Dark Energy and Neutrino Model in SUSY

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PLB 633 (2006) 675 & [hep-ph/0603204]

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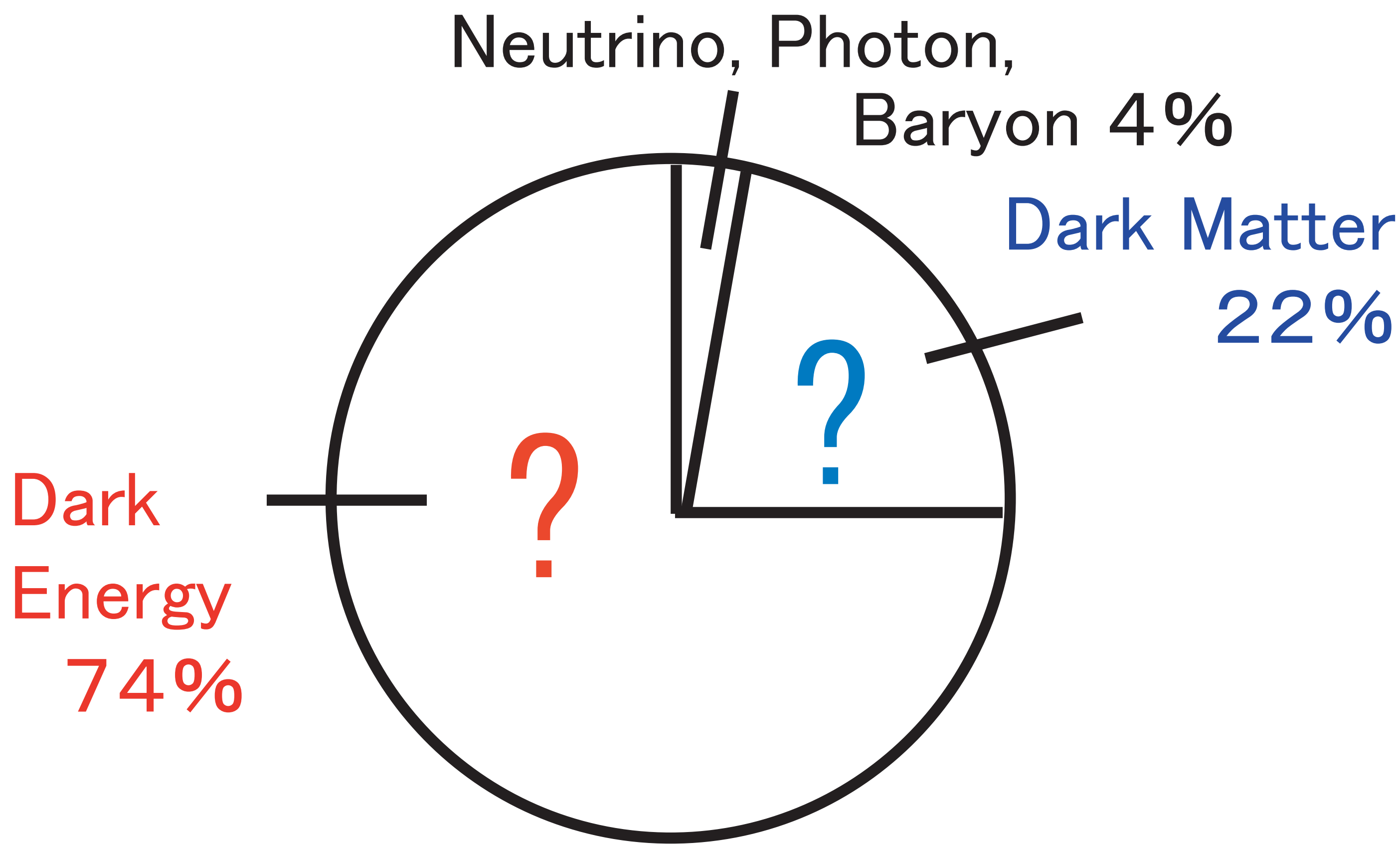
1. Introduction

