One-loop Renormalisation of $\mathcal{N} = \frac{1}{2}$ Supersymmetry

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Introduction

- Introduction to $\mathcal{N} = \frac{1}{2}$.
- $U(N)$ and $SU(N)$ versions of the pure gauge theory.
- $\mathcal{N} = \frac{1}{2}$ with chiral matter.
- Some $\mathcal{N} = 1$.
- Massive $\mathcal{N} = \frac{1}{2}$ theory.
- $\mathcal{N} = \frac{1}{2}$ in the adjoint representation.
- Summary.

- Based upon
  - I. Jack, D.R.T. Jones and L.W., “One-loop renormalisation of $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory,” Phys. Lett. B611 199 (2005), hep-th 0412009.
  - I. Jack, D.R.T. Jones and L.W., “One-loop renormalisation of general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory,” Phys. Rev. D72 065002 (2005), hep-th 0505248.
  - I. Jack, D.R.T. Jones and L.W., “Renormalisation of supersymmetric gauge theory in the uneliminated component formalism,” Phys. Rev. D72 107701 (2005), hep-th 0509089.
  - I. Jack, D.R.T. Jones and L.W., “One-loop renormalisation of $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory in the adjoint representation,” in preparation.
  - I. Jack, D.R.T. Jones and L.W., “One-loop renormalisation of massive $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory,” in preparation.

$\mathcal{N} = \frac{1}{2}$ Supersymmetry

- $\mathcal{N} = \frac{1}{2}$ SUSY arises when superspace is deformed, i.e. the superspace coordinates no longer anticommute, but instead satisfy the condition

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \qquad \alpha = 1, 2.$$

- N. Seiberg, "Noncommutative Superspace, $\mathcal{N} = \frac{1}{2}$ Supersymmetry, Field Theory and String Theory," hep-th 0305248.
- This condition means that products of functions of θ should be reordered according to the star product

$$f(\theta) * g(\theta) = f(\theta) exp\left(- \frac{C^{\alpha\beta}}{2} \frac{\overleftarrow{\partial}}{\partial\theta^\alpha} \frac{\overrightarrow{\partial}}{\partial\theta^\beta} \right) g(\theta).$$

- The star product leads to new terms in the action involving the non-anti-commutativity parameter C.
- Can use the alternative form of the NAC parameter, $C^{\mu\nu}$:

$$
\begin{aligned}
C^{\mu\nu} &= C^{\alpha\beta}\epsilon_{\beta\gamma}(\sigma^{\mu\nu})_{\alpha}{}^{\gamma}, \\
\sigma^{\mu\nu} &= \tfrac{1}{4}(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}),
\end{aligned}
$$

$$
\sigma^{\mu} \equiv (1, \sigma^{i}), \qquad \bar{\sigma}^{\mu} \equiv (1, -\sigma^{i}).
$$

- The supersymmetry transformations are modified from the $\mathcal{N} = 1$ case.

$\mathcal{N}=\frac{1}{2}$ Pure Gauge Theory

- Original lagrangian for pure $\mathcal{N}=\frac{1}{2}$ supersymmetric gauge theory is

$$
\begin{aligned}
S = & \int d^4x \Big[tr\{-\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu} - 2i\bar\lambda\bar\sigma^\mu(D_\mu\lambda) + D^2\} \\
& - 2igC^{\mu\nu}tr\{F_{\mu\nu}\bar\lambda\bar\lambda\} + g^2|C|^2 tr\{(\bar\lambda\bar\lambda)^2\}\Big].
\end{aligned}
$$

(T. Araki, K. Ito and A. Ohtsuka, hep-th 0307076)

- This action is invariant under the transformations

$$
\begin{aligned}
\delta A^A_\mu &= -i\bar\lambda^A\bar\sigma_\mu\epsilon, \\
\delta\lambda^A_\alpha &= i\epsilon_\alpha D^A + (\sigma^{\mu\nu}\epsilon)_\alpha\left[F^A_{\mu\nu} + \tfrac{1}{2}iC_{\mu\nu}d^{ABC}\bar\lambda^B\bar\lambda^C\right], \\
\delta\bar\lambda^A_{\dot\alpha} &= 0, \\
\delta D^A &= -\epsilon\sigma^\mu D_\mu\bar\lambda^A.
\end{aligned}
$$

