QCD corrections to neutralino annihilation

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- V. Barger, W.-Y. Keung, HEL, G. Shaughnessy, *work in progress*
Why calculate neutralino annihilation?

- Cross section controls dark matter relic abundance
  
  [Kolb & Turner]

- Cross section controls indirect detection rates
  
  EGRET/GLAST
  
  IceCube

[Olive et al]

[Bertone, Hooper & Silk 2004]
Why calculate QCD corrections?

Because they are significant in some regions of parameter space.

Neutralino annihilation xsec: \( \sigma v_{\text{rel}} = a + b v_{\text{rel}}^2 \)
- Early universe (dark matter freeze-out): \( v_{\text{rel}} \sim 1/3 \)
- Present day (halo): \( v_{\text{rel}} \sim 10^{-3} \)

Where \( \chi\chi \rightarrow \) light fermions dominates:
- s-wave cross section \( a \) is helicity-suppressed by \( m_f^2/m_\chi^2 \)
- Neutralinos are p-wave annihilators in the early universe

- Hard QCD radiation and \( \chi\chi \rightarrow gg \) through a loop lift the \( m_f^2 \) suppression
  Big effect on the s-wave cross section; not so much on the p-wave cross section.

Corrections tend to be most relevant for indirect-detection rates:
s-wave dominates at present day \( (v_{\text{rel}} \sim 10^{-3}) \).
Consider $\chi\chi$ annihilation through squark exchange

Some typical diagrams:

The first diagram above can be reduced to an effective vertex described by a dimension-six operator suppressed by the squark mass $M_{\tilde{q}}$:

$$\mathcal{L} = \frac{c}{M_{\tilde{q}}^2} \mathcal{O}_6,$$

$$\mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)$$

This is valid in the limit $m_\chi \ll M_{\tilde{q}}$. 
In the $v_{\text{rel}} \to 0$ limit the neutralinos behave like a pseudoscalar: $\mathcal{O}_6$ is related to the divergence of the axial vector current of the quarks:

$$\mathcal{O}_6 = (\bar{\chi} \gamma_\mu \gamma_5 \chi)(\bar{q} \gamma^\mu \gamma_5 q) \to \left[ \bar{\chi} \frac{i \gamma_5}{2m_\chi} \chi \right] \left[ \partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) \right]$$

- If $m_q = 0$, the axial vector current is conserved at tree level:
  $$\partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) = 0$$
  This is the $m_f^2/m_\chi^2$ suppression showing up.

- There are two ways to lift this suppression:
  1. Go beyond leading order in $\alpha_s$ to include the anomalous triangle diagram.
  2. Go to dimension-eight (or higher) by including hard gluon radiation.
Anomalous triangle diagram

The lifting of the $m_f^2$ suppression here is due to the well-known Partially Conserved Axial Current (PCAC):

$$\partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) \neq 0 \text{ due to the anomaly, even when } m_q = 0.$$

The anomaly condition reads:

$$(\bar{q} \gamma^\mu \gamma_5 q) = 2m_q \bar{q} i \gamma_5 q + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$

Neglecting $m_q$, we can write the zero-velocity dimension-six $\chi\chi$ annihilation amplitude in the form

$$\mathcal{L}_{\text{eff}} = \left( \frac{c/m_\chi}{2M_q^2} \right) (\bar{\chi} i \gamma_5 \chi) \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$

This is $\chi\chi$ annihilation into gluons.

[Still working in $M_q^{-2}$ approximation for squark propagator.]

Expression describes one massless quark running around the loop.
Anomalous triangle diagram

- Calculation first done for $\chi\chi \rightarrow \gamma\gamma$ \cite{Rudaz 1989; Bergstrom 1989}

- Easy to extend to $\chi\chi \rightarrow gg$ \cite{Flores, Olive, Rudaz 1989}

$m_{q'} = 0$ result: (sum is over 5 light quarks; top decouples)

$$v_{rel} \sigma(\chi\chi \rightarrow gg) = \frac{\alpha_s^2}{32\pi^3 m_\chi^2} \left[ \sum_{q'} \frac{|g_\ell|^2}{M_{q'}^2} + \frac{|g_r|^2}{M_{q'}^2} \right]^2$$

where

$$g_\ell = -\sqrt{2}N_{11}g' (T_3 - Q) + \sqrt{2}N_{12}gT_3, \quad g_r = -\sqrt{2}N_{11}g' Q.$$ 

We neglect left-right squark mixing ($m_{q'} = 0$ approximation)

- Full $m_{q'}$, $M_{q'}$ dependence is also known \cite{Drees, Jungman, Kamionkowski, Nojiri 1993}
What about beyond leading order?