Content of the Universe



Properties of the Dark Energy

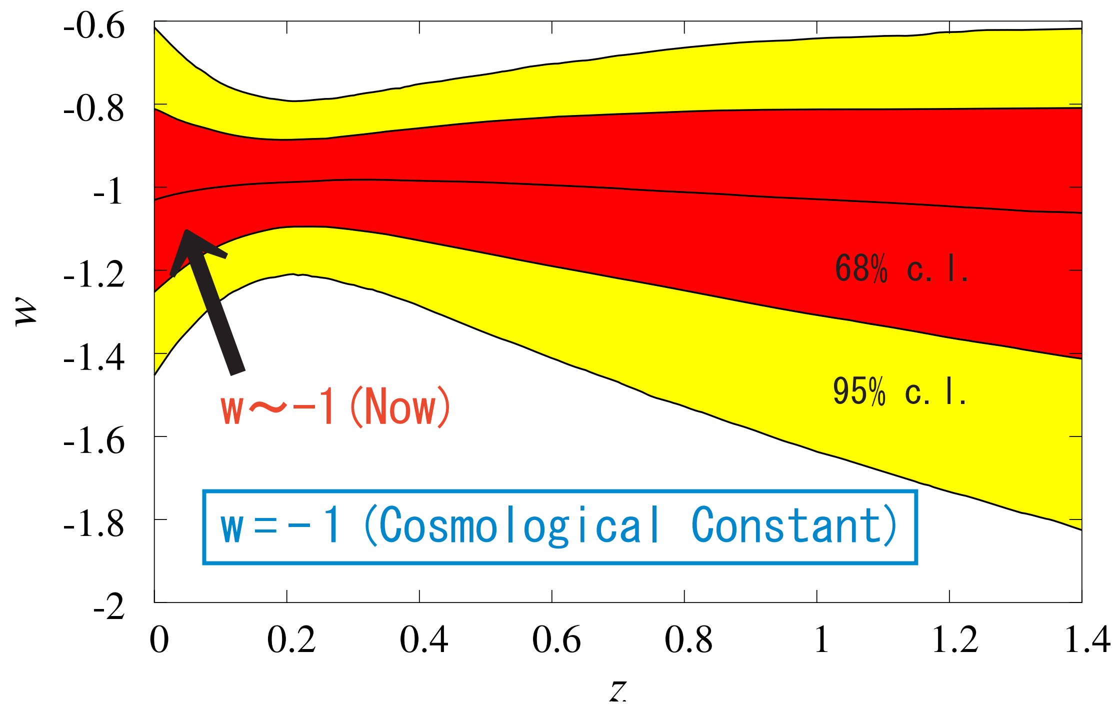
Positive energy density, Negative pressure

Fluid having negative pressure $\Rightarrow$ Cosmic acceleration

Important Parameter

The equation of state :

$$w = \frac{p_{\rm DE}}{\rho_{\rm DE}} \quad (p : \text{Pressure})$$



(U. Seljak, *et al.*, Phys. Rev. D71, 103515 (2005))

Candidates of the Dark Energy

- Quintessence (scalar field: $m_\phi \sim 10^{-33}$eV)
- Phantom energy ($w < -1$)
- Mass Varying Neutrinos (MaVaNs)

⋮

2. Mass Varying Neutrinos Scenario

Gu, Wang, Zhang, PRD 68(2003)087301

Fardon, Nelson, Weiner, JCAP 10(2004)005

Why do we relate the dark energy with neutrinos?

- The energy density and masses of neutrinos are uncertain compared with other components of the Universe (*e.g.* baryons...).

- The mass scale of the neutrino is close to the dark energy scale.

Neutrino mass scale

$$\Delta m^2_{\rm sol} \sim 8.0 \times 10^{-5}\ {\rm eV}^2 \quad , \quad \Delta m^2_{\rm atm} \sim 2.5 \times 10^{-3}\ {\rm eV}^2$$

