Supersymmetry, naturalness and environmental selection

G.F. Giudice

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

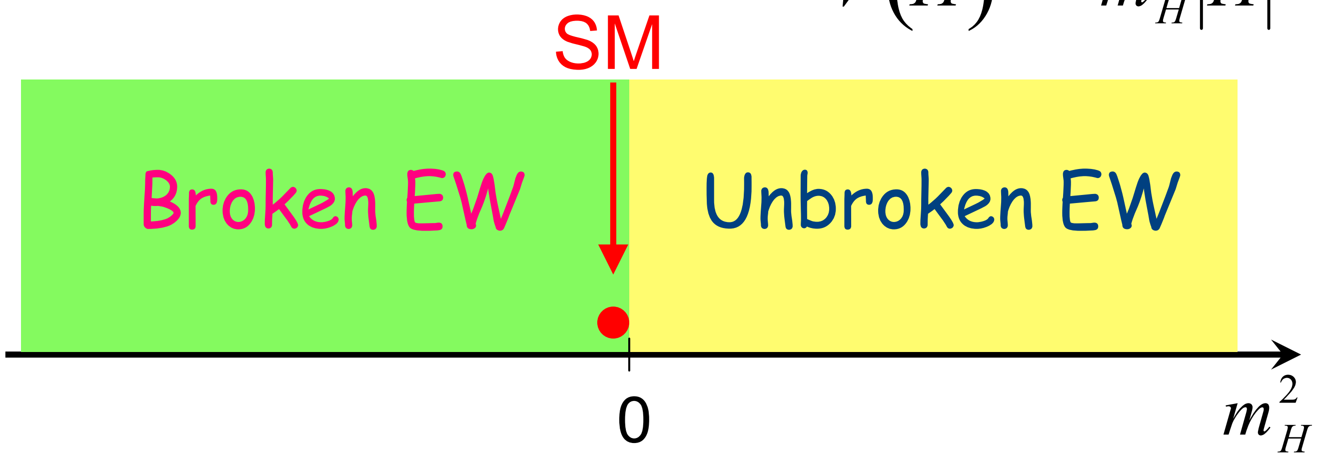
G.F.G., R. Rattazzi, *hep-ph/0606105*
[& N. Arkani-Hamed, A. Delgado, G.F.G., *NPB 741, 108 (2006)*]

Hierarchy problem

- Guiding principle for physics BSM
- One of the main motivations for LHC

Formulation in terms of criticality:

$$V(H) = -m_H^2 |H|^2 + \lambda |H|^4$$



Why is nature so close to the critical line?

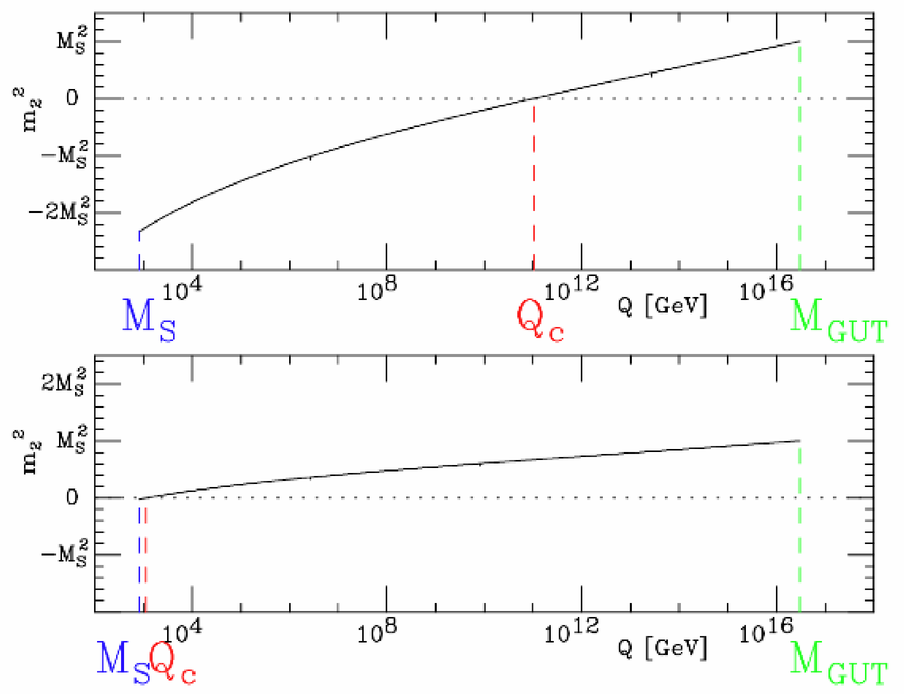
Supersymmetry:

- Exact susy (and $\mu=0$) $\Rightarrow$ critical line
- Dynamical susy breaking $M_S \sim M_P\, e^{-1/\alpha} \Rightarrow$
  - small departure from critical line
  - stabilization of flat direction $|H_1|=|H_2|$
- For "generic" parameters $\Rightarrow m_H^2 \sim -M_S^2$

Expectations for discovery at LEP: unfulfilled!

“generic” supersymmetry: $M_S << Q_C << M_P$

$Q_C \sim e^{-1/\alpha} M_P$
- unrelated to M_S (depends on c_i, α_a)
- much smaller than UV scale



M_S and Q_C equal to few %

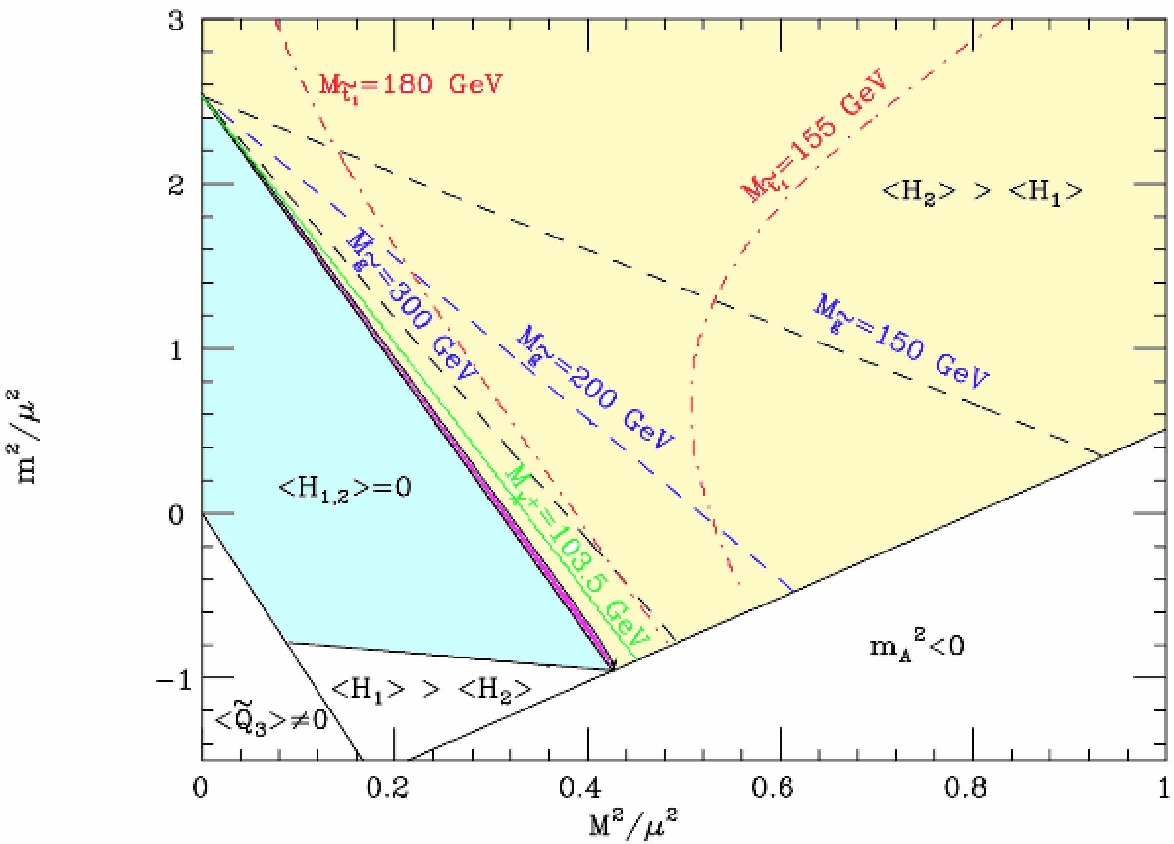
“tuned” supersymmetry: $M_S \sim Q_C << M_P$

