Resonantly Enhanced CP Violation and Relaxation in EWB

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Outline

- Quantum Transport Equations from Closed Time Path QFT
- Resonant Enhancements in:
  - CP Violating Sources
  - Relaxation Rates
- Solution for Baryon Asymmetry
  - EDM constraints

Goal

- **Derive transport equations for particle densities:**
  - **Thermal expectation values of currents:**

$$\langle \partial_\mu j^\mu(x) \rangle \quad \text{where} \quad \langle \mathcal{O}(x) \rangle = \frac{1}{Z}\,\mathrm{Tr}\left[e^{-\beta\mathcal{F}}\mathcal{O}(x)\right]$$

Bosons $$j^\mu_\phi = i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)$$

Fermions $$j^\mu_\psi = i\bar{\psi}\gamma^\mu\psi$$

Densities in MSSM Baryogenesis

- Densities for supermultiplets:

$$Q = n_{t_L} + n_{\tilde{t}_L} + n_{b_L} + n_{\tilde{b}_L}$$
$$T = n_{t_R} + n_{\tilde{t}_R}$$
$$B = n_{b_R} + n_{\tilde{b}_R}$$
$$H = n_{H_u} + n_{\tilde{H}_u} - n_{H_d} - n_{\tilde{H}_d}$$
$$h = n_{H_u} + n_{\tilde{H}_u} + n_{H_d} + n_{\tilde{H}_d}$$

Assume supergauge interactions keep superpartners in chemical equilibrium

- Relate particle densities to chemical potentials:

$$n_i = g_i \int \frac{d^3k}{(2\pi)^3} [n_{B,F}(\omega_{\mathbf{k}}, \mu_i) - n_{B,F}(\omega_{\mathbf{k}}, -\mu_i)]$$

$$\Rightarrow n_i = \frac{k_i T^2}{6} \mu_i$$

Transport Equations

- Coupled transport equations for particle densities:

Huet, Nelson (1995):

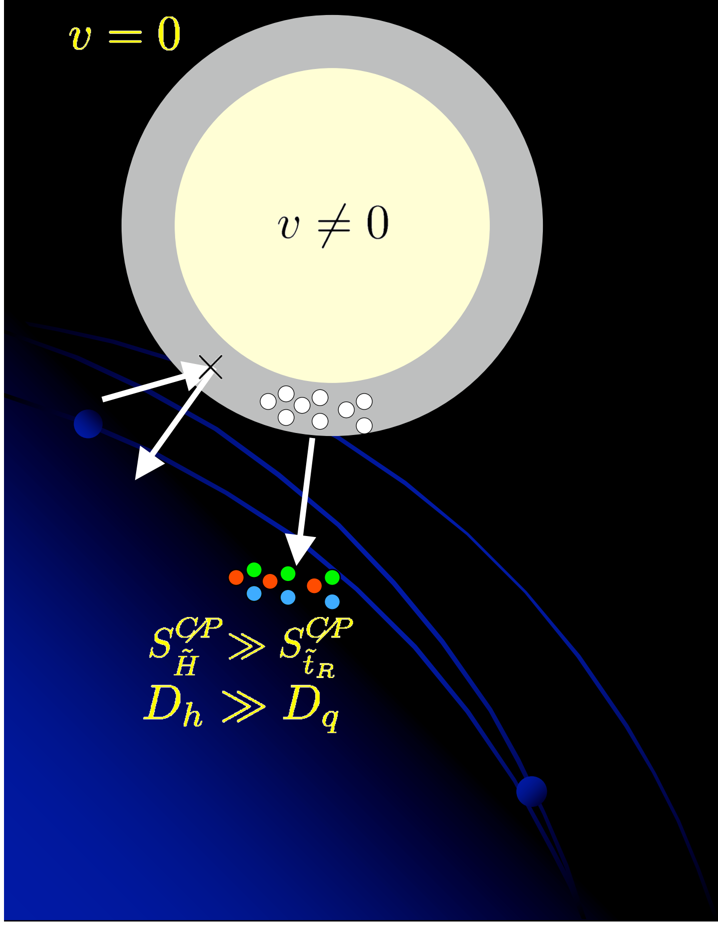
$$\partial_\mu J_i^\mu = S_i^{CP}(\mu_i) + S_i^{\not{C}\not{P}} + S_i^{\text{s.s.}}$$