$\chi\chi \rightarrow gg$ is order $\alpha_s^2$: large scale dependence at leading order.

Set scale $\mu_0 = 2m_\chi$, vary between $\mu_0/2...2\mu_0$: $v\sigma$ varies by $\pm 16\%$.

At NLO, must include:

1. gluon splitting into quark or gluon pairs

2. radiation of a 3rd gluon off of the internal $q'$ line

3. virtual corrections: gluons crossing the box, gluons connecting the box to a gluon leg

4. renormalization; e.g., gluon propagator bubbles containing quarks and gluons

The calculation is big and ugly.
Luckily we can use a trick to do it!
The trick:

Recall anomaly equation:

$$\partial_\mu (\bar{q}' \gamma^\mu \gamma_5 q') = 2m_q' \bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$

- $m_q' \to 0$ limit:

$$\partial_\mu (\bar{q}' \gamma^\mu \gamma_5 q') \simeq \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$

- If we take the opposite limit, $m_q' \gg m_\chi$, then the anomaly equation relates a pseudoscalar coupling to the same two-gluon operator:

$$0 \simeq 2m_q' \bar{q}' i\gamma_5 q' + \frac{\alpha_s}{4\pi} G^{(a)}_{\mu\nu} \tilde{G}^{(a)\mu\nu}$$

(term on left-hand side becomes negligible in $m_q' \gg m_\chi$ limit)
\[ 0 \simeq 2m_{q'} \bar{q}' i \gamma_5 q' + \frac{\alpha_s}{4\pi} G_{\mu\nu}^{(a)} \tilde{G}^{(a)\mu\nu} \]

This describes pseudoscalar decay through a heavy quark triangle in the limit \( m_Q \gg m_A \).

This helps us because of the Adler-Bardeen theorem, which tells us that the anomaly equation holds to all orders in \( \alpha_s \).

Should be able to relate \( \chi\chi \rightarrow gg \) at NLO to \( A \rightarrow gg \) at NLO.

\( A \rightarrow gg \) at NLO calculated by Spira, Djouadi, Graudenz, Zerwas (1995):

\[
\Gamma_{\text{NLO}}(A \rightarrow gg) = \Gamma_{\text{LO}}(A \rightarrow gg) \times \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{97}{4} - \frac{7}{6} N_f + \frac{33 - 2N_f}{6} \log \frac{\mu^2}{4m^2_\chi} \right) \right]
\]

Correction is multiplicative in the \( m_Q \gg m_A \) approximation.
How can we use this?

Start with the bare Yukawa Lagrangian for interactions of $A^0$ with quarks: following Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)

\[
\mathcal{L} = -\frac{A}{v} \left[ \sum_{i=1}^{n_l} m_{q_i}^0 \bar{q}_i^0 i\gamma_5 q_i^0 + m_t^0 \bar{t}^0 i\gamma_5 t^0 \right]
\]

Taking the limit $m_t \rightarrow \infty$ and setting $m_{q_i} = 0$ for the light quarks, we can write this as a combination of pseudoscalar operators:

\[
\mathcal{L} = -\frac{A}{v} \left[ C_1^0 O_1^0 + C_2^0 O_2^0 + \cdots \right]
\]

where

\[
O_1^0 = G_{\mu\nu}^0, a \tilde{G}_{\mu\nu}^0, a, O_2^0 = \partial_\mu J_5^{0,\mu} \quad \text{with} \quad J_5^{0,\mu} = \sum_{i=1}^{n_l} \bar{q}_i^0 \gamma^\mu \gamma_5 q_i^0
\]