$M_S < Q_C$ broken EW; $M_S > Q_C$ unbroken EW

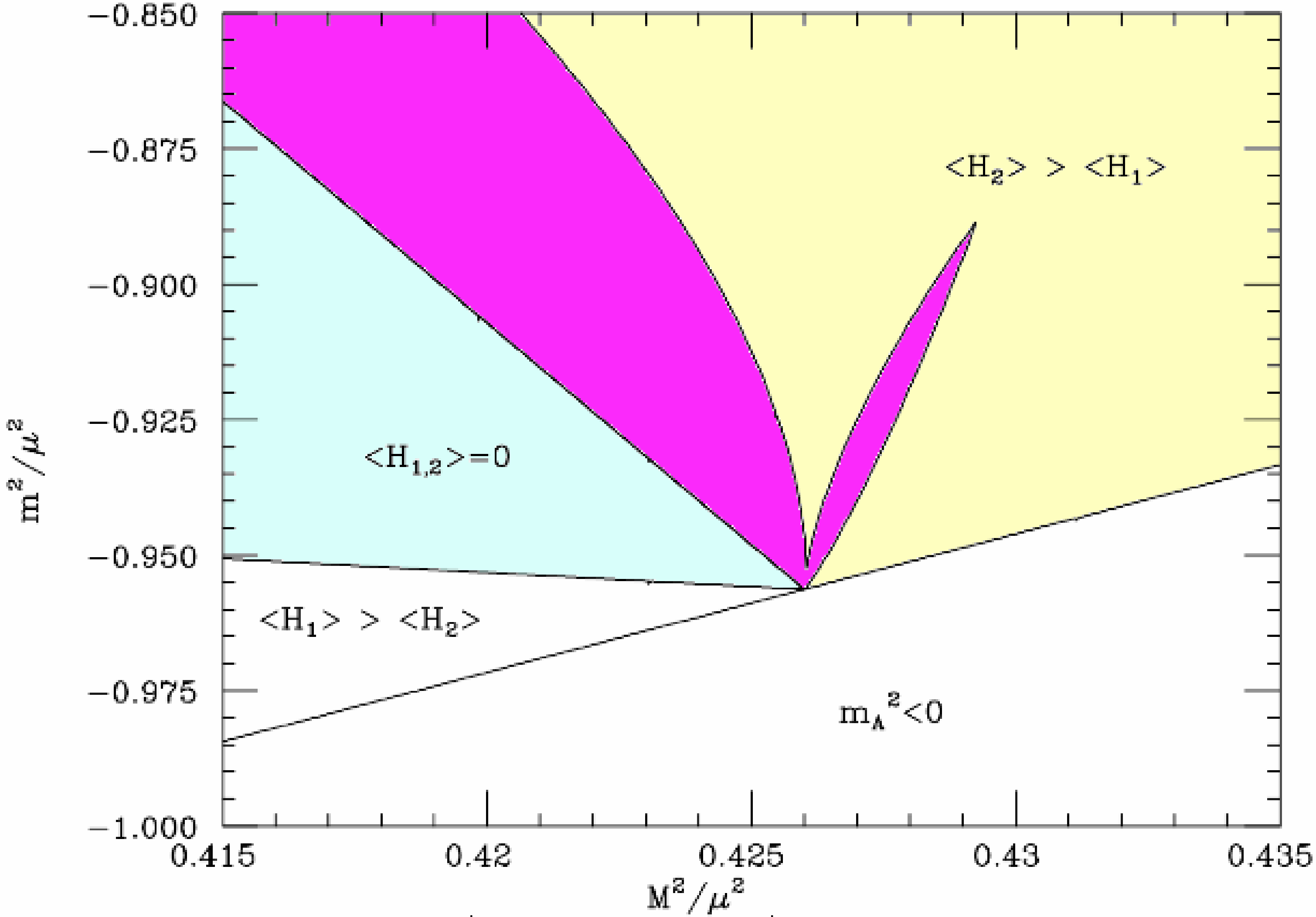
Why supersymmetry should prefer to be near critical?

