1 M_1 \kappa_1}_{J_2 M_2 \kappa_2 \; JM\rho} \int dt \; \mathrm{Tr}(\psi^\dagger_{J_1 M_1 \kappa_1 A} [A_{JM\rho}, \psi^A_{J_2 M_2 \kappa_2}])$$

$$G^{J_1 M_1 +}_{J_2 M_2 + \; JM+}$$

$$= (-1)^{\frac{\rho}{2}} \sqrt{6(2J_2+1)(2J_2+2)(2J+1)(2J+3)} \left\{ \begin{array}{ccc} J_1+1/2 & J_1 & 1/2 \\ J_2+1/2 & J_2 & 1/2 \\ J+1 & J & 1 \end{array} \right\} C^{J_1+1/2 \; m_1}_{J_2+1/2 \; m_2 \; J_1+1 \; m} C^{J_1 \tilde{m}}_{J_2 \tilde{m}_2 \; J \tilde{m}}$$

KK tower



$\frac{11}{2}$
5
$\frac{9}{2}$
4
$\frac{7}{2}$
3
$\frac{5}{2}$
2
$\frac{3}{2}$
1
mass
A_{JM+}
ψ^{A}_{JM+}
X^{AB}_{JM}
ψ_{JM-A}
A_{JM-}
$(\frac{5}{2}, 2, 4)$
$(2, \frac{5}{2}, \bar{4})$
$(\frac{5}{2}, \frac{3}{2}, 1)$
$(2, 2, 6)$
$(\frac{3}{2}, \frac{5}{2}, 1)$
$(2, \frac{3}{2}, 4)$
$(\frac{3}{2}, 2, \bar{4})$
$(2, 1, 1)$
$(\frac{3}{2}, \frac{3}{2}, 6)$
$(1, 2, 1)$
$(\frac{3}{2}, 1, 4)$
$(1, \frac{3}{2}, \bar{4})$
$(\frac{3}{2}, \frac{1}{2}, 1)$
$(1, 1, 6)$
$(\frac{1}{2}, \frac{3}{2}, 1)$
$(1, \frac{1}{2}, 4)$
$(\frac{1}{2}, 1, \bar{4})$
$(1, 0, 1)$
$(\frac{1}{2}, \frac{1}{2}, 6)$
$(0, 1, 1)$
$(\frac{1}{2}, 0, 4)$
$(0, \frac{1}{2}, \bar{4})$
$(0, 0, 6)$

Consistent truncations

Keep only KK modes that are invariant under action of subgroup

$Z_k \quad : \ \tilde{m} = \frac{k}{2} Z \quad \Longrightarrow \quad \mathcal{N}=4$ SYM on $R \times S^3/Z_k$

$U(1) \quad : \ \tilde{m} = 0 \quad \Longrightarrow \quad 2+1$ SYM on $R \times S^2$

$\tilde{SU}(2) \quad : \ \tilde{J} = 0, \ \tilde{m} = 0 \quad \Longrightarrow \quad$ plane wave matrix model

We can examine the original and truncated theories at the same time at least for the trivial vacuum

comparison for $2+1$ SYM on $R \times S^2$

$$S_2 = \frac{1}{g'^2} \int dt \frac{d\Omega'}{\mu^2} \mathrm{Tr}\{ -\frac{1}{4} F_{a'b'} F^{a'b'} - \frac{1}{2}(D_{a'} X_m)^2 - \frac{\mu^2}{8} X_m^2 - \frac{1}{2}(D_{a'}\Phi)^2 - \frac{\mu^2}{2}\Phi^2 + \frac{1}{4}[X_m, X_n]^2 + \frac{1}{2}[\Phi, X_m]^2 - \mu \Phi F_{12} + \cdots \}$$

