QCD corrections to neutralino annihilation

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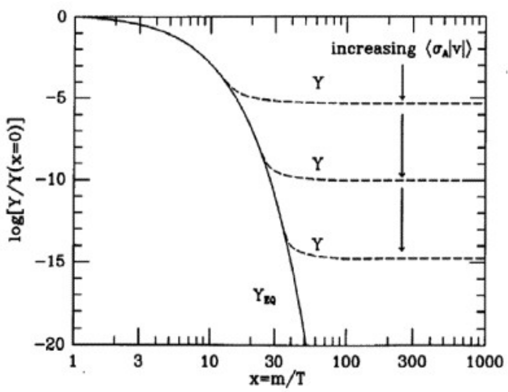
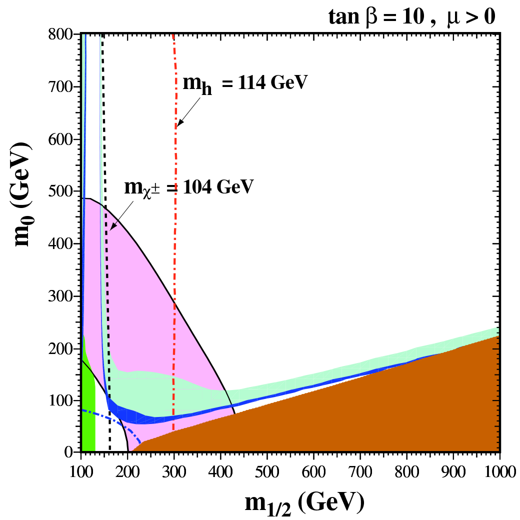
• V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, A. Tregre, Phys. Lett. B633 (2006) 98-105 [hep-ph/0510257]
• V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, *work in progress*

Why calculate neutralino annihilation?

- Cross section controls dark matter relic abundance



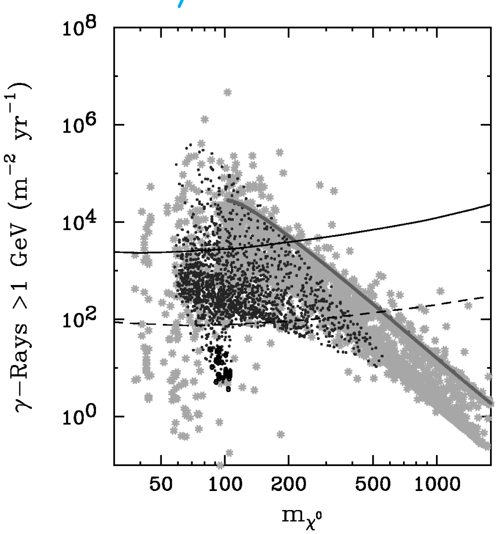
[Kolb & Turner]



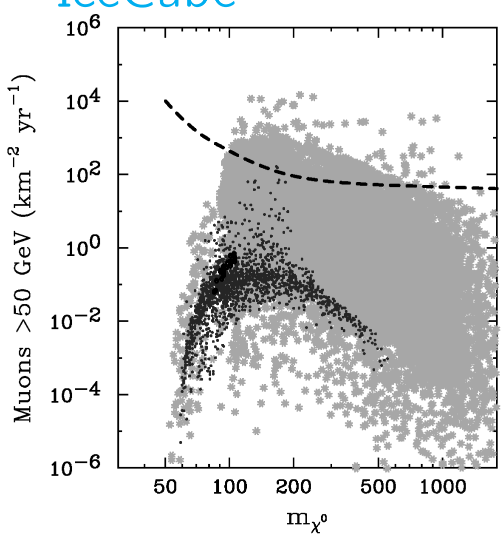
[Olive et al]

- Cross section controls indirect detection rates

EGRET/GLAST



IceCube



[Bertone, Hooper & Silk 2004]

Why calculate QCD corrections?

Because they are significant in some regions of parameter space.