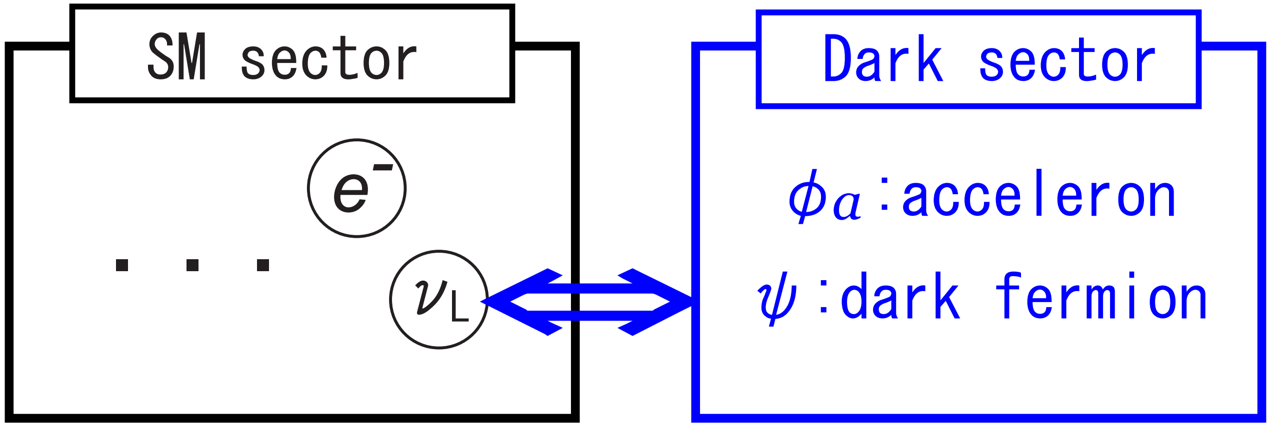
[KamLAND, SNO] [K2K, SK]

Dark energy scale

$$\Lambda_{\rm DE} \sim (2 \times 10^{-3}\ {\rm eV})^4$$

—Mass Varying Neutrinos Scenario—

Assumption



Example)

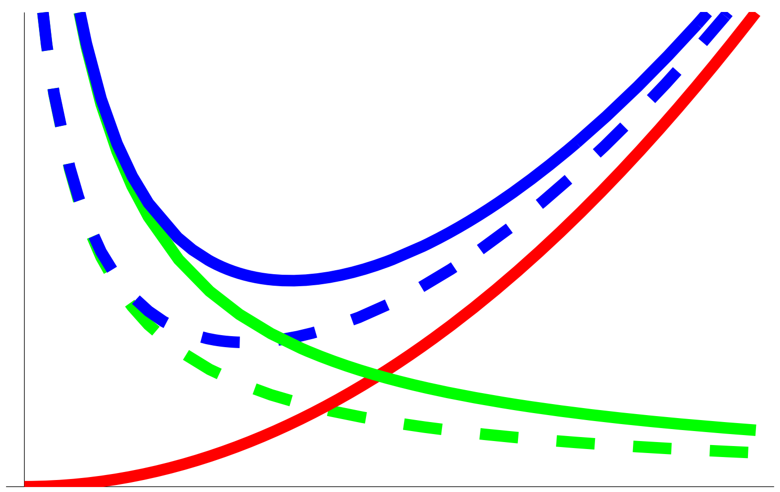
$$\mathcal{L}_{mass} = \lambda\phi_a\psi\psi + m_D\bar{\nu}_L\psi + h.c. \Rightarrow m_\nu = m_\nu(\phi_a) = \frac{m_D^2}{\lambda\phi_a}$$

Dark Energy = Scalar Potential + Neutrinos

$$\rho_{\rm DE} = V(\phi_a) + \rho_\nu(m_\nu(\phi_a))$$

Consequences

♠ As the universe expands, neutrinos dilute.



$$\rho_{\rm DE} = V(\phi_a) + \rho_\nu(m_\nu(\phi_a))$$
$$= V(\phi_a) + m_\nu n_\nu \quad \text{(non-rel.)}$$
$$= V(\phi_a) + \frac{m_D^2}{\lambda\phi_a} n_\nu$$

— $\rho_{\rm DE}$ ⇒ - - $\rho'_{\rm DE}$

— ρ_ν ⇒ - - ρ'_ν

— $V(\phi_a)$

- m_ν and $\rho_{\rm DE}$ also vary on cosmological time scales.
- m_ν is dynamical parameter.

$$\frac{\partial \rho_{\rm DE}}{\partial \phi_a} = 0 \implies \frac{\partial \rho_{\rm DE}}{\partial m_\nu} \cdot \frac{\partial m_\nu}{\partial \phi_a} = 0$$

