N-flationary magnetic fields

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M. Anber, LS, in preparation

Looking for inflation in String Theory...

...as in Field Theory...

radiative stability

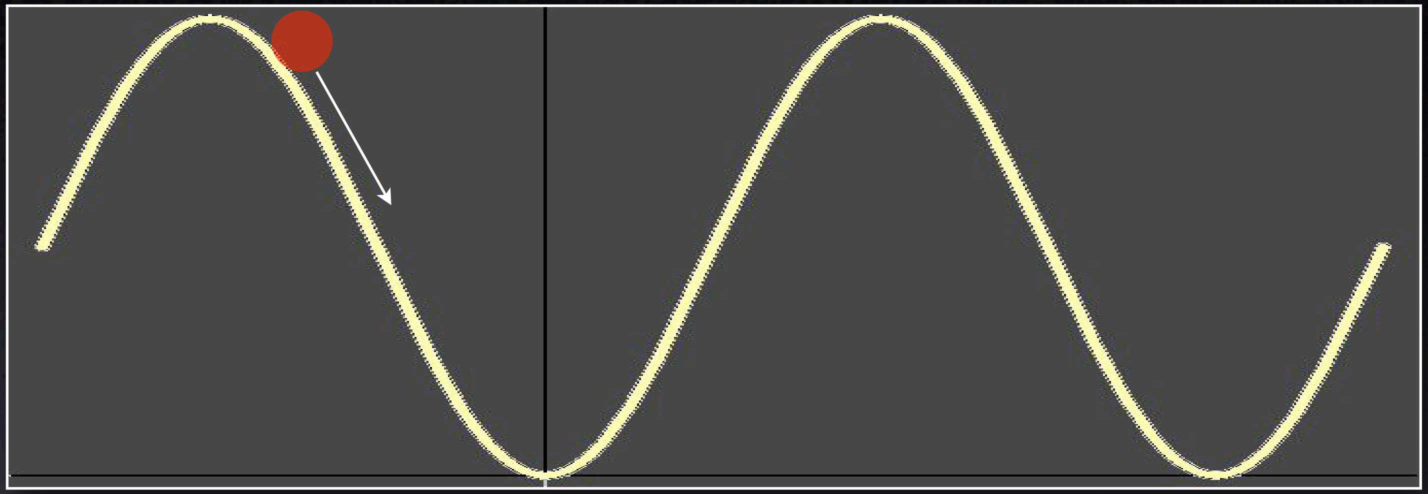
of the inflaton potential is required!

As usual, symmetries help:

a *shift symmetry* can keep the inflaton potential flat

Natural Inflation

natural inflaton
=
pseudo Nambu-Goldstone boson (pNBG)



$$V(\phi) = \Lambda^4 [1 + \cos(\phi/f)]$$

WMAP requires $f > 3\ M_P$...

Freese and Kinney 2004

...while String Theory seems to tolerate only $f < M_P$!

A way out:

more than one pNGB

- Mixture of a pNGB and a modulus
  Blanco-Pillado et al 2004
- Two pNGBs
  Kim, Nilles and Peloso 2004
- A lot of pNGBs
  Dimopoulos, Kachru, McGreevy and Wacker 2005

⇓

N-flation

How does N-flation work?

Start from N pNGBs:

$$\mathcal{L} = -\sqrt{-g} \sum_{i=1}^{N} \left\{ \frac{1}{2} (\partial \phi_i)^2 + \Lambda_i^4 \left[1 + \cos(\phi_i / f_i)\right] \right\}$$

Assume that all the ϕ_i, all the f_i and all the Λ_i are equal:

$$\mathcal{L} = -\sqrt{-g} \left\{ \frac{N}{2} (\partial \phi)^2 + N \, \Lambda^4 \left[1 + \cos(\phi / f)\right] \right\}$$

Canonically normalized field $\Phi = \sqrt{N} \phi$

$$\mathcal{L} = -\sqrt{-g} \left\{ \frac{1}{2} (\partial \Phi)^2 + N \Lambda^4 \left[1 + \cos\left(\frac{\Phi}{\sqrt{N} f} \right) \right] \right\}$$

Can be $>M_P$!

N>600 required by WMAP

Kim and Liddle 2006

pNGBs are coupled to the electromagnetic field!