Phase diagram of supersymmetric SM

$$V = \frac{g^2 + g'^2}{8}\left(|H_1|^2 - |H_2|^2\right)^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + \text{h.c.})$$



- A measure of the fine tuning
- A characterization of the tuning



$$Z \to hZ \quad \propto \quad \sin^2(\beta-\alpha) = \frac{1}{2}\left[1 + \frac{m_A^2 - (m_Z^2 + \Delta)}{m_H^2 - m_h^2}\right]$$

$$Z \to hA \quad \propto \quad \cos^2(\beta-\alpha)$$

Need large stop corrections $\Delta \Rightarrow$ close to criticality

STATISTICAL CRITICALITY

Assume soft terms are environmental parameters

Simplest case: $m_i = c_i M_S$ and M_S scans in multiverse

$Q_C = M_P \times F(c_i, \alpha_a, \lambda_t)$ is fixed

Two possibilities:

1) $M_S > Q_C$: unbroken EW

2) $M_S < Q_C$: broken EW

Impose prior that EW is broken

(analogy with Weinberg)

In "field-theoretical landscapes" we expect $N \propto M_S^n$

Probability distribution **dP =** $\begin{cases} n\left(\frac{M_S}{Q_C}\right)^n \frac{dM_S}{M_S} & \text{for } M_S < Q_C \\ 0 & \text{for } M_S > Q_C \end{cases}$

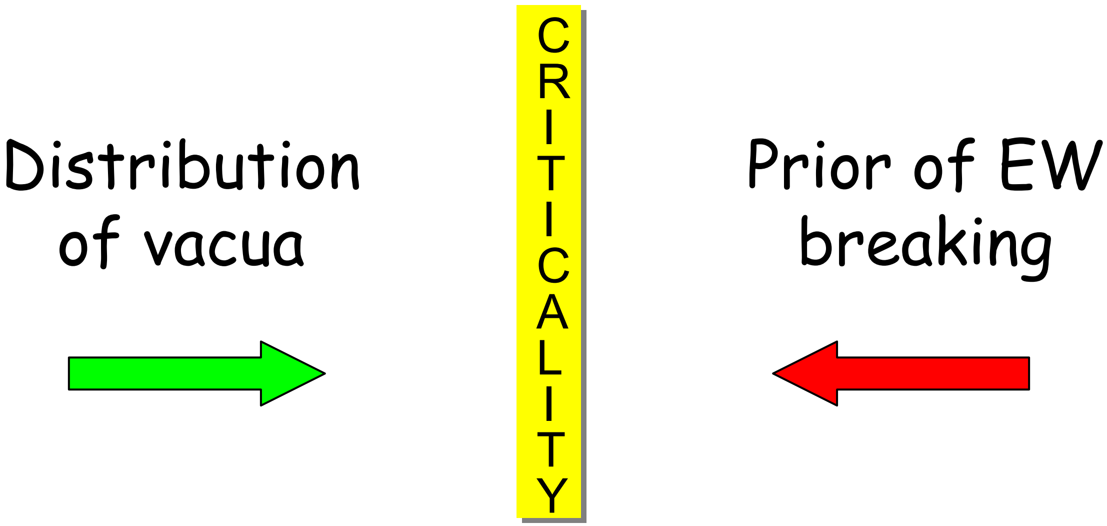
$$\left\langle \frac{M_Z^2}{M_S^2} \right\rangle = \frac{2\,dm_2^2}{M_S^2\,d\ln Q} \left\langle \ln \frac{Q_C}{M_S} \right\rangle$$