Neutralino annihilation xsec: $\sigma v_{\rm rel} = a + b v_{\rm rel}^2$

- Early universe (dark matter freeze-out): $v_{\rm rel} \sim 1/3$
- Present day (halo): $v_{\rm rel} \sim 10^{-3}$

Where $\chi\chi \to$ light fermions dominates:

- s-wave cross section a is helicity-suppressed by m_f^2/m_χ^2
- Neutralinos are p-wave annihilators in the early universe

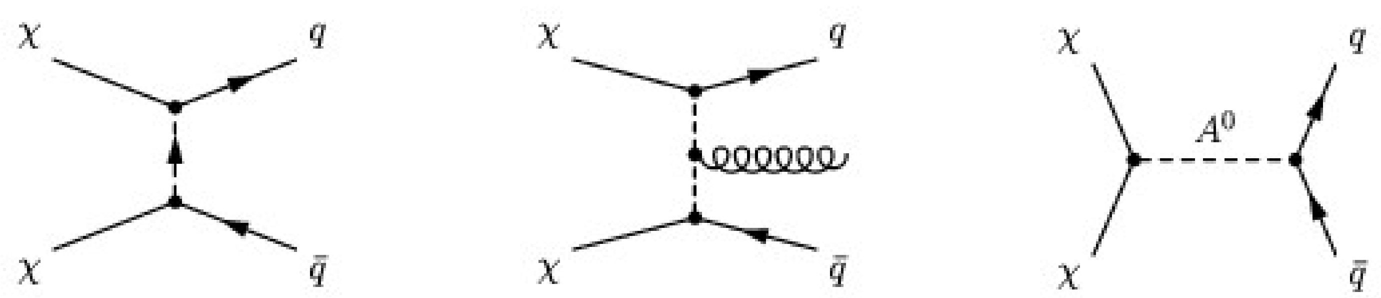
• Hard QCD radiation and $\chi\chi \to gg$ through a loop lift the m_f^2 suppression

Big effect on the s-wave cross section;
not so much on the p-wave cross section.

Corrections tend to be most relevant for indirect-detection rates:
s-wave dominates at present day ($v_{\rm rel} \sim 10^{-3}$).

Consider $\chi\chi$ annihilation through squark exchange

Some typical diagrams:



etc.

The first diagram above can be reduced to an effective vertex described by a dimension-six operator suppressed by the squark mass $M_{\tilde{q}}$:

$$\mathcal{L} = \frac{c}{M_{\tilde{q}}^2}\mathcal{O}_6, \qquad \mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)\,(\bar{q}\gamma^\mu\gamma_5 q)$$

This is valid in the limit $m_\chi \ll M_{\tilde{q}}$.

In the $v_{\text{rel}} \to 0$ limit the neutralinos behave like a pseudoscalar: $\mathcal{O}_6$ is related to the divergence of the axial vector current of the quarks:

$$\mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q) \to \left[\bar{\chi}\frac{i\gamma_5}{2m_\chi}\chi\right]\left[\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q)\right]$$

- If $m_q = 0$, the axial vector current is conserved at tree level:
  $\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 0$
  This is the m_f^2/m_χ^2 suppression showing up.

- There are two ways to lift this suppression:

(1) Go beyond leading order in α_s to include the anomalous triangle diagram.

(2) go to dimension-eight (or higher) by including hard gluon radiation.

Anomalous triangle diagram

The lifting of the m_f^2 suppression here is due to the well-known Partially Conserved Axial Current (PCAC):

$\partial_\mu (\bar{q}\gamma^\mu\gamma_5 q) \neq 0$ due to the anomaly, even when $m_q = 0$.