Renormalize the bare Lagrangian:

\[
\mathcal{L} = -\frac{A}{v} \left[ C_1 O_1 + C_2 O_2 + \cdots \right],
\]

\[
O_1 = Z_{11} O_1^0 + Z_{12} O_2^0, \quad O_2 = Z_{22} O_2^0.
\]
$A \rightarrow gg$ is the imaginary part of the $A \rightarrow A$ amplitude, described by correlators $\langle O_i O_j \rangle$:

$$\Gamma(A \rightarrow gg) = \frac{\sqrt{2}G_F}{M_A} \left[ C_1^2 \text{Im}\langle O_1 O_1 \rangle + 2C_1C_2 \text{Im}\langle O_1 O_2 \rangle + C_2^2 \text{Im}\langle O_2 O_2 \rangle \right]$$

- $\langle O_1 O_1 \rangle \sim \alpha_s^0 + \cdots$  Diagram $\longrightarrow$
- $\langle O_1 O_2 \rangle \sim \alpha_s^1 + \cdots$ [Need to radiate a gluon from $q\bar{q}$ in $O_2$ and split a gluon into quarks in $O_1$.]
- $\langle O_2 O_2 \rangle \sim \alpha_s^2 + \cdots$ [Kinematics kills $\langle O_2 O_2 \rangle$ at leading order for $m_q = 0$. Need to make two boxes and connect the gluons.]

- $C_1 \sim \alpha_s^1$, with no higher order corrections: Adler-Bardeen theorem! $[A G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$ is generated by the top loop.]
- $C_2 \sim \alpha_s^2 + \cdots$ $[A \partial_\mu J_{5\mu}^a$ is generated at two loops by attaching a quark line to the gluons that were generated by the top loop.]

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\[ C_1^2 \text{Im}\langle O_1 O_1 \rangle \sim \alpha_s^2 + \cdots \]
\[ C_1 C_2 \text{Im}\langle O_1 O_2 \rangle \sim \alpha_s^4 + \cdots \]
\[ C_2^2 \text{Im}\langle O_2 O_2 \rangle \sim \alpha_s^6 + \cdots \]

Non-renormalization of \( C_1 \) means we can take the universal QCD corrections to \( \text{Im}\langle O_1 O_1 \rangle \) from \( A \rightarrow gg \) over to \( \chi\chi \rightarrow gg \).

The \( A \rightarrow gg \) calculation transfers directly over to \( \chi\chi \rightarrow gg \) at NLO only:

- **LO**: want \( C_1^2 \text{Im}\langle O_1 O_1 \rangle \) at leading \( \alpha_s^2 \) order.
  - This is just LO \( A \rightarrow gg \).

- **NLO**: want \( C_1^2 \text{Im}\langle O_1 O_1 \rangle \) at NLO, \( \alpha_s^3 \).
  - This is just NLO \( A \rightarrow gg \).

- **NNLO**: want \( C_1^2 \text{Im}\langle O_1 O_1 \rangle \) at NNLO, \( \alpha_s^4 \).
  - Cannot get this simply from \( A \rightarrow gg \), since \( C_1 C_2 \text{Im}\langle O_1 O_2 \rangle \) also contributes at this order.
We get $\chi\chi \rightarrow gg$ at NLO “for free”:

$$v_{\text{rel}}\sigma_{\text{NLO}}(\chi\chi \rightarrow gg) = v_{\text{rel}}\sigma_{\text{LO}}(\chi\chi \rightarrow gg) \times \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{97}{4} - \frac{7}{6}N_f + \frac{33 - 2N_f}{6} \log \frac{\mu^2}{4m_{\chi}^2} \right) \right] = v_{\text{rel}}\sigma_{\text{LO}}(\chi\chi \rightarrow gg) [1 + 0.62]$$

where the last line is for $\mu = 2m_{\chi} = 2 \times (100 \text{ GeV})$ and $N_f = 5$. 

\[\begin{array}{c}
\text{v}_{\text{rel}} \sigma (\text{pb}) \\
\end{array}\]

\[\begin{array}{c}
\text{v}_{\text{rel}} \sigma_{\text{LO}} (\text{pb}) \\
\text{v}_{\text{rel}} \sigma_{\text{NLO}} (\text{pb}) \\
\end{array}\]

\[\begin{array}{c}
\text{m}_{\chi} \text{(GeV)} \\
\end{array}\]

$\rightarrow$ NLO: scale uncertainty $\pm 9$

$\rightarrow$ LO: scale uncertainty $\pm 16$

Bino; $M_{\tilde{q}} = 200$ GeV

[Barger, Keung, HEL, Shaughnessy, Tregre 2005]

$\chi\chi \rightarrow gg$ cross section is increased by $\sim 60\%$ at NLO.
Dimension-eight amplitude

Remember there were two ways to lift the $m_f^2/m_X^2$ suppression:
(1) using the anomaly
(2) going to dimension-eight.