1 generation model

Energy density of neutrinos

$$\rho_\nu = T^4 F(\xi) \qquad \text{(For non-relativistic neutrinos : } \rho_\nu = m_\nu n_\nu)$$

$$\xi = \frac{m_\nu}{T}$$

$$F(\xi) = \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2\sqrt{y^2+\xi^2}}{e^y+1}$$

Equation of state

- Energy conservation law : $\dot{\rho}_{\rm DE} = -3H(\rho_{\rm DE} + p_{\rm DE})$
- Stationary condition : $\frac{\partial \rho_{\rm DE}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu} = 0$

$$w + 1 = \frac{4 - h(\xi)}{3\left[1 + \frac{V(\phi_a(m_\nu))}{T^4 F(\xi)}\right]} \Longrightarrow \frac{m_\nu n_\nu}{\rho_{\rm DE}} \quad \text{(Non-rel. limit)}$$

$$h(\xi) \equiv \frac{\xi(\partial F(\xi)/\partial \xi)}{F(\xi)}$$

Stationary condition

$$\frac{\partial \rho_{\rm DE}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu} = 0$$

$$= T^3 \frac{\partial F}{\partial \xi} + \frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu} = 0$$

$$\Downarrow$$

♠Once a scalar potential is given, one can find the temperature (time) dependence of the neutrino mass and w.

However...

there are two severe constraints on the scalar potential.

[Smallness] & [Flatness]

Constraints on MaVaNs model (Scalar potential)

- Observation : $\Omega^0_{\rm DE} = \rho^0_{\rm DE}/\rho_c \simeq 0.73$

$$\Rightarrow V(\phi^0_a(m^0_\nu)) = 0.73\rho_c - \rho^0_\nu \simeq 2.93 \times 10^{-11}\,{\rm eV}^4$$

[Smallness]

- Stationary condition (@ the present)

$$\Rightarrow \left.\frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu}\right|_{m_\nu = m^0_\nu} = -T^3 \left.\frac{\partial F}{\partial \xi}\right|_{m_\nu = m^0_\nu,\ T = T_0} \simeq -n^0_\nu \simeq -8.81 \times 10^{-13}\,{\rm eV}^3$$

[Flatness]

♠ The present value of a scalar potential must be small, and its gradient must be flat !!

Constraints on MaVaNs model (m_ν^0 & $m_{\phi_a}^0$)

♠ In order that the acceleron does not vary significantly on distance of inter-neutrino spacing, the present acceleron mass must be less than $\mathcal{O}(10^{-4})$ (eV) ($\sim (n_\nu^0)^{1/3}$).

- Present acceleron mass:

$$m_{\phi_a}^0 = 10^{-4}\,\mathrm{eV}$$

- Present neutrino mass (Typical value):

$$m_\nu^0 = f(\phi_a^0) = 10^{-2}\,\mathrm{eV}$$

Constraints on MaVaNs model (Speed of sound)

♠ MaVaNs contains a catastrophic instability which occurs when ν become nonrelativistic ($\rho_\nu = m_\nu n_\nu$).

Afshordi, Zaldarriage, Kohri, PRD 72(2005)065024

$$c_s^2 = \frac{\dot{p}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \frac{\dot{w}\rho_{\rm DE} + w\dot{\rho}_{\rm DE}}{\dot{\rho}_{\rm DE}} \simeq \frac{\dot{m}_\nu n_\nu}{m_\nu \dot{n}_\nu} < 0 \quad \text{(at the non-rel. limit)}$$

$$\Downarrow$$

However, the next leading term is impotant.

$$\Downarrow$$

$$\rho_\nu = \frac{T^4}{\pi^2}\int_0^\infty \frac{dy y^2\sqrt{y^2+\xi^2}}{e^y+1} \simeq \frac{\hat{n}_\nu}{T}\xi + a\frac{\hat{n}_\nu}{T^3}\frac{1}{\xi} + \cdots$$

$$\xi \equiv \frac{m_\nu}{T}, \quad \hat{n}_\nu \equiv \frac{T^3}{\pi^2}\int_0^\infty \frac{dy y^2}{e^y+1}, \quad a \equiv \frac{\int_0^\infty \frac{dy y^4}{e^y+1}}{2\int_0^\infty \frac{dy y^2}{e^y+1}} \simeq 6.47$$

$$\Downarrow$$

$$c_s^2 \simeq \underbrace{\frac{\frac{\partial m_\nu}{\partial z}\hat{n}_\nu}{m_\nu \frac{\partial \hat{n}_\nu}{\partial z}}}_{\Uparrow\ \text{Negative}} + \underbrace{\frac{\frac{5}{3}a\hat{n}_\nu \left(\frac{5T_0}{\xi}\right) - \frac{T}{\xi^2}\frac{\partial m_\nu}{\partial z}}{m_\nu \frac{\partial \hat{n}_\nu}{\partial z}}}_{\Uparrow\ \text{Positive}}$$