The anomaly condition reads: (including m_q and only QCD interactions)

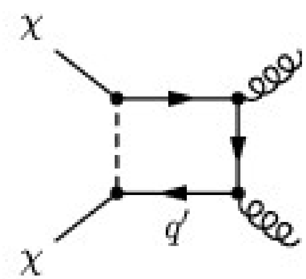
$$\partial_\mu (\bar{q}\gamma^\mu\gamma_5 q) = 2m_q\bar{q}i\gamma_5 q + \frac{\alpha_s}{4\pi}G^{(a)}_{\mu\nu}\widetilde{G}^{(a)\mu\nu}$$

Neglecting m_q, we can write the zero-velocity dimension-six $\chi\chi$ annihilation amplitude in the form

$$\mathcal{L}_{\rm eff} = \left(\frac{c/m_\chi}{2M^2_{\tilde{q}}}\right)(\bar{\chi}i\gamma_5\chi)\frac{\alpha_s}{4\pi}G^{(a)}_{\mu\nu}\widetilde{G}^{(a)\mu\nu}$$

This is $\chi\chi$ annihilation into gluons.

[Still working in $M^{-2}_{\tilde{q}}$ approximation for squark propagator.]



Expression describes one massless quark running around the loop.

Anomalous triangle diagram

- Calculation first done for $\chi\chi \to \gamma\gamma$ [Rudaz 1989; Bergstrom 1989]

- Easy to extend to $\chi\chi \to gg$ [Flores, Olive, Rudaz 1989]

$m_{q'} = 0$ result: (sum is over 5 light quarks; top decouples)

$$v_{\rm rel}\sigma(\chi\chi \to gg) = \frac{\alpha_s^2}{32\pi^3} m_\chi^2 \left[\sum_{q'} \frac{|g_\ell|^2}{M^2_{\tilde{q}'_L}} + \frac{|g_r|^2}{M^2_{\tilde{q}'_R}} \right]^2$$

where

$$g_\ell = -\sqrt{2} N_{11} g'(T_3 - Q) + \sqrt{2} N_{12} g T_3, \qquad g_r = -\sqrt{2} N_{11} g' Q.$$

We neglect left-right squark mixing ($m_{q'} = 0$ approximation)

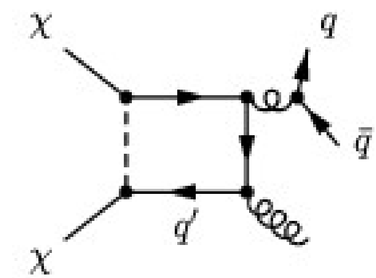
- Full $m_{q'}$, $M_{\tilde{q}}$ dependence is also known

  [Drees, Jungman, Kamionkowski, Nojiri 1993]

What about beyond leading order?

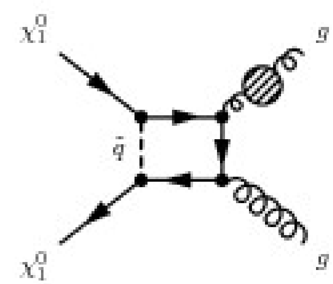
$\chi\chi \to gg$ is order α_s^2: large scale dependence at leading order.
Set scale $\mu_0 = 2m_\chi$, vary between $\mu_0/2 \ldots 2\mu_0$: $v\sigma$ varies by $\pm 16\%$.

At NLO, must include:



(1) gluon splitting into quark or gluon pairs

(2) radiation of a 3rd gluon off of the internal q' line



(3) virtual corrections: gluons crossing the box, gluons connecting the box to a gluon leg

(4) renormalization; e.g., gluon propagator bubbles containing quarks and gluons

The calculation is big and ugly.
Luckily we can use a trick to do it!

The trick:

Recall anomaly equation:

$$\partial_\mu(\bar{q}'\gamma^\mu\gamma_5 q') \;=\; 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

- $m_{q'} \to 0$ limit:

$$\partial_\mu(\bar{q}'\gamma^\mu\gamma_5 q') \;\simeq\; \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

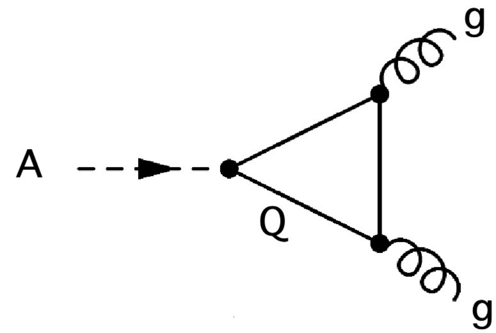
- If we take the opposite limit, $m_{q'} \gg m_\chi$, then the anomaly equation relates a pseudoscalar coupling to the same two-gluon operator:

$$0 \;\simeq\; 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}$$

(term on left-hand side becomes negligible in $m_{q'} \gg m_\chi$ limit)

$$0 \simeq 2m_{q'}\bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$

This describes pseudoscalar decay through a heavy quark triangle in the limit $m_Q \gg m_A$.