- Problem: $U(N)$ pure gauge theory is NOT renormalisable.
- Solution: $SU(N)$ theory used instead. Proposed action:

$$S = \frac{1}{2}\int d^4x \Big[\{ -\tfrac{1}{2} F^{\mu\nu a} F^a_{\mu\nu} - 2i\bar{\lambda}^a \bar{\sigma}^\mu (D_\mu \lambda)^a + D^a D^a \} - igC^{\mu\nu} d^{abc} F^a_{\mu\nu} \bar{\lambda}^b \bar{\lambda}^c + \tfrac{1}{4} g^2 |C|^2 [d^{abe} d^{cde} (\bar{\lambda}^a \bar{\lambda}^b)(\bar{\lambda}^c \bar{\lambda}^d) + \tfrac{2}{N} h (\bar{\lambda}^a \bar{\lambda}^a)(\bar{\lambda}^b \bar{\lambda}^b)] \Big].$$

- This action is invariant under $SU(N)$ $\mathcal{N} = \frac{1}{2}$ transformations.

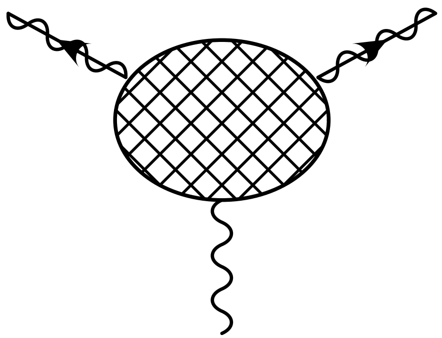
Renormalisation

- Our aim is to determine whether:

  the divergences that appear in 1-loop diagrams can simply be removed by replacing fields and couplings with bare fields and bare couplings, without the need to add further terms to make the result finite.

Calculation of Divergences

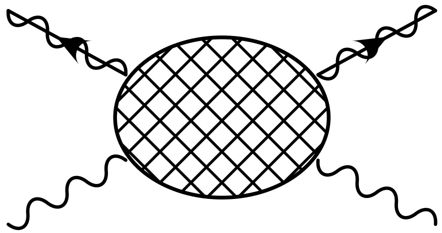
- One-loop graphs contributing to standard terms in lagrangian are as in the $\mathcal{N} = 1$ case.
- Terms containing C's are all that need to be considered.
- For example: term $C^{\mu\nu} d^{abc} \partial_\mu A^a_\nu \bar{\lambda}^b \bar{\lambda}^c$:



This set of diagrams gives anomalous terms of the form

$$C^{\mu\nu} d^{abc} A^a_\rho \bar{\lambda}^b \bar{\sigma}^\rho_\nu \partial_\mu \bar{\lambda}^c.$$

- Similarly, the term $C^{\mu\nu} d^{abe} f^{cde} A_\mu^c A_\nu^d \bar{\lambda}^a \bar{\lambda}^b$



produces anomalous results of the kind

$$C^{\mu\rho} d^{cde} f^{abe} A_\mu^c A_\nu^d \bar{\lambda}^a \bar{\sigma}_\rho^\nu \bar{\lambda}^b.$$

- These terms can be removed by an unusual divergent field redefinition

$$\delta\lambda^a = -\tfrac{1}{2}NLg^2C^{\mu\nu}d^{abc}\sigma_\mu\bar{\lambda}^cA^b_\nu \quad L = \frac{1}{16\pi^2\epsilon}\ .$$

which results in a change in the action:

$$\begin{aligned}\delta S_\lambda &= NLg^2\int d^4x\Big[-\tfrac{1}{4}iC^{\mu\nu}(d^{abc}g\partial_\mu A^a_\nu\bar{\lambda}^b\bar{\lambda}^c - d^{abe}f^{cde}g^2A^c_\mu A^d_\nu\bar{\lambda}^a\bar{\lambda}^b) \\ &+ id^{abc}gC^\mu{}_\nu A^a_\mu\bar{\lambda}^b\bar{\sigma}^{\nu\rho}\partial_\rho\bar{\lambda}^c - \tfrac{1}{2}id^{cde}f^{abe}g^2C^{\mu\rho}A^c_\mu A^d_\nu\bar{\lambda}^a\bar{\sigma}^\nu_\rho\bar{\lambda}^b\end{aligned}$$

- All C-dependent terms are now in the desired form.