$$\Downarrow$$

$$\partial_\mu T^\mu = S_{\tilde{t}_R}^{\not{C}\not{P}} - S^{\text{s.s.}} + \Gamma_M^-(\mu_Q - \mu_T) + \Gamma_Y(\mu_Q - \mu_T + \mu_H)$$

$$\partial_\mu Q^\mu = -S_{\tilde{t}_R}^{\not{C}\not{P}} + 2S^{\text{s.s.}} - \Gamma_M^-(\mu_Q - \mu_T) - \Gamma_Y(\mu_Q - \mu_T + \mu_H)$$

$$\partial_\mu H^\mu = S_{\tilde{H}}^{\not{C}\not{P}} - \Gamma_H \mu_H - \Gamma_Y(\mu_Q - \mu_T + \mu_H)$$



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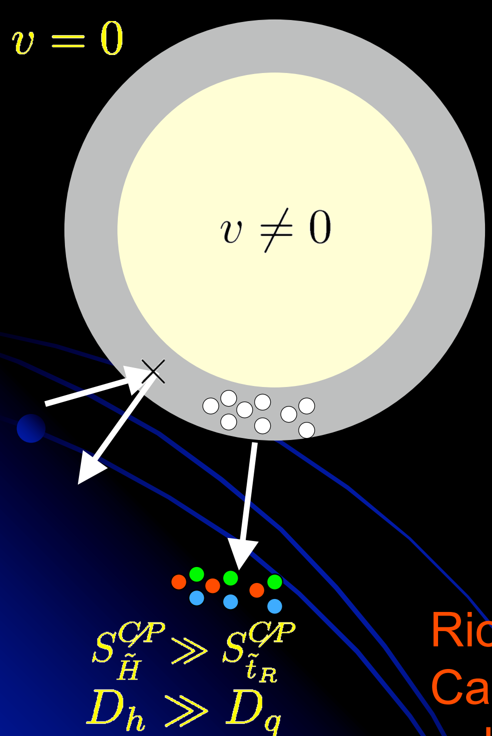
$$v_w T' - D_q T'' = S_{\tilde{t}_R}^{C\not{P}} - S^{\text{s.s.}} + \Gamma_M^-(\mu_Q - \mu_T) + \Gamma_Y(\mu_Q - \mu_T + \mu_H)$$

$$v_w Q' - D_q Q'' = -S_{\tilde{t}_R}^{C\not{P}} + 2S^{\text{s.s.}} - \Gamma_M^-(\mu_Q - \mu_T) - \Gamma_Y(\mu_Q - \mu_T + \mu_H)$$

$$v_w H' - D_h H'' = S_{\tilde{H}}^{C\not{P}} - \Gamma_H \mu_H - \Gamma_Y(\mu_Q - \mu_T + \mu_H)$$



Riotto; Carena et al.: resonant enhancements from CTP

This talk: Resonant relaxation from CTP

Next talk: Role of flavor transfer

(Higgsino source is dominant, but will illustrate calculation of squark source in this talk.)

Nonequilibrium QFT

- Expectation value in given "in" state:

$$\langle n | S_{\rm int}^\dagger T\{\mathcal{O}(x) S_{\rm int}\} | n \rangle$$

$$S_{\rm int} = T \exp\left(i \int d^4x \, \mathcal{L}_{\rm int} \right)$$

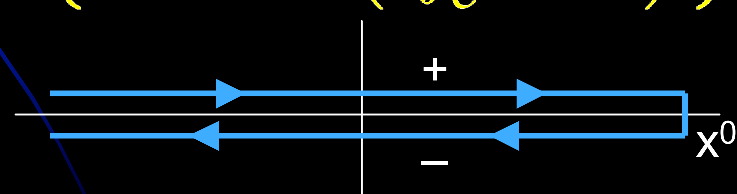
$$\Downarrow$$

$$\langle n | \mathcal{P} \left\{ \mathcal{O}_+(x) \exp\left(i \int d^4x \, \mathcal{L}_+ - i \int d^4x \, \mathcal{L}_- \right) \right\} | n \rangle$$