This helps us because of the Adler-Bardeen theorem, which tells us that the anomaly equation holds to all orders in α_s.

Should be able to relate $\chi\chi \to gg$ at NLO to $A \to gg$ at NLO.

$A \to gg$ at NLO calculated by
Spira, Djouadi, Graudenz, Zerwas (1995):

$$\Gamma_{\rm NLO}(A \to gg) = \Gamma_{\rm LO}(A \to gg) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_f + \frac{33 - 2N_f}{6}\log\frac{\mu^2}{4m_\chi^2}\right)\right]$$

Correction is multiplicative in the $m_Q \gg m_A$ approximation.

How can we use this?

Start with the bare Yukawa Lagrangian for interactions of A^0 with quarks: following Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)

$$\mathcal{L} = -\frac{A}{v}\left[\sum_{i=1}^{n_l} m^0_{q_i}\bar{q}^0_i i\gamma_5 q^0_i + m^0_t \bar{t}^0 i\gamma_5 t^0\right]$$

Taking the limit $m_t \to \infty$ and setting $m_{q_i} = 0$ for the light quarks, we can write this as a combination of pseudoscalar operators:

$$\mathcal{L} = -\frac{A}{v}\left[C^0_1 O^0_1 + C^0_2 O^0_2 + \cdots\right]$$

where $O^0_1 = G^{0,a}_{\mu\nu}\widetilde{G}^{0,a\mu\nu}$,

$O^0_2 = \partial_\mu J^{0,\mu}_5$ with $J^{0,\mu}_5 = \sum_{i=1}^{n_l} \bar{q}^0_i \gamma^\mu \gamma_5 q^0_i$

Renormalize the bare Lagrangian:

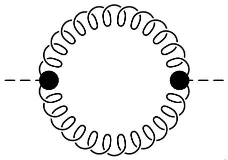
$$\mathcal{L} = -\frac{A}{v}\left[C_1 O_1 + C_2 O_2 + \cdots\right],$$

$$O_1 = Z_{11} O^0_1 + Z_{12} O^0_2, \qquad O_2 = Z_{22} O^0_2.$$

$A \to gg$ is the imaginary part of the $A \to A$ amplitude, described by correlators $\langle O_i O_j \rangle$:

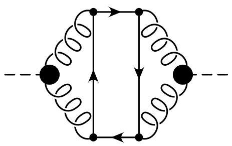
$$\Gamma(A \to gg) = \frac{\sqrt{2} G_F}{M_A} \left[C_1^2 \mathrm{Im}\langle O_1 O_1 \rangle + 2 C_1 C_2 \mathrm{Im}\langle O_1 O_2 \rangle + C_2^2 \mathrm{Im}\langle O_2 O_2 \rangle \right]$$

- $\langle O_1 O_1 \rangle \sim \alpha_s^0 + \cdots$ Diagram $\longrightarrow$



- $\langle O_1 O_2 \rangle \sim \alpha_s^1 + \cdots$ [Need to radiate a gluon from $q\bar{q}$ in O_2 and split a gluon into quarks in O_1.]
- $\langle O_2 O_2 \rangle \sim \alpha_s^2 + \cdots$ [Kinematics kills $\langle O_2 O_2 \rangle$ at leading order for $m_q = 0$. Need to make two boxes and connect the gluons.]