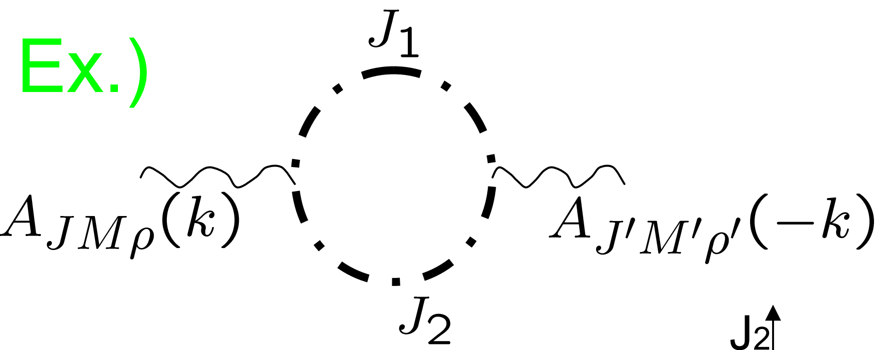
$\mu^{-1} =$ radius of $S^2 = \frac{1}{2}$

$$\left\{ \begin{array}{l} \Phi = \sum_{Jm} \Phi_{Jm} Y_{Jm}, \qquad A_i = \sum_{Jm} A^t_{Jm} Y^t_{Jmi} \\ iA_{(J-1)m+} = \sqrt{\frac{1+J}{1+2J}} A^t_{Jm} + \sqrt{\frac{J}{1+2J}} \Phi_{Jm}, \qquad (\text{for } J \geq 1) \\ iA_{Jm-} = -\sqrt{\frac{J}{1+2J}} A^t_{Jm} + \sqrt{\frac{1+J}{1+2J}} \Phi_{Jm}, \qquad (\text{for } J \geq 0) \end{array} \right.$$

etc.

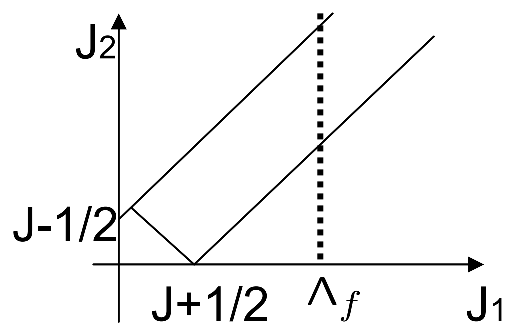
1-loop calculation in original theory

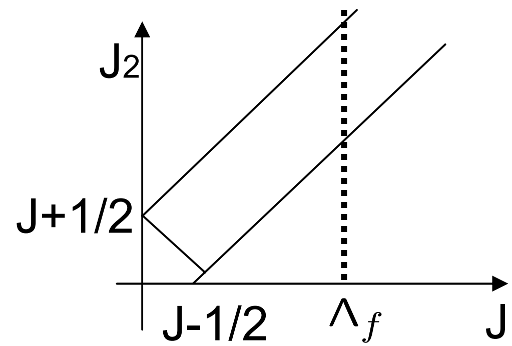
Ex.)



$$\sim \frac{32}{3}\Lambda_f^2 + \frac{64}{3}\Lambda_f + \frac{4}{3}\left(k^2 - 4(J+1)^2\right)\sum_P^{2\Lambda_f}\frac{1}{P}$$

All divergences are local





Log divergences

$$\Pi_J^A(k)_{log\ div} = \frac{4}{3}(k^2 - {\omega_J^A}^2)\log(2\Lambda)$$

$$\Pi_J^X(k)_{log\ div} = (k^2 - {\omega_J^X}^2)\log(2\Lambda)$$

$$\Pi_J^X(k)_{log\ div} = (k^2 - {\omega_J^X}^2)\log(2\Lambda)$$

1-loop log div. of vertices

$\Longrightarrow$ Ward identity OK

beta function vanishes

Determination of counter terms

counter terms

ex.) quadratic in X

$$\alpha_X \mathrm{Tr}\left(\frac{1}{2}\partial_0 X_{AB}\partial_0 X^{AB} + \frac{1}{2}X_{AB}\nabla^2 X^{AB} - \frac{1}{2}X_{AB}X^{AB}\right)$$
$$+\frac{\beta_X}{2}\mathrm{Tr}(X_{AB}\nabla^2 X^{AB}) - \frac{\gamma_X}{2}\mathrm{Tr}(X_{AB}X^{AB}),$$

on R^4 | on $R \times S^3$

chiral primary operator $\Longleftrightarrow$ chiral primary state

$\mathrm{Tr}(Z^2)$ | $\frac{2}{N}\mathrm{Tr}(a_{00}^{34\dagger}a_{00}^{34\dagger})|0\rangle$

↓ descendant

$\mathrm{Tr}(\psi^3 Z)$ $\mathrm{Tr}(\psi^4 Z)$ | $\frac{\sqrt{2}}{N}\mathrm{Tr}(d_{0M}^{3\dagger}\alpha_{00}^{34\dagger})|0\rangle$ $\frac{\sqrt{2}}{N}\mathrm{Tr}(d_{0M}^{4\dagger}\alpha_{00}^{34\dagger})|0\rangle$

↓

1-loop energy shift vanishes

$$\Longrightarrow \gamma_X = g^2 N\left(\sqcap_{J=0}^{X}(1) + \frac{1}{2}\right)$$

etc.