$$= \frac{9\lambda_t^2}{4\pi^2} \times \frac{1}{n} \approx \frac{0.15}{n}$$

- Susy prefers to be broken at high scale
- Prior sets an upper bound on M_S

} Susy near-critical

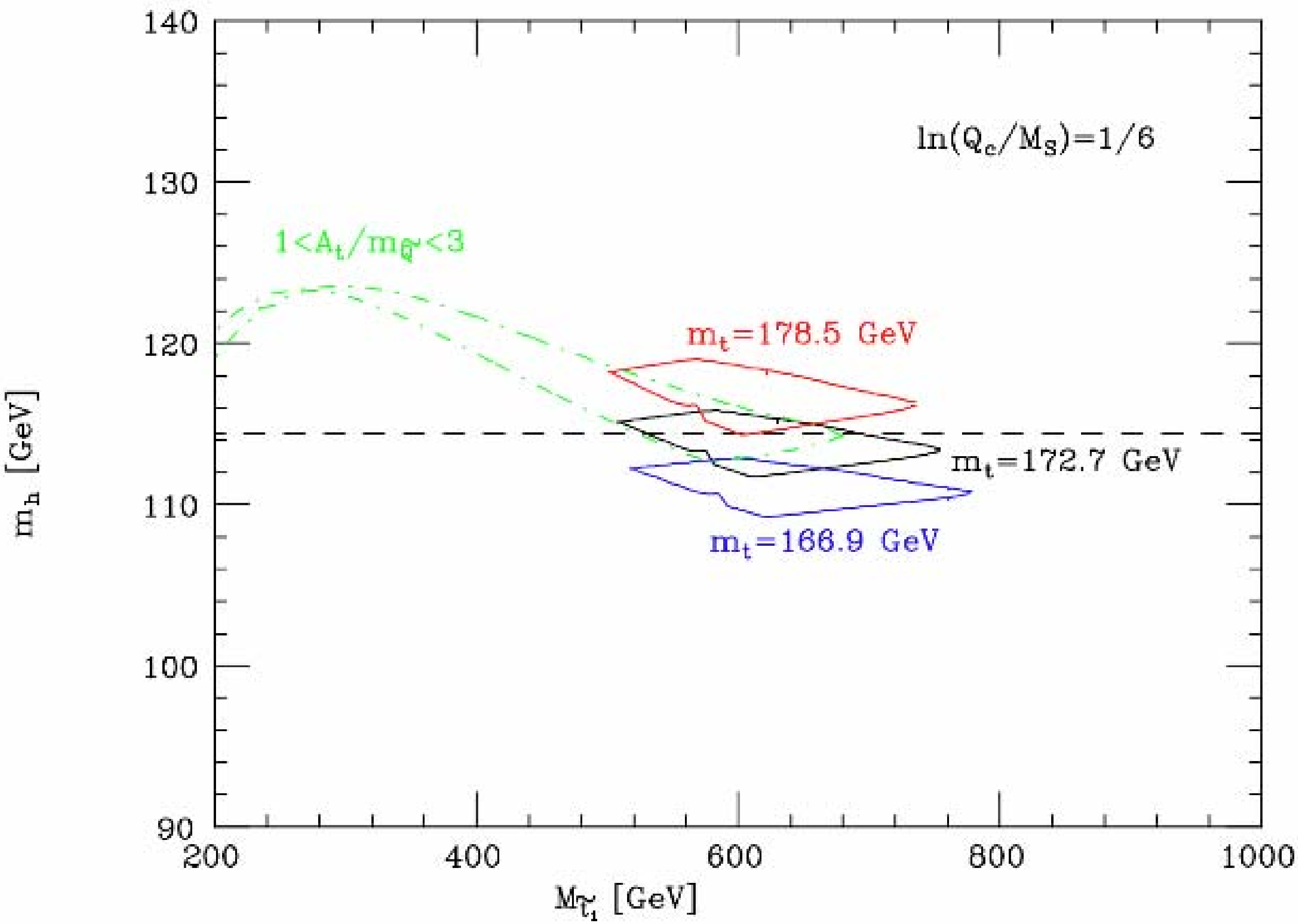
Little hierarchy: Supersymmetry visible at LHC, but not at LEP (*post*-diction)



Supersymmetry looks tuned because there many more vacua with $\langle H \rangle = 0$ than with $\langle H \rangle \neq 0$