- Positive c_s^2 condition

R.T., Tanimoto, [astro-ph/0601119]

$$\frac{\partial m_\nu}{\partial z}\underbrace{\left(1 - \frac{5aT}{3m_\nu^2}\right)}_{\Uparrow\ \text{Negative}} + \underbrace{\frac{25aT_0^2(z+1)}{3m_\nu}}_{\Uparrow\ \text{Positive}} > 0$$

$\partial m_\nu / \partial z$ depends on a model!

3. Supersymmetric Mass Varying Neutrinos model

♠A chiral superfield is assumed to be in the dark sector.

One superfield model R.T., M. Tanimoto, (2005)

Superpotential

$$W = \frac{\lambda}{3}A^3 + m_D L A$$

A; Chiral superfield, Gauge singlet

Scalar component : ϕ_a ("acceleron")

Spinor component : ψ_a (sterile neutrino)

L; Left-handed lepton doublet

Scalar potential

$$V(\phi_a) = \lambda^2|\phi_a|^4 + m_D^2|\phi_a|^2$$

Lagrangian

$$\mathcal{L} = 2\lambda\phi_a\psi_a\psi_a + m_D\bar{\nu}_L\psi_a + h.c.$$

Mass matrix

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & 2\lambda\phi_a \end{pmatrix} \begin{matrix} \nu_L \\ \psi_a \end{matrix}$$

$$\qquad\qquad \nu_L \quad \psi_a$$

♠From two constraints on the scalar potential,

$$\left.\frac{V(\phi_a(m_\nu))}{\frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu}}\right|_{m_\nu = m_\nu^0} \simeq \frac{2.93 \times 10^{-11}}{-8.81 \times 10^{-13}} = -33.3 \ \mathrm{eV}$$

$\Downarrow$ Putting $m_\nu^0 = 10^{-2}\,\mathrm{eV}$

$$\frac{m_\nu^0}{4} \cdot \frac{1 - \frac{m_D^4}{(m_\nu^0)^4}}{1 + \frac{m_D^4}{(m_\nu^0)^4}} \neq -33.3\,\mathrm{eV} \quad \left(\because \text{LHS} > -\frac{m_\nu^0}{4}\right) \Rightarrow \times$$

One superfield + Right-handed neutrino

R.T., M. Tanimoto, [hep-ph/0603204]

Superpotential

$$W = \frac{\lambda_1}{6} A^3 + m_D LA + M_D LR + \frac{M_A}{2} AA + \frac{\lambda_2}{2} ARR + \frac{M_R}{2} RR$$

R; Right-handed neutrino superfield, Gauge singlet

Scalar potential $(\tilde{l}, \tilde{\nu}_R = 0)$

$$V(\phi_a) = \frac{\lambda_1^2}{4} |\phi_a|^4 + m_D^2 |\phi_a|^2 + M_A^2 |\phi_a|^2$$

Lagrangian

$$\mathcal{L} = \lambda_1 \phi_a \psi_a \psi_a + M_A \psi_a \psi_a + m_D \bar{\nu}_L \psi_a + M_D \bar{\nu}_L \nu_R + \lambda_2 \phi_a \nu_R \nu_R + M_R \nu_R \nu_R + h.c..$$

$\Downarrow$

$\Downarrow$ Integrating out the right-handed neutrinos...