Bare Quantities

- The divergences are now removed by replacing fields and couplings with bare fields and bare couplings.

$$\lambda_B^a = Z_\lambda^{\frac{1}{2}} \lambda^a, \qquad A_B^{\mu a} = Z_A^{\frac{1}{2}} A^{\mu a},$$
$$g_B = Z_g g, \qquad C^{\mu\nu} = Z_c C^{\mu\nu}.$$

- The renormalisation constants for the fields are the same as in the $\mathcal{N} = 1$ supersymmetric theory and are given by

$$\begin{aligned} Z_\lambda &= 1 - g^2 L(2\alpha N + 2), \\ Z_A &= 1 + g^2 L[(3 - \alpha)N - 2], \\ Z_g &= 1 + g^2 L(1 - 3N). \end{aligned}$$

- The only renormalisation constant left to determine is Z_C and we find

$$Z_C = 1$$

  That is, the non-anti-commutativity parameter C is not renormalised.

- Combining the field redefinition and the replacement of the fields and couplings with the bare quantities, we find that

$\mathcal{N} = \frac{1}{2}$ pure gauge theory is renormalisable.

$\mathcal{N} = \frac{1}{2}$ Theory with Matter

- $U(N)$ theory?
- $SU(N)$ theory?
- We propose a $U(1) \times SU(N)$ action:

$$
\begin{aligned}
S = \int & d^4x \Big[(\mathcal{N} = 1 \text{ terms}) - \tfrac{1}{2} i C^{\mu\nu} d^{ABC} e^{ABC} F^A_{\mu\nu} \bar{\lambda}^B \bar{\lambda}^C \\
+ & \tfrac{1}{8} g^2 |C|^2 d^{abe} d^{cde} (\bar{\lambda}^a \bar{\lambda}^b)(\bar{\lambda}^c \bar{\lambda}^d) + \frac{1}{4N} \frac{g^4}{g_0^2} |C|^2 (\bar{\lambda}^a \bar{\lambda}^a)(\bar{\lambda}^b \bar{\lambda}^b) \\
+ & \sqrt{2} C^{\mu\nu} D_\mu \bar{\phi} \bar{\hat{\lambda}} \bar{\sigma}_\nu \psi + i C^{\mu\nu} \bar{\phi} \hat{F}_{\mu\nu} F + \tfrac{1}{8} |C|^2 d^{ABC} \bar{\phi} R^A \bar{\hat{\lambda}}^B \bar{\hat{\lambda}}^C F \\
+ & \tfrac{1}{N} \gamma_1 g_0^2 |C|^2 (\bar{\lambda}^a \bar{\lambda}^a)(\bar{\lambda}^0 \bar{\lambda}^0) - \gamma_2 C^{\mu\nu} g \Big(\sqrt{2} D_\mu \bar{\phi} \bar{\lambda}^a R^a \bar{\sigma}_\nu \psi \\
+ & \sqrt{2} \bar{\phi} \bar{\lambda}^a R^a \bar{\sigma}_\nu D_\mu \psi + \bar{\phi} F^a_{\mu\nu} R^a F \Big)
\end{aligned}
$$

where

$$\hat{A}_\mu = \hat{A}_\mu^A R^A = g A_\mu^a R^a + g_0 A_\mu^0 R^0$$

and

$$e^{abc} = g, \qquad e^{a0b} = e^{ab0} = e^{000} = g_0, \qquad e^{0ab} = \frac{g^2}{g_0}.$$