SO(6) spin chain

on R^4

$$\mathrm{Tr}(X_{m_1}(x)X_{m_2}(x)\cdots X_{m_L}(x))$$

on $R \times S^3$

$$\mathrm{Tr}((a^{00}_{m_1})^\dagger (a^{00}_{m_2})^\dagger \cdots (a^{00}_{m_L})^\dagger)|0\rangle$$

1-loop anomalous dimension ⟺ 1-loop energy shift

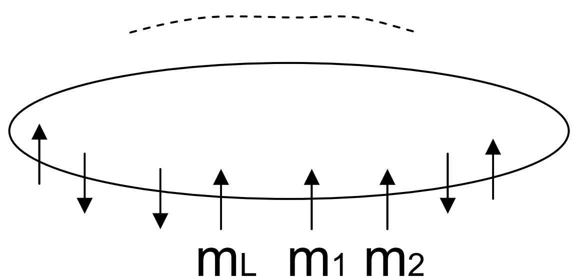
$$\Delta - L = \Delta_{1-loop} + \cdots$$

$$E - L = E_{1-loop} + \cdots$$

Minahan-Zarembo

← using γ_X

SO(6) Heisenberg model
~integrable



We can consider the same states in the truncated theories and show that they are mapped to the same SO(6) spin chain

1-loop effective action around time-dep. BPS solution

$$Z_{00} = \begin{pmatrix} \xi\, e^{it} & \\ & 0 \end{pmatrix}$$

time-dep. half-BPS solution in original and truncated theories

In original theory

$$\begin{aligned}
&\Gamma_{eff}^{1-loop} \\
&= \int dt \Big[-g^2(N-1) \sum_{J=0}^{\Lambda} (4(2J+1)^2\sqrt{(2J+1)^2+2\xi^2} + (2J+1)^2\sqrt{(2J+2)^2+2\xi^2}\,) \\
&\quad -g^2(N-1) \sum_{J=1/2}^{\Lambda} (2J+1)^2\sqrt{4J^2+2\xi^2} \\
&\quad -2g^2(N-1) \sum_{J=0}^{\Lambda_v} (2J+1)(2J+3)\sqrt{(2J+2)^2+2\xi^2} \\
&\quad +4g^2(N-1) \sum_{J=0}^{\Lambda_f} (2J+1)(2J+2)(\sqrt{(2J+1)^2+2\xi^2} + \sqrt{(2J+2)^2+2\xi^2}\,) \Big] \\
&= 0 \leftarrow \text{tuning of } X^4 \text{ counter term}
\end{aligned}$$

1-loop stable ! Also in the truncated theories

Summary

- We made a harmonic expansion of N=4 SYM on $R \times S^3$ including the interaction terms.
- We obtained each of the truncated theories by keeping a part of KK modes and made a comparison for 2+1 SYM on $R \times S^2$.
- We calculated the 1-loop radiative corrections in the original theory in terms of the angular momentum cut-off scheme.

  and saw the consistency of this scheme
- We determined some counter terms in the original and truncated theories by using the BPS states
- We verified that the 1-loop dilatation operator for the SO(6) spin chain is obtained as the 1-loop energy shift in the original theory and the same SO(6) spin chain is obtained for the truncated theories
- We showed that the 1-loop effective action around the time-dependent BPS solution vanishes in the original and truncated theories

Outlook

- Study of nontrivial vacua
- Beyond 1-loop for SU(2) spin chain around trivial and nontrivial vacua   cf.)Fischbacher-Klose-Plefka
- Other spin chains
- Non-BPS solution around trivial and nontrivial vacua   interaction bet. branes   cf.)Shin-Yoshida
- Comparison with gravity duals
- Finite temperature and black hole thermodynamics