- $C_1 \sim \alpha_s^1$, with no higher order corrections: Adler-Bardeen theorem! [$A G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$ is generated by the top loop.]
- $C_2 \sim \alpha_s^2 + \cdots$ [$A \partial_\mu J_5^\mu$ is generated at two loops by attaching a quark line to the gluons that were generated by the top loop.] Diagram $\longrightarrow$



$C_1^2\text{Im}\langle O_1 O_1\rangle \sim \alpha_s^2 + \cdots$

$C_1 C_2\text{Im}\langle O_1 O_2\rangle \sim \alpha_s^4 + \cdots$

$C_2^2\text{Im}\langle O_2 O_2\rangle \sim \alpha_s^6 + \cdots$

Non-renormalization of C_1 means we can take the universal QCD corrections to $\text{Im}\langle \mathcal{O}_1 \mathcal{O}_1\rangle$ from $A \to gg$ over to $\chi\chi \to gg$.

The $A \to gg$ calculation transfers directly over to $\chi\chi \to gg$ at NLO only:

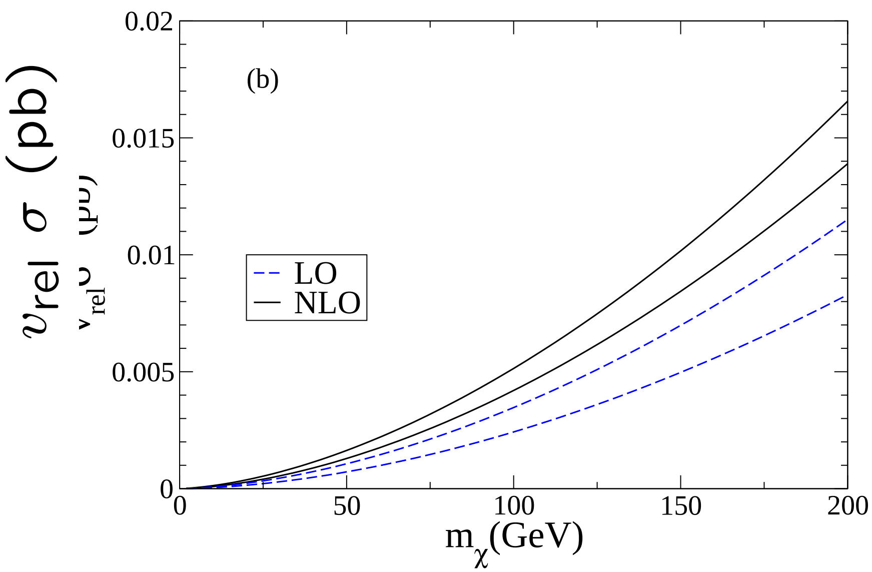
- LO: want $C_1^2\text{Im}\langle O_1 O_1\rangle$ at leading α_s^2 order.
  - This is just LO $A \to gg$.
- NLO: want $C_1^2\text{Im}\langle O_1 O_1\rangle$ at NLO, α_s^3.
  - This is just NLO $A \to gg$.
- NNLO: want $C_1^2\text{Im}\langle O_1 O_1\rangle$ at NNLO, α_s^4.
  - Cannot get this simply from $A \to gg$, since $C_1 C_2\text{Im}\langle O_1 O_2\rangle$ also contributes at this order.

We get $\chi\chi \to gg$ at NLO "for free":

$$v_{\rm rel}\sigma_{\rm NLO}(\chi\chi \to gg) = v_{\rm rel}\sigma_{\rm LO}(\chi\chi \to gg) \times \left[1 + \frac{\alpha_s}{\pi}\left(\frac{97}{4} - \frac{7}{6}N_f + \frac{33-2N_f}{6}\log\frac{\mu^2}{4m_\chi^2}\right)\right]$$

$$= v_{\rm rel}\sigma_{\rm LO}(\chi\chi \to gg)\,[1+0.62]$$

where the last line is for $\mu = 2m_\chi = 2 \times (100\ \text{GeV})$ and $N_f = 5$.