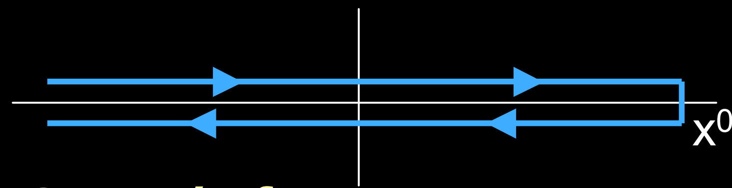
"path-ordering"

$$\Downarrow$$

$$\langle n | \mathcal{P} \left\{ \mathcal{O}(x) \exp\left(i \int_{\mathcal{C}} d^4x \, \mathcal{L} \right) \right\} | n \rangle$$



Green's Functions



- Four possible Green's functions on contour:

$$G^{>}(x,y) = \langle \phi_-(x)\phi_+^*(y)\rangle$$

$$G^{<}(x,y) = \langle \phi_+^*(y)\phi_-(x)\rangle$$

$$G^{t}(x,y) = \langle T\phi_+(x)\phi_+^*(y)\rangle$$

$$G^{\bar{t}}(x,y) = \langle \bar{T}\phi_-(x)\phi_-^*(y)\rangle$$

$$\widetilde{G}(x,y) = \begin{pmatrix} G^{t}(x,y) & -G^{<}(x,y) \\ G^{>}(x,y) & -G^{\bar{t}}(x,y) \end{pmatrix}$$

Schwinger-Dyson Equations

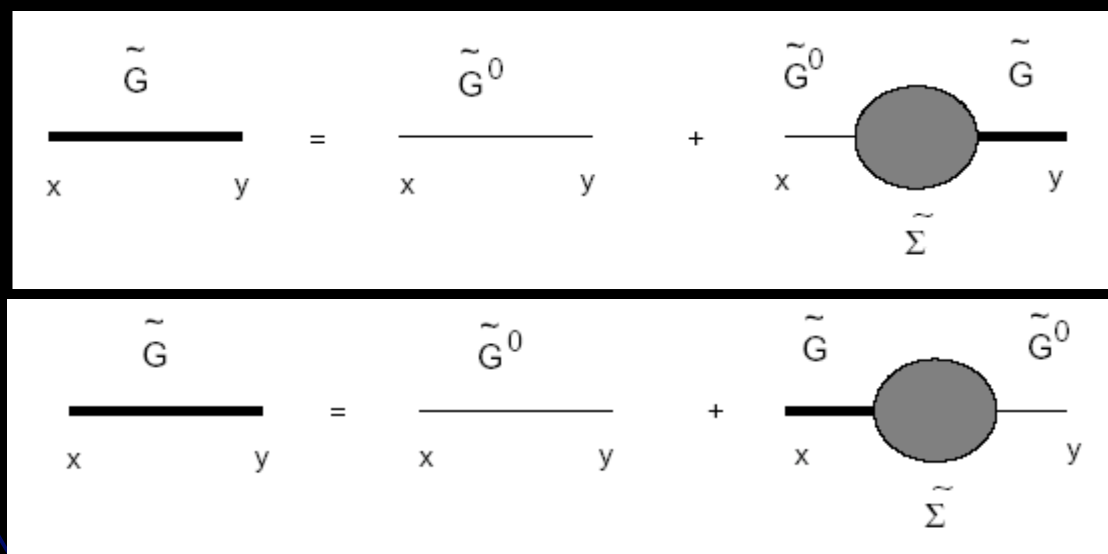
1. Apply:

$$(\Box_x + m^2): \quad \tilde{G}(x,y) = \tilde{G}^0(x,y) + \int d^4z \int d^4w \, \tilde{G}^0(x,z)\tilde{\Sigma}(z,w)\tilde{G}(w,y)$$

$$(\Box_y + m^2): \quad \tilde{G}(x,y) = \tilde{G}^0(x,y) + \int d^4z \int d^4w \, \tilde{G}(x,z)\tilde{\Sigma}(z,w)\tilde{G}^0(w,y)$$