- The action is invariant under the transformations

$$\begin{aligned} \delta\phi &= \sqrt{2}\epsilon\psi, \qquad \delta\bar{\phi} = 0, \\ \delta\psi^\alpha &= \sqrt{2}\epsilon^\alpha F, \qquad \delta\bar{\psi}_{\dot{\alpha}} = -i\sqrt{2}(D_\mu\bar{\phi})(\epsilon\sigma^\mu)_{\dot{\alpha}}, \\ \delta F &= 0, \qquad \delta\bar{F} = -i\sqrt{2}D_\mu\bar{\psi}\bar{\sigma}^\mu\epsilon - 2i\bar{\phi}\epsilon\hat{\lambda} + 2C^{\mu\nu}D_\mu(\bar{\phi}\epsilon\sigma_\nu\bar{\hat{\lambda}}) \end{aligned}$$

More Anomalous Terms...

- More anomalous results - terms that violate gauge invariance.
- Similar solution to before?
- Make a redefinition of λ
- Problem: terms linear in F are still problematic.
- Solution: make a divergent redefinition of $\bar{F}$.

A Detour to $\mathcal{N} = 1$

- Renormalisation already studied in detail.
- Action in the uneliminated formalism:

$$
\begin{aligned}
S_{unel} \quad = \quad & \int d^4x \Big[-\tfrac{1}{4} F^{\mu\nu A} F^A_{\mu\nu} - i\bar{\lambda}^A \bar{\sigma}^\mu (D_\mu \lambda^A) + \tfrac{1}{2}(D^A)^2 \\
& F^i F_i - i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi - D^\mu \bar{\phi} D_\mu \phi + g\bar{\phi} R^A \phi D^A + i\sqrt{2} g(\bar{\phi}\lambda\psi - \bar{\psi}\bar{\lambda}\phi) \\
& F^i W_i + F_i W^i - \tfrac{1}{2} W_{ij} \psi^i \psi^j - \tfrac{1}{2} W^{ij} \psi_i \psi_j \Big]
\end{aligned}
$$

where $W(\phi) = \frac{1}{6} Y^{ijk} \phi_i \phi_j \phi_k$ is the superpotential.

- Diagrams present for which there are no counter terms to cancel the divergences.

- $F\phi^2$ and $D\bar{\phi}\phi$ terms also not finite after replacement of fields and couplings by bare fields and couplings.
- Divergences cancel if we use the equations of motion for D^A and F^i OR we can make a nonlinear renormalisation of the auxiliary fields:

$$
\begin{aligned}
(F_B)_i &= (Z_F^{\frac{1}{2}}F)_i + \tfrac{1}{2}(\alpha+3)g^2L[C(R)]_i^l Y_{ljk}\phi^j\phi^k \\
(D_B)^A &= (Z_D^{\frac{1}{2}}D)^A + (\alpha+2)C(G)g^3L\bar{\phi}R^A\phi
\end{aligned}
$$

- $\mathcal{N} = \frac{1}{2}$ results not so surprising?

Back to $\mathcal{N} = \frac{1}{2}$

- Redefinition of λ, redefinition of $\bar{F}$ and taking
$$Z_1 = 1 - 3g^2 NL, \qquad Z_2 = 1 - g^2 NL, \qquad Z_C = Z_{|C|^2} = 1,$$
gives a finite result.
- So $U(1) \times SU(N)$ theory is renormalisable and the $\mathcal{N} = \frac{1}{2}$ supersymmetry is preserved
- but only with the inclusion of γ_1 and γ_2 terms that do not appear in the original theory.
- No renormalisation of the NAC parameter and redefinitions required confirmed in superfield formalism
M.T. Grisaru, S. Penati and A. Romagnoni, JHEP 0602 (2006) 043, hep-th 0510175.