$\Downarrow$

Mass matrix

$$\mathcal{M} \simeq \begin{pmatrix} -\frac{M_D^2}{M_R} + \frac{\lambda_2 \phi_a M_D^2}{M_R^2} & m_D \\ m_D & M_A + \lambda_1 \phi_a \end{pmatrix} \begin{matrix} \nu_L \\ \psi_a \end{matrix}$$

$$\Downarrow$$

$$\Downarrow \quad c \equiv \frac{-M_D^2}{M_R} \quad , \quad \lambda_2' \equiv \frac{-\lambda_2 c}{M_R}$$

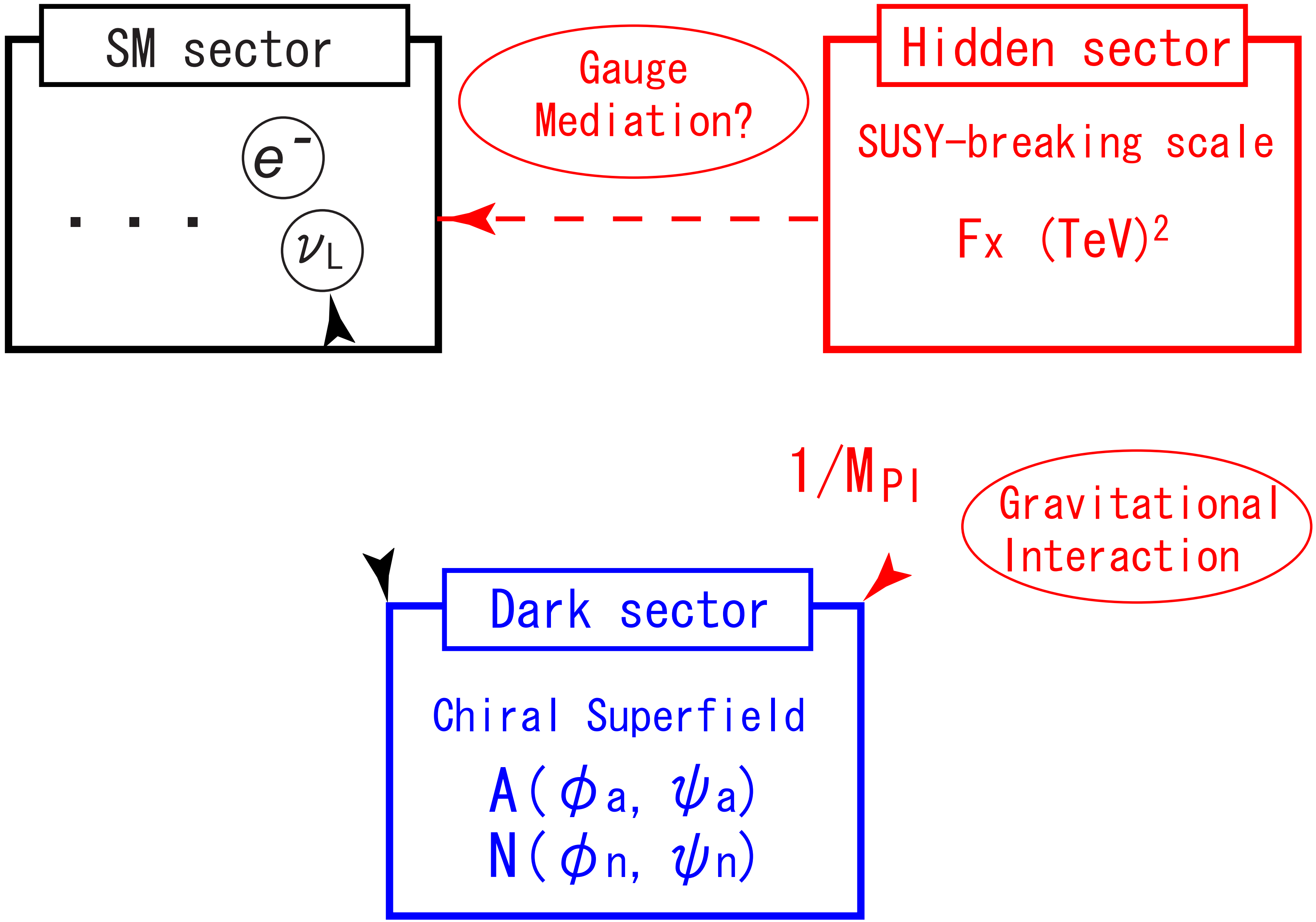
$$\Downarrow$$

$$= \begin{pmatrix} c + \lambda_2' \phi_a & m_D \\ m_D & M_A + \lambda_1 \phi_a \end{pmatrix}$$

$$(\lambda_1 \phi_a \ll M_D \ll M_R)$$

4. Effects of the SUSY-breaking

Our Scheme



Operator

$$\int d^4\theta \frac{X^\dagger X}{M_{\rm Pl}^2} A^\dagger A, \qquad \int d^4\theta \frac{X^\dagger + X}{M_{\rm Pl}} A^\dagger A$$