The dimension-eight amplitude was calculated for $\chi\chi \to f\bar{f}\gamma$ in
[Flores, Olive, Rudaz 1989]

The full calculation was done in [Drees, Jungman, Kamionkowski, Nojiri 1993].

For $m_q \approx 0$, the leading $1/M_\tilde{q}^8$ part is

$$v_{\text{rel}}\sigma(\chi\chi \to q\bar{q}g) = \frac{4\alpha_s}{15} \frac{m_X^6}{16\pi^2} \left[ \frac{|g_\ell|^4}{M_\tilde{q}^8 q_L} + \frac{|g_r|^4}{M_\tilde{q}^8 q_R} \right].$$
Interference term between
(1) dimension-eight $\chi\chi \rightarrow q\bar{q}g$, and
(2) dimension-six $\chi\chi \rightarrow q\bar{q}g$ through the box
with gluon splitting to $q\bar{q}$

- Interference term is order $\alpha_s^2$ – same order as LO $\chi\chi \rightarrow gg$
- Interference term is order $1/M_\tilde{q}^6$ – more suppressed than $\chi\chi \rightarrow gg$ but less suppressed than pure dimension-eight cross section.

\[
v_{\text{rel}} \sigma = \frac{\alpha_s m_\chi^6 N_f}{\pi M_\tilde{q}^8} \frac{|g_\ell|^4 + |g_r|^4}{60\pi} \quad (\chi\chi \rightarrow q\bar{q}g \text{ tree level})
\]

\[
+ \left( \frac{\alpha_s}{\pi} \right)^2 \frac{m_\chi^2 N_f^2}{M_\tilde{q}^4} \frac{|g_\ell|^2 + |g_r|^2}{32\pi} \left[ 1 \quad (\chi\chi \rightarrow gg \text{ LO})
\right.
\]

\[
\sim 0.6 \rightarrow + \frac{\alpha_s 221}{\pi 12} \quad (\chi\chi \rightarrow gg \text{ NLO})
\]

preliminary $\rightarrow - \frac{m_\chi^2 2}{M_\tilde{q}^2 3} \quad ($interference term$)$

Degenerate squarks, $\mu = 2m_\chi$, $N_f = 5$; $v_{\text{rel}} \rightarrow 0$ limit

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Where is this useful?

- Early universe: $\chi\chi \rightarrow gg$ typically only a small contribution to the total annihilation cross section. Not particularly important.

- Present day: $\chi\chi \rightarrow gg$ can be the dominant annihilation mode. Corrections are important for total annihilation cross section and branching fractions → can affect indirect DM detection rates.

![Graph showing the relationship between $v_{rel}$ and $\sigma$ for different annihilation modes.](image)

\( \chi\chi \rightarrow q\bar{q} \) in early universe

\( \chi\chi \rightarrow gg \) (includes NLO)

\( \chi\chi \rightarrow q\bar{q} \) today

Bino; $M_{q} = 200$ GeV Interference term not included – still prelim.

[Barger, Keung, HEL, Shaughnessy, Tregre 2005]
Conclusions

- Precision cosmology motivates calculation at higher orders.

- Majorana neutralinos $\rightarrow$ s-wave annihilation helicity suppressed. Processes that lift the suppression can have a big impact on present-day annihilation rates.

- We calculated NLO QCD corrections to $\chi\chi \rightarrow gg$ by using the Adler-Bardeen theorem and known NLO QCD corrections to $A \rightarrow gg$: about a $+60\%$ effect.

- Interference term between $\chi\chi \rightarrow g^*g \rightarrow q\bar{q}g$ and tree-level $\chi\chi \rightarrow q\bar{q}g$ (in preparation):
  - Same $\alpha_s$ order as LO $\chi\chi \rightarrow gg$
  - Relative $m_\chi^2/M_q^2$ suppression
  - Destructive interference

- Implications for indirect detection still need to be worked out.