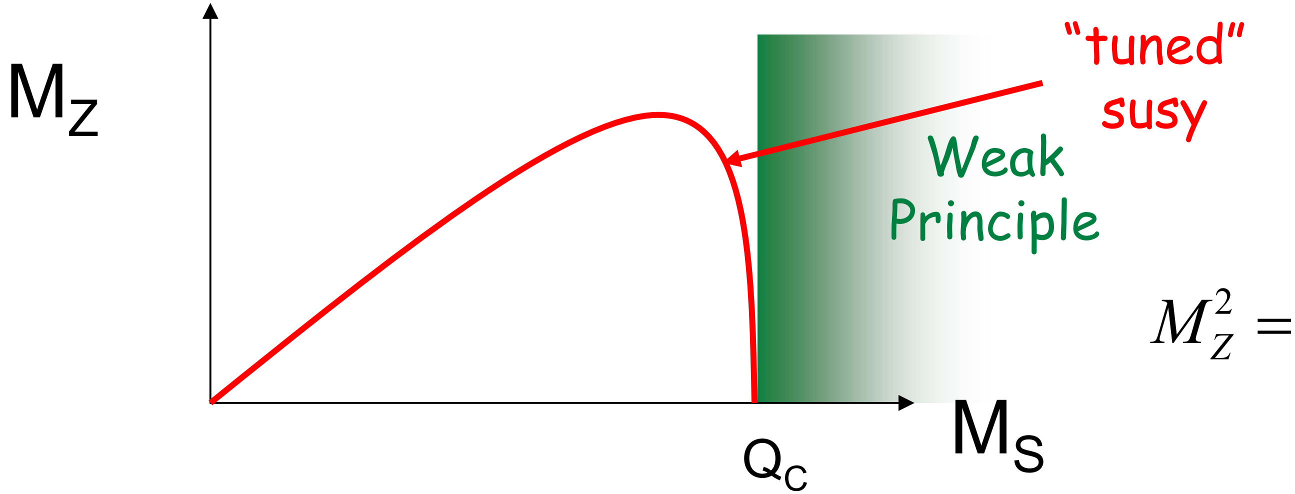
The level of tuning is dictated by RG running, and it is of the order of a one-loop factor

TESTING STATISTICAL CRITICALITY:



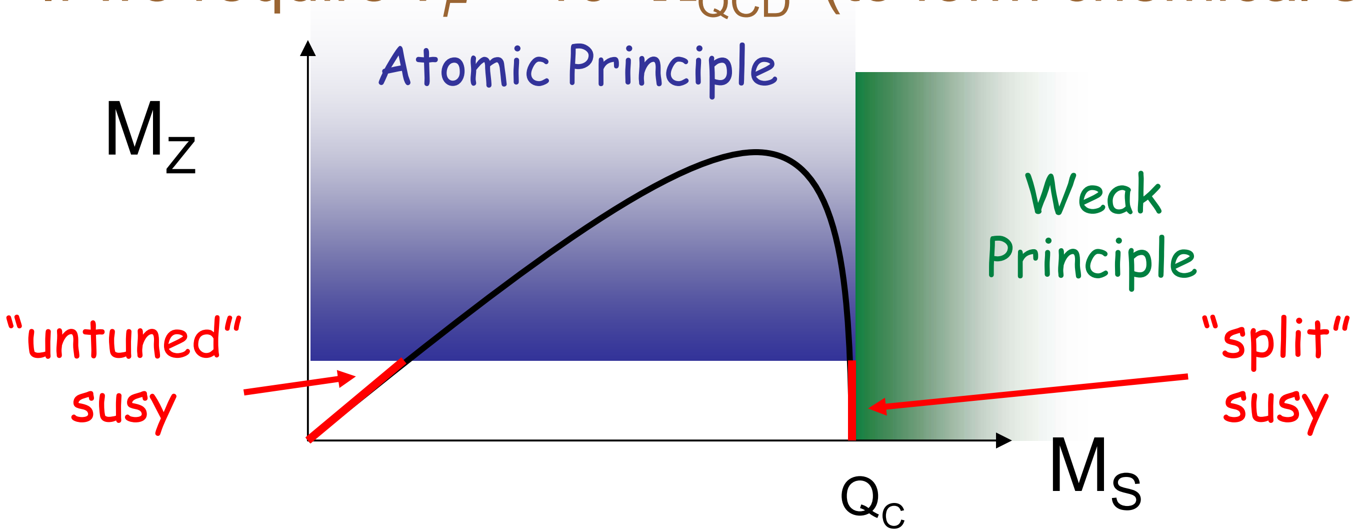
$$0.8 < \frac{A_t}{\tilde{m}_Q} < 1.0 \quad \left(\text{taking } 0 < \frac{m^2}{M^2} < 1 \quad \text{and} \quad \left| \frac{A}{M} \right| < 1 \right)$$