Soft mass

$$\frac{F_X}{M_{\rm Pl}} \equiv m \simeq \frac{({\rm TeV})^2}{10^{18}{\rm GeV}} \simeq \mathcal{O}(10^{-2}\text{-}10^{-3})\,({\rm eV})$$

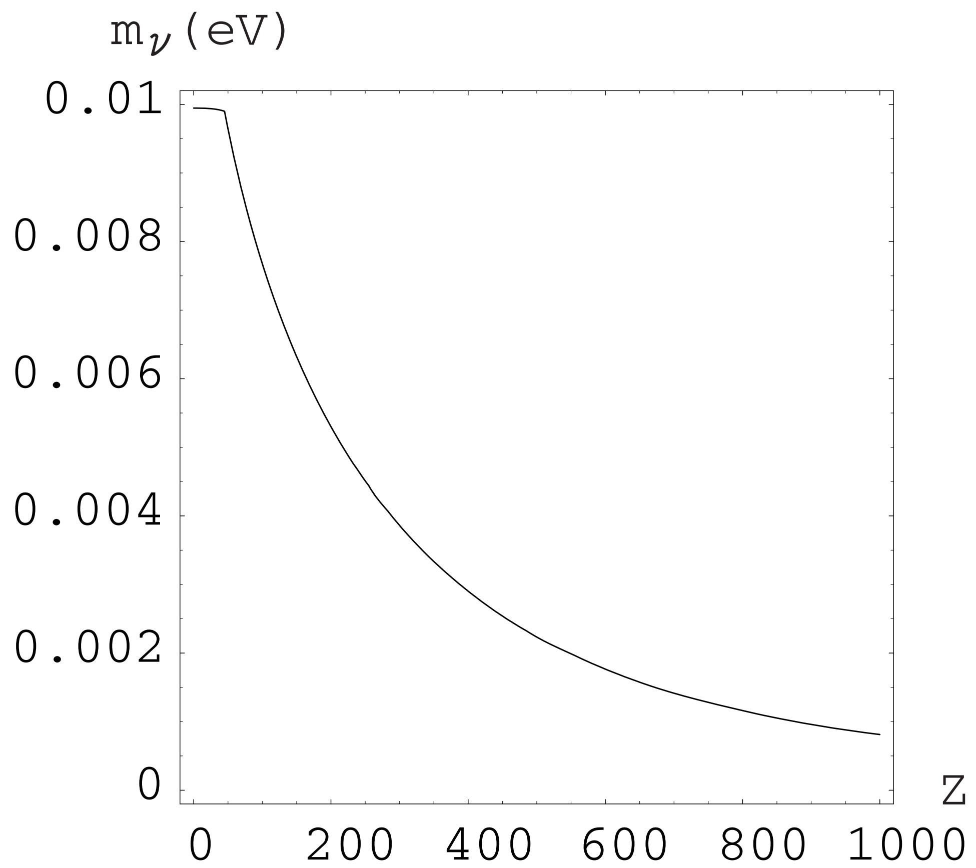
(Chacko, Hall, Nomura, JCAP 0410, 011 (2004))