Massive $\mathcal{N} = \frac{1}{2}$ Gauge Theory

- Only superpotential terms possible:
  - cubic terms for chiral matter fields, but not antichiral fields,
  - mass terms for chiral and antichiral fields.
- We propose

$$S_{mass} = \int d^4x\, m[(\phi\tilde{F} + F\tilde{\phi} - \psi\tilde{\psi}) + h.c. + iC^{\mu\nu}\bar{\phi}\hat{F}_{\mu\nu}\bar{\tilde{\phi}}].$$

$$\delta\bar{F} = -i\sqrt{2}D_\mu\bar{\psi}\bar{\sigma}^\mu\epsilon - 2i\bar{\phi}\epsilon\hat{\lambda} + 2C^{\mu\nu}D_\mu(\bar{\phi}\epsilon\sigma_\nu\bar{\hat{\lambda}})$$

- $\{\phi, \psi, F\}$ transform according to fundamental representation, $\{\tilde{\phi}, \tilde{\psi}, \tilde{F}\}$ transform according to its conjugate, with $C^{\mu\nu} \to -C^{\mu\nu}$ for the conjugate representation.
- As before:
  - divergent redefinition of λ required,
  - modified redefinition of the auxiliary field.
- Theory is renormalisable at one-loop only if a separate coupling is introduced in order to absorb the remaining divergences - but this means the $\mathcal{N} = \frac{1}{2}$ supersymmetry is broken.

Adjoint representation

- Repeat $\mathcal{N} = \frac{1}{2}$ with chiral matter in the adjoint representation.
- Additional superpotential terms possible in adjoint representation:

$$
\begin{aligned}
S_{int} \;&=\; \int d^4x tr\Big\{\lambda[\phi^2 F - \frac{1}{2}\psi^2\phi + \bar{\phi}^2\bar{F} - \frac{1}{2}\bar{\psi}^2\bar{\phi} \\
&\quad + \frac{4}{3} igC^{\mu\nu}\bar{\phi}^3\hat{F}_{\mu\nu} + \frac{2}{3}C^{\mu\nu}D_\mu\bar{\phi}D_\nu\bar{\phi}\bar{\phi}] + m[\phi F - \frac{1}{2}\psi\psi \\
&\quad + \bar{\phi}\bar{F} - \frac{1}{2}\bar{\psi}\bar{\psi} + igC^{\mu\nu}\bar{\phi}\hat{F}_{\mu\nu}\bar{\phi} - \frac{1}{8}C^{\mu\nu}\bar{\phi}\bar{\lambda}^F\bar{\lambda}^F\bar{\phi}]\Big\}
\end{aligned}
$$

- Need usual λ redefinition.
- Need modified auxiliary field redefinition.

- Also: need to modify the bare action

$$
\begin{aligned}
S_B &= S_{0B} \\
&\quad -Z_2 C^{\mu\nu} g d^{abc}\left(\sqrt{2}D_\mu\bar{\phi}^b\bar{\lambda}^a\bar{\sigma}_\nu\psi^c + \sqrt{2}\bar{\phi}^b\bar{\lambda}^a\bar{\sigma}_\nu D_\mu\psi^c + i\bar{\phi}^b F^a_{\mu\nu}F^c\right) \\
&\quad -Z_3 C^{\mu\nu} g d^{ab0}\left(\sqrt{2}D_\mu\bar{\phi}^0\bar{\lambda}^a\bar{\sigma}_\nu\psi^b + \sqrt{2}\bar{\phi}^b\bar{\lambda}^a\bar{\sigma}_\nu D_\mu\psi^0\right)
\end{aligned}
$$

- Results are now finite BUT $\mathcal{N} = \frac{1}{2}$ supersymmetry is broken.

Summary

- $U(N)$ $\mathcal{N} = \frac{1}{2}$ SUSY can't be renormalised.
- $SU(N)$ $\mathcal{N} = \frac{1}{2}$ pure gauge theory can be renormalised only by implementing a divergent field redefinition.
- $U(1) \times SU(N)$ gauge theory with chiral matter can be renormalised with the divergent field redefinition and a redefinition of the auxiliary field.
- We found it was possible to construct an $\mathcal{N} = \frac{1}{2}$ theory with mass terms - but the $\mathcal{N} = \frac{1}{2}$ supersymmetry is broken under renormalisation.
- In the adjoint representation the theory is renormalisable at one loop, but the $\mathcal{N} = \frac{1}{2}$ supersymmetry is again lost.