STABILITY UNDER DIFFERENT PRIORS



$$M_Z^2 = \frac{\alpha}{\pi} M_S^2 \ln\frac{Q_C}{M_S}$$

If we require $v_F < 10^3\,\Lambda_{QCD}$ (to form chemical structures)



If also Λ_{QCD} scans, we go back to "tuned" supersymmetry

"Tuned" susy is obtained if enough parameters scan

Statistical solution to μ problem

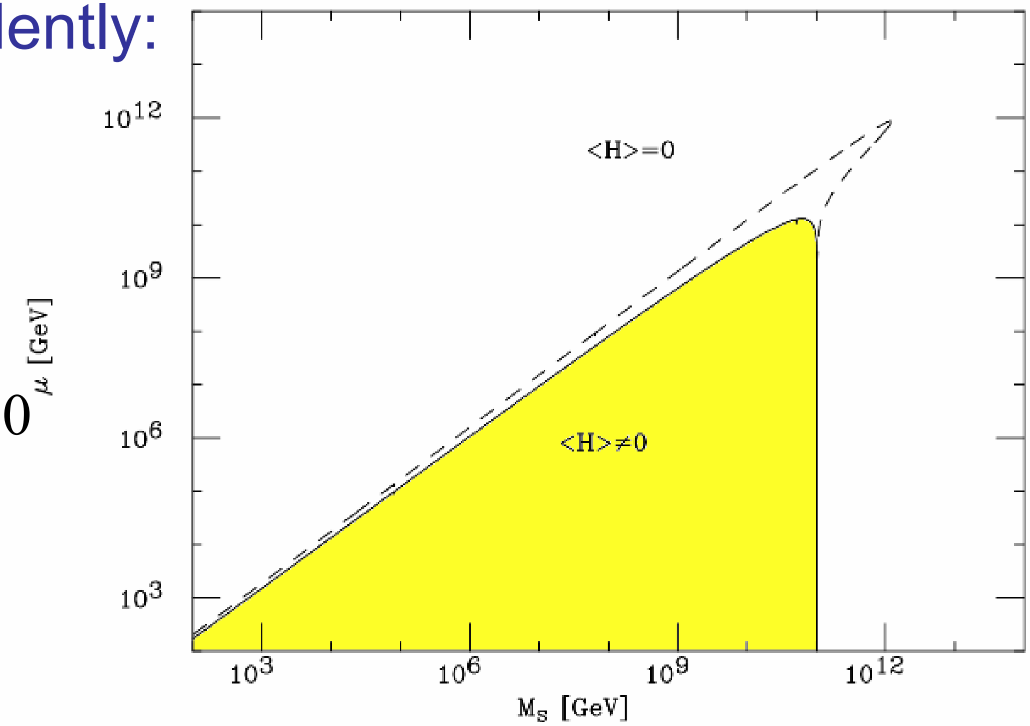
If μ and M_S scan independently:

$$M^2 = \begin{pmatrix} \tilde{m}_1^2 + \mu^2 & B\mu \\ B^*\mu & \tilde{m}_2^2 + \mu^2 \end{pmatrix}$$

Critical line:

$$\tilde{m}_1^2 \tilde{m}_2^2 + \left(\tilde{m}_1^2 + \tilde{m}_2^2 - |B|^2\right)\mu^2 + \mu^4 = 0$$

$$\mu^2 \approx \alpha\, M_S^2 \ln \frac{Q_C}{M_S}$$



Assume $dN \propto dM_S^n\, d\mu^m$

$$\left\langle \frac{m_Z^2}{M_S^2} \right\rangle = \frac{\alpha}{n+m} \qquad \left\langle \frac{\mu^2}{M_S^2} \right\rangle = \frac{\alpha\, m}{n+m}$$

$$\left\langle \frac{\mu}{M_S} \right\rangle \approx \frac{1}{\langle \tan\beta \rangle} \approx \sqrt{\text{loop}} \approx \frac{1}{5-10}$$

- solution to μ problem
- prediction for μ and tanβ
- compatible with well-tempered bino-higgsino