M. Anber, LS

$$\mathcal{L} \supset \sum_{i=1}^{N} \alpha \frac{\phi_i}{4\, M_P} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$(\alpha = \mathcal{O}(1))$

Magnetic fields can be produced by the rolling pNGBs at inflation

Cosmological magnetic fields

- Observed with intensities of order μGauss
- Coherence lengths of 10s of kpcs
- ***Unknown origin***

Can be amplified by a *dynamo mechanism*

Seed field required ~ 10^{-30} G

Davis, Lilley and Tornkvist 1999

back to our model...

$$\mathcal{L} \supset \sum_{i=1}^{N} \alpha \frac{\phi_i}{4\,M_P} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Electromagnetic field coupled to the *sum* of the pNGBs

the *direction* of rolling of the pNGBs matters:

define $\gamma=(N_+-N_-)/N$ where

N_+= # of pNGBs with $\dot{\phi}>0$

N_-= # of pNGBs with $\dot{\phi}<0$

[$-1<\gamma<1$]

The main equation

$$\frac{\partial^2 F_\pm}{\partial \tau^2} + \left(k^2 \pm \frac{\alpha\,\gamma\sqrt{N}}{M_P}\,\frac{d\Phi}{d\tau}\,k \right) F_\pm = 0$$

($F_\pm$= >ve and <ve helicity modes of the magnetic field)

The result depends only on one combination of parameters

$$\xi \equiv |\alpha\gamma|\,\sqrt{N\epsilon/2}$$

where ε is the slow-roll parameter

The main result

$$F(\tau,\vec{k}) \simeq \sqrt{\frac{k}{2}}\left(\frac{k}{2\xi\, aH}\right)^{1/4} e^{-2\sqrt{2\xi\, k/aH}}\, e^{\pi\,\xi}$$

Exponential amplification term!

A Constraint...

The energy in the magnetic field should not exceed the energy in the inflaton condensate!

If insist on COBE normalization ($H \sim 10^{13}$GeV),

$\xi < 7$

If require just $H > 10^{-3}$eV,

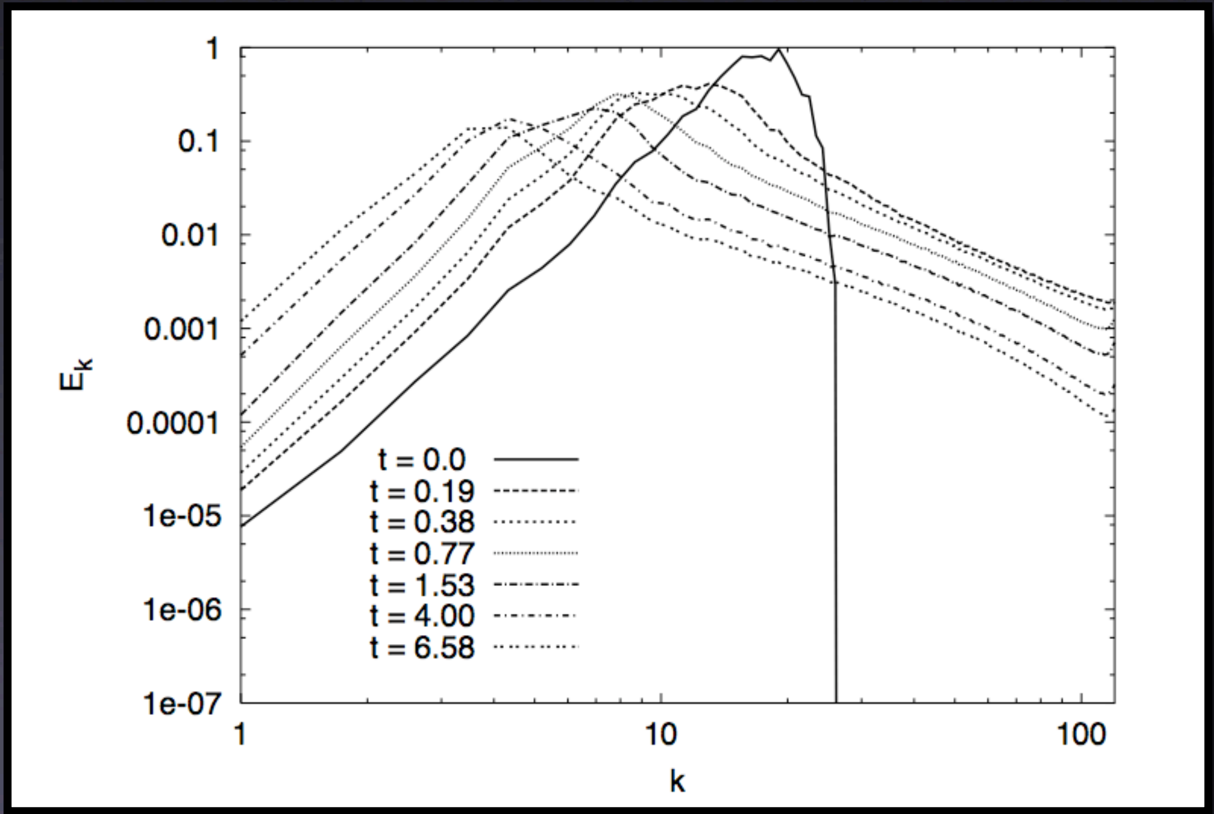
$\xi < 25$

Cf. Garretson, Field and Carroll 1992

Evolving the field in the cosmic plasma

The magnetic field produced has *maximal helicity*

MagnetoHydroDynamic processes *(inverse cascade)* transfer power to large scales



From Jedamzik and Banerjee 2004

Final value of the magnetic field

(before the dynamo)

$$B \simeq 10^{-33}\, \frac{e^{\pi\,\xi}}{\xi^{17/12}} \left(\frac{T_{\rm RH}}{10^9 \mathrm{GeV}}\right)^{11/36} \left(\frac{l_{\rm phys}}{10\,\mathrm{kpc}}\right)^{-9/4} \mathrm{G}$$