Scalar potential

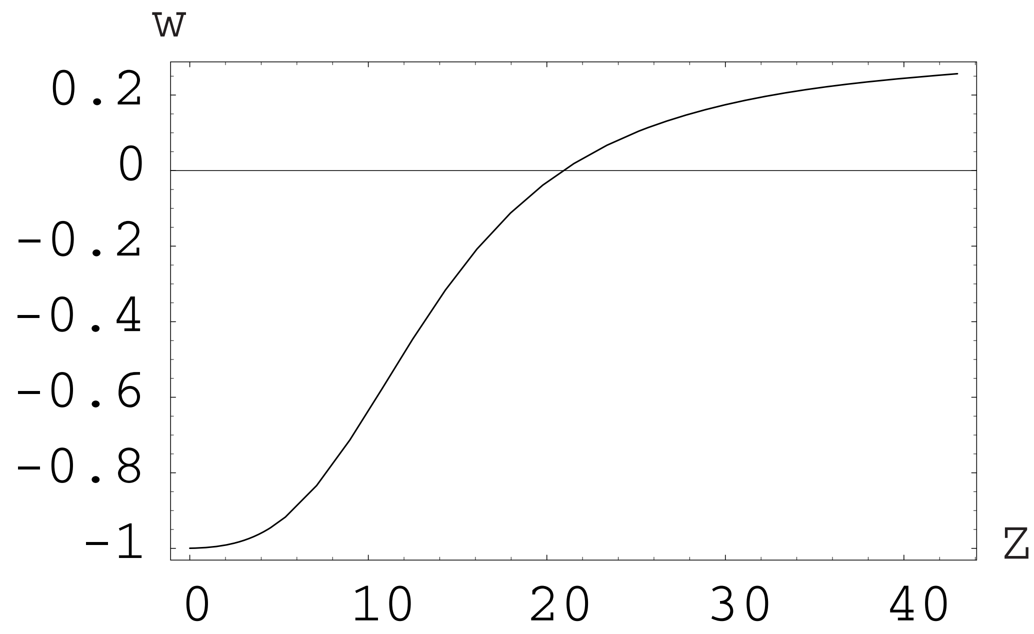
$$V(\phi_a) = \frac{\lambda_1^2}{4}|\phi_a|^4 + m_D^2|\phi_a|^2 + M_A^2|\phi_a|^2 - m^2|\phi_a|^2$$

$$\begin{cases} \lambda_1 = 1 & \phi_a^0 \simeq -3.25 \times 10^{-3}\,{\rm eV} \\ \lambda_2' = 10^{-14} & c \simeq 10^{-2}\,{\rm eV} \\ m_\nu^0 = 10^{-2}\,{\rm eV} & M_A \simeq 1.33 \times 10^{-2}\,{\rm eV} \\ m_D = 10^{-12}\,{\rm eV} & m \simeq 1.33 \times 10^{-2}\,{\rm eV} \end{cases}$$

Time evolution of m_{ν_s}



Time evolution of w



- m_{ν_s} had varied ($m_{\nu_L} \simeq$ const).

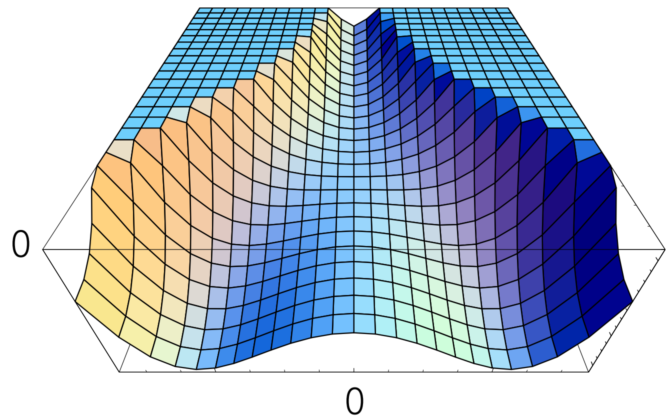
$$\frac{\partial m_\nu}{\partial z} \simeq 0 \, (z < 50) \Rightarrow c_s^2 > 0$$

Other models of supersymmetric MaVaNs

Two chiral superfields model

$$W = \kappa ANN + m_D LA$$

Fardon, Nelson, Weiner,
JHEP 0603 (2006) 042



Two chiral superfields + Right hended neutrino model

$$W = \lambda ANN + m_D LA + m'_D LN + M_D LR + M_R RR$$

R.T., Tanimoto, PLB 633 (2006) 675

5. Summary

- We have built a supersymmetric MaVaNs model.
  - SUSY was broken at TeV scale in the hidden sector, and the effects are mediated by the gravity.
  - A chiral superfield A couples to both the left- and right-handed neutrino.
  - m_{ν_s} had varied ($m_{\nu_L} \sim$ const).

Future work

- BBN constraints on m_{ν_s}

Weiner & Zurek, [hep-ph/0509201]