2. Subtract two equations

3. Take x=y limit…



Quantum Transport Equations

- Obtain:

CP-violating and conserving interactions enter self-energies

$$\frac{\partial n}{\partial X^0} + \nabla \cdot \mathbf{j}(X)$$
$$= \int d^3z \int_{-\infty}^{X^0} dz^0 [\Sigma^>(X,z)G^<(z,X) - G^>(X,z)\Sigma^<(z,X)$$
$$-\Sigma^<(X,z)G^>(z,X) + G^<(X,z)\Sigma^>(z,X)]$$

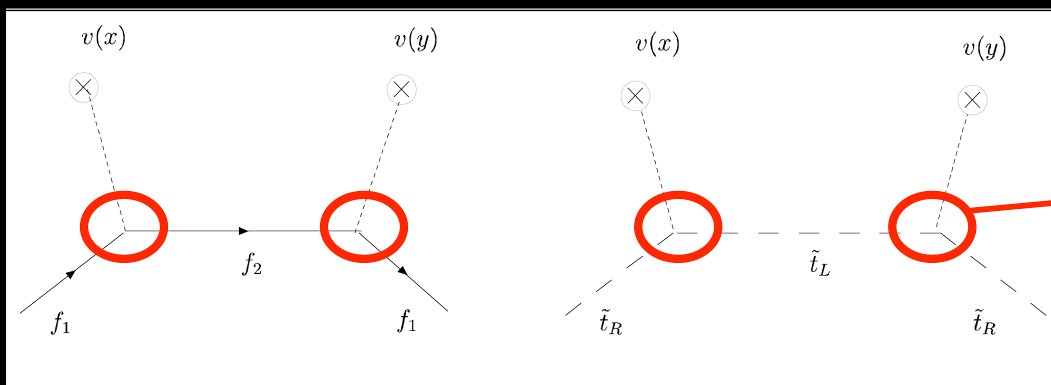
- Spectral representation of Green's functions:

$$G^>_{\tilde{t}_R}(x,y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)}[1 + n_B(k^0 - \mu_{\tilde{t}_R})]\rho_{\tilde{t}_R}(k)$$

- Expand in μ:
  - CP-violating source appears at zeroth order
  - Relaxation terms appear at linear order

Examples of Self-Energies

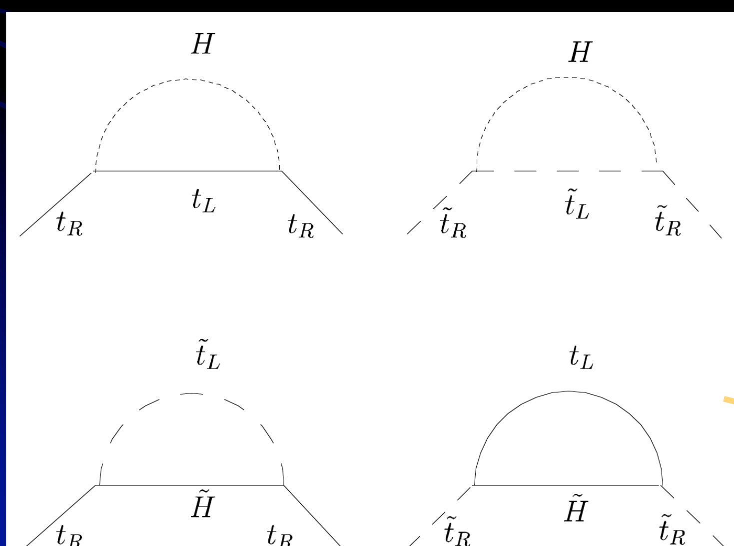
- Fermion and scalar interactions with Higgs vevs, Higgs particles, and Higgsinos



contribution to $S^{\not{C}\!P}_{\tilde{t}_R}$ $\Gamma_{M,H}$

$$\mathcal{L}_{\mathrm{MSSM}} \supset y_t \tilde{t}_L (A_t H_u^0 - \mu^* H_d^{0*}) \tilde{t}_R^*$$

$$A_t = |A_t| e^{i\phi_A} \qquad \mu = |\mu| e^{i\phi_\mu}$$



contribution to Γ_Y

CP-violating Source

Riotto

- CP-violating contribution to source:

$\propto \sin(\phi_\mu + \phi_A)$

$\propto \frac{v_w}{c}$ "Decoherence"

$$S^{\not{CP}}_{\tilde{t}_R}(x) = \frac{N_C y_t^2}{2\pi^2} \mathrm{Im}(\mu A_t) v(x)^2 \dot{\beta}(x)$$

$$\times \int_0^\infty \frac{k^2 dk}{\omega_{\tilde{t}_R}\omega_{\tilde{t}_L}} \mathrm{Im}\left\{ \frac{n_B(\mathcal{E}_R^*) - n_B(\mathcal{E}_L)}{(\mathcal{E}_L - \mathcal{E}_R^*)^2} + \frac{1 + n_B(\mathcal{E}_R) + n_B(\mathcal{E}_L)}{(\mathcal{E}_L + \mathcal{E}_R)^2} \right\}$$

$\propto \frac{\Gamma_{\tilde{t}}}{m_{\tilde{t}}}$

"Degeneracy"

Resonant enhancement when $m_{\tilde{t}_L} = m_{\tilde{t}_R}$

$$\mathcal{E} = \omega - i\Gamma$$

Relaxation Terms

Cirigliano, CL, Ramsey-Musolf

- CP-conserving contribution to source:

Keep chemical potential to first order in $\frac{\mu}{T}$

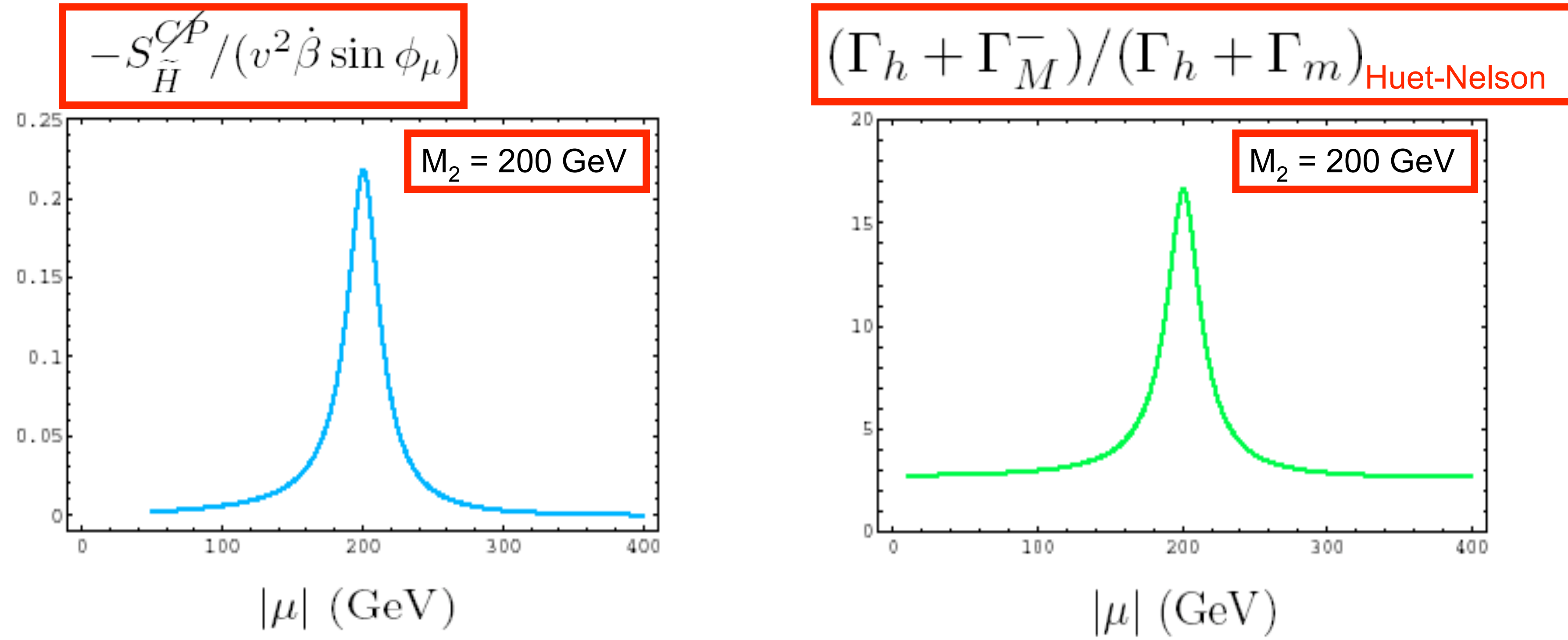
$$S_{\tilde{t}_R}^{CP} = \Gamma^{+}_{\tilde{t}_R}(\mu_{\tilde{t}_L} + \mu_{\tilde{t}_R}) + \Gamma^{-}_{\tilde{t}_R}(\mu_{\tilde{t}_L} - \mu_{\tilde{t}_R})$$

$$\Gamma^{\pm}_{\tilde{t}_R} = -\frac{1}{T}\frac{N_C y_t^2}{4\pi^2}|A_t v_u(x) - \mu^* v_d(x)|^2$$
$$\times \int_0^\infty \frac{k^2 dk}{\omega_{\tilde{t}_R}\omega_{\tilde{t}_L}} \mathrm{Im}\left\{\frac{h_B(\mathcal{E}_L) \mp h_B(\mathcal{E}_R^*)}{\mathcal{E}_L - \mathcal{E}_R^*} - \frac{h_B(\mathcal{E}_L) \mp h_B(\mathcal{E}_R)}{\mathcal{E}_L + \mathcal{E}_R}\right\}$$

Resonant enhancement when $m_{\tilde{t}_L} = m_{\tilde{t}_R}$

$$h_B(x) = T\frac{d}{dx}n_B(x)$$

Higgsino Source and Relaxation



Solution for Baryon Density

- Insert solution for $n_L(\bar{z})$ into equation for $\rho_B(\bar{z})$

$$D_q \rho_B''(\bar{z}) - v_w \rho_B'(\bar{z}) - \theta(-\bar{z})\mathcal{R}\rho_B(\bar{z}) = \theta(-\bar{z})\frac{n_F}{2}\Gamma_{\rm ws} n_L(\bar{z})$$

- Baryon density left over inside bubble of broken EW phase:

$$\rho_B(\bar{z} > 0) \propto \frac{S_{\widetilde{H}}^{C\!\!\!/P\!\!\!/} + S_{\tilde{t}_R}^{C\!\!\!/P\!\!\!/}}{\sqrt{\Gamma_h + \Gamma_M^-}}$$

(constant)

Resonant enhancements of relaxation terms mitigate but do not cancel out those of CP-violating sources

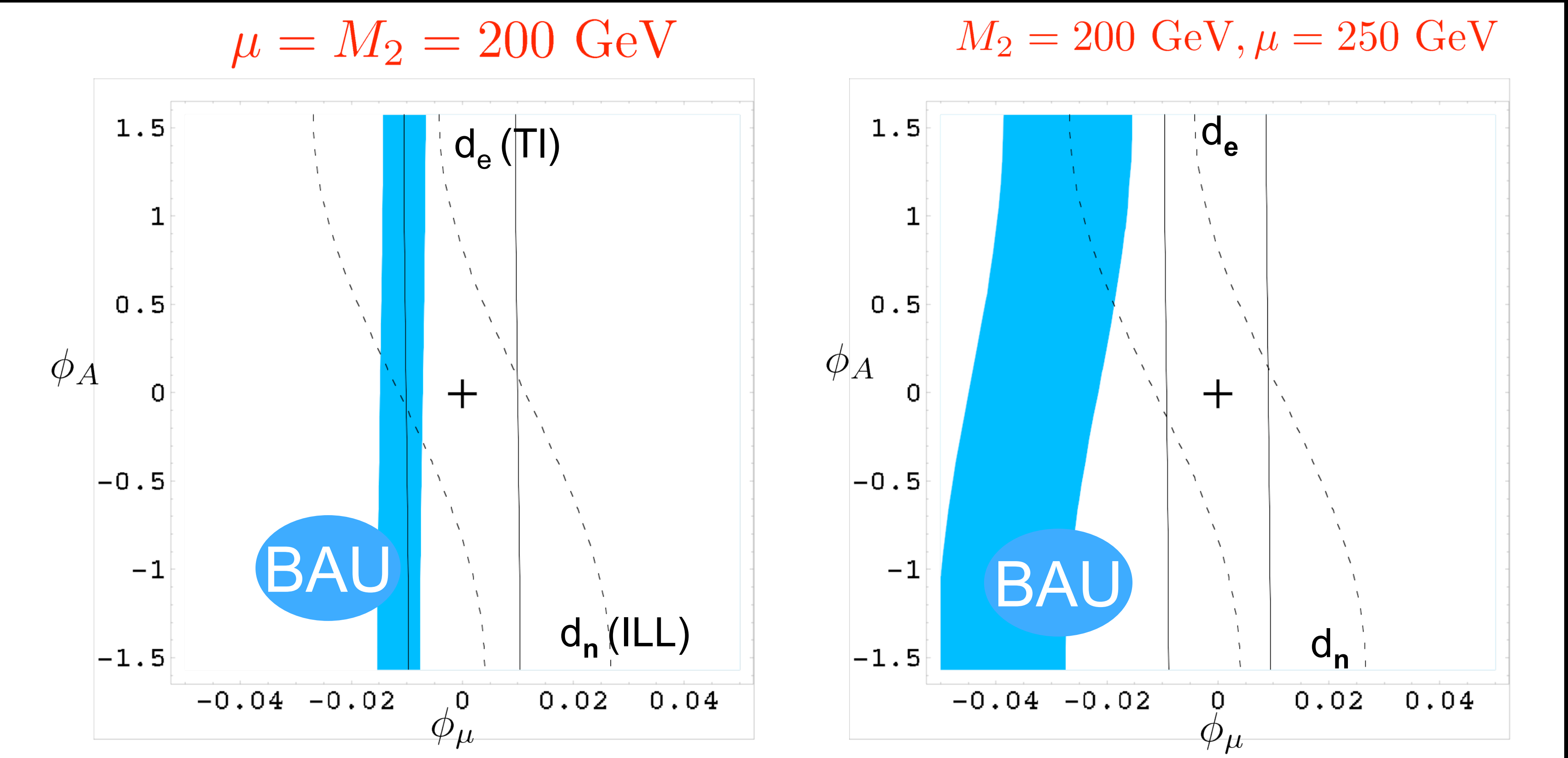
Combined BAU & EDM constraints

(1-loop; for 2-loop, see S. Profumo talk)

$$\mathcal{L}_{\rm SUSY} \supset y_t \tilde{t}_L (A_t H_u^0 - \mu^* H_d^{0*}) \tilde{t}_R^* + \mu(\widetilde{H}_d^0 \widetilde{H}_u^0 - \widetilde{H}_d^- \widetilde{H}_u^+)$$

$$A_t = |A_t| e^{i\phi_A}$$

$$\mu = |\mu| e^{i\phi_\mu}$$



The Future of Baryogenesis

- Progress towards more complete, consistent calculation of BAU with non-equilibrium QFT
  - To do:
    - Resummation of Higgs vev insertions, account for mixing of flavor eigenstates (cf. Konstandin et al., Carena et al.)
    - Relax approximation of gauge or superpartner equilibrium
- Next talk: Effect of Γ_Y terms in transport equations