$\xi \gtrsim 2$

is sufficient to initiate the dynamo

In terms of the original parameters

$$\alpha\, y \sqrt{N} \gtrsim 10$$

Enough magnetic field for α and/or $y \sqrt{N}$ of O(few)!

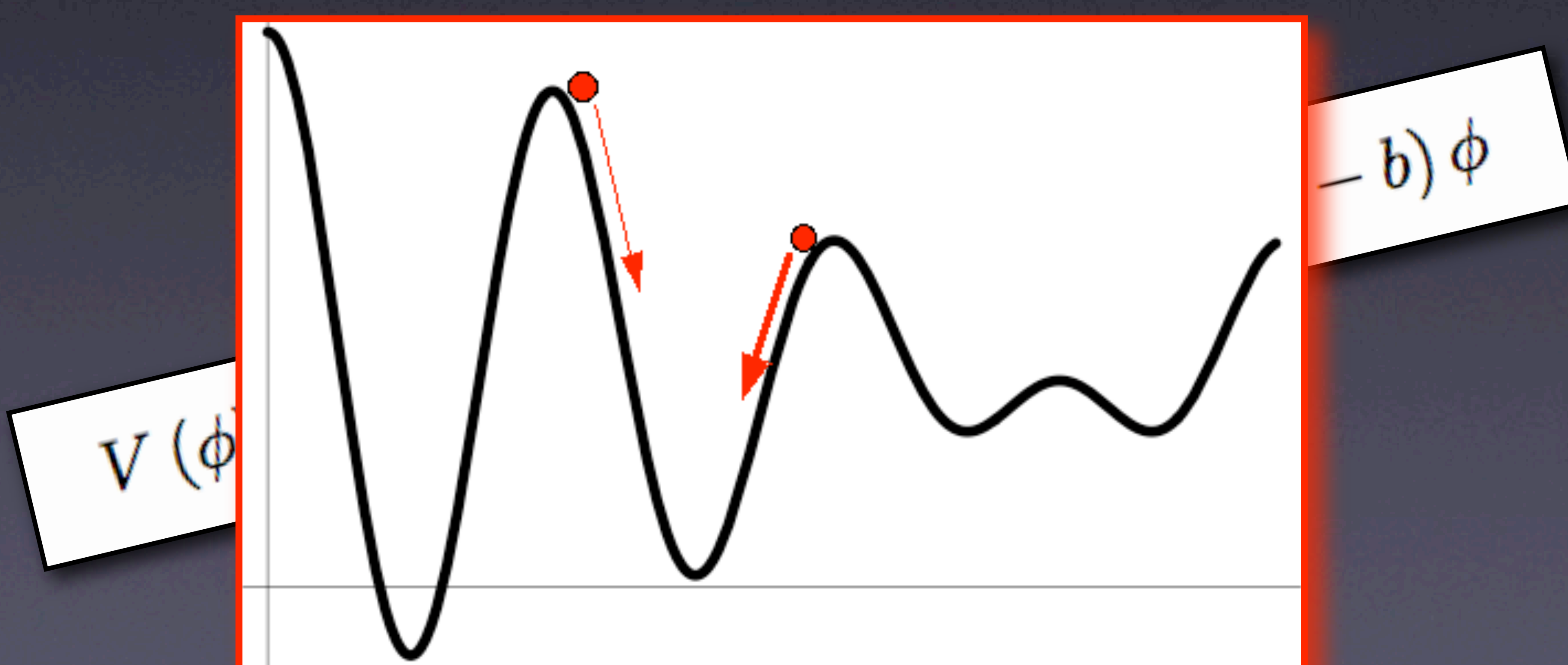
Discussion...

- One obvious possibility: N=few, $\alpha \sim 10$
- More difficult: insist on $\alpha = 1$

e.g. for N=600, need $N_+ \sim 420$ and $N_- \sim 180$...
...rather improbable, if the theory is exactly symmetric wrt $\phi_i \to -\phi_i$

...but an asymmetry can exist:

(Blanco-Pillado et al 2004)



Conclusions

- Models of inflation in string theory could naturally lead to the observed magnetic fields
- Overproduction of magnetic fields could kill some of these models
- $O(1)$ coefficients matter!