← NLO: scale uncertainty ±9%

← LO: scale uncertainty ±16%

Bino; $M_{\tilde q} = 200$ GeV

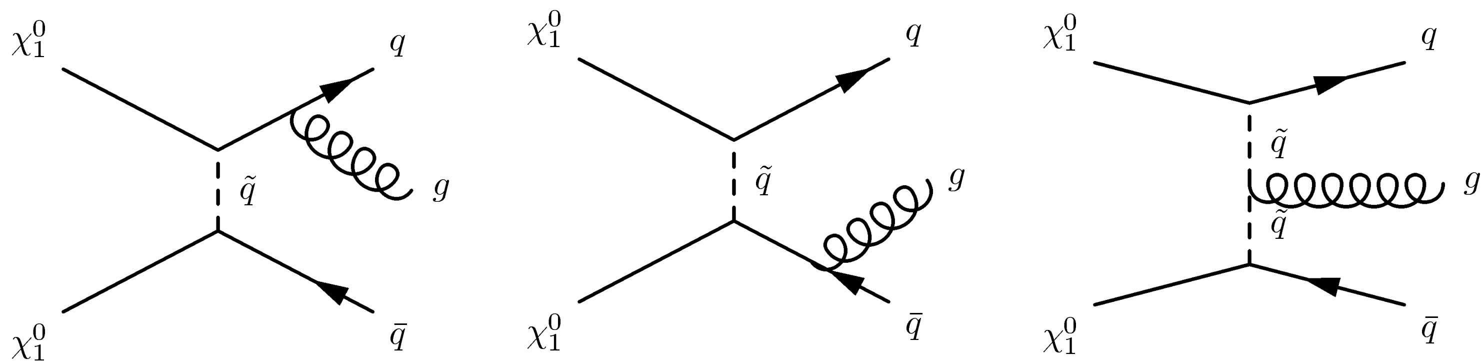
[Barger, Keung, HEL, Shaughnessy, Tregre 2005]

$\chi\chi \to gg$ cross section is increased by $\sim 60\%$ at NLO.

Dimension-eight amplitude

Remember there were two ways to lift the m_f^2/m_χ^2 suppression:
(1) using the anomaly
(2) going to dimension-eight.

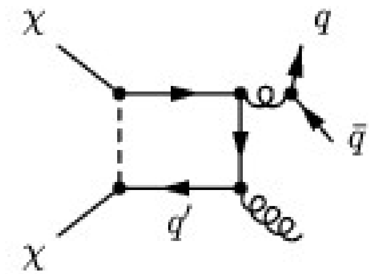
The dimension-eight amplitude was calculated for $\chi\chi \to f\bar{f}\gamma$ in [Flores, Olive, Rudaz 1989]



The full calculation was done in [Drees, Jungman, Kamionkowski, Nojiri 1993].

For $m_q \simeq 0$, the leading $1/M_{\tilde{q}}^8$ part is

$$v_{\rm rel}\sigma(\chi\chi \to q\bar{q}g) = \frac{4\alpha_s}{15}\frac{m_\chi^6}{16\pi^2}\left[\frac{|g_\ell|^4}{M_{\tilde{q}_L}^8} + \frac{|g_r|^4}{M_{\tilde{q}_R}^8}\right].$$



Interference term between
(1) dimension-eight $\chi\chi \to q\bar{q}g$, and
(2) dimension-six $\chi\chi \to q\bar{q}g$ through the box with gluon splitting to $q\bar{q}$

- Interference term is order α_s^2 – same order as LO $\chi\chi \to gg$
- Interference term is order $1/M_{\tilde{q}}^6$ – more suppressed than $\chi\chi \to gg$ but less suppressed than pure dimension-eight cross section.

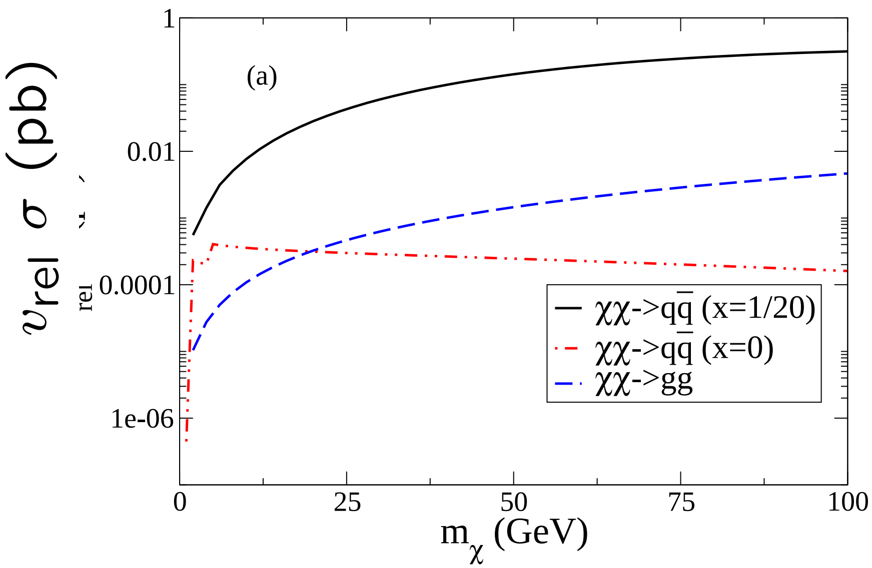
$$
\begin{aligned}
v_{\rm rel}\,\sigma \;=\;& \frac{\alpha_s}{\pi}\frac{m_\chi^6}{M_{\tilde{q}}^8}\frac{N_f(|g_\ell|^4+|g_r|^4)}{60\pi} && (\chi\chi \to q\bar{q}g \text{ tree level})\\
&+\left(\frac{\alpha_s}{\pi}\right)^2 \frac{m_\chi^2}{M_{\tilde{q}}^4}\frac{N_f^2(|g_\ell|^2+|g_r|^2)^2}{32\pi}\Bigg[1 && (\chi\chi \to gg \text{ LO})\\
&\sim 0.6 \to \quad +\frac{\alpha_s}{\pi}\frac{221}{12} && (\chi\chi \to gg \text{ NLO})\\
&\text{preliminary} \to \quad -\frac{m_\chi^2}{M_{\tilde{q}}^2}\frac{2}{3} && (\text{interference term})\Bigg].
\end{aligned}
$$

Degenerate squarks, $\mu = 2m_\chi$, $N_f = 5$; $v_{\rm rel} \to 0$ limit

Where is this useful?

• Early universe: $\chi\chi \to gg$ typically only a small contribution to the total annihilation cross section. Not particularly important.

• Present day: $\chi\chi \to gg$ can be the dominant annihilation mode. Corrections are important for total annihilation cross section and branching fractions $\to$ can affect indirect DM detection rates.



$\leftarrow$ $\chi\chi \to q\bar{q}$ in early universe

$\leftarrow$ $\chi\chi \to gg$ (includes NLO)

$\leftarrow$ $\chi\chi \to q\bar{q}$ today

Bino; $M_{\tilde{q}} = 200$ GeV

Interference term not included – still prelim.

[Barger, Keung, HEL, Shaughnessy, Tregre 2005]

Conclusions

- Precision cosmology motivates calculation at higher orders.

- Majorana neutralinos $\rightarrow$ s-wave annihilation helicity suppressed. Processes that lift the suppression can have a big impact on present-day annihilation rates.

- We calculated NLO QCD corrections to $\chi\chi \rightarrow gg$ by using the Adler-Bardeen theorem and known NLO QCD corrections to $A \rightarrow gg$: about a +60% effect.

- Interference term between $\chi\chi \rightarrow g^* g \rightarrow q\bar{q}g$ and tree-level $\chi\chi \rightarrow q\bar{q}g$ (in preparation):
  - Same α_s order as LO $\chi\chi \rightarrow gg$
  - Relative $m_\chi^2 / M_{\tilde{q}}^2$ suppression
  - Destructive interference

- Implications for indirect detection still need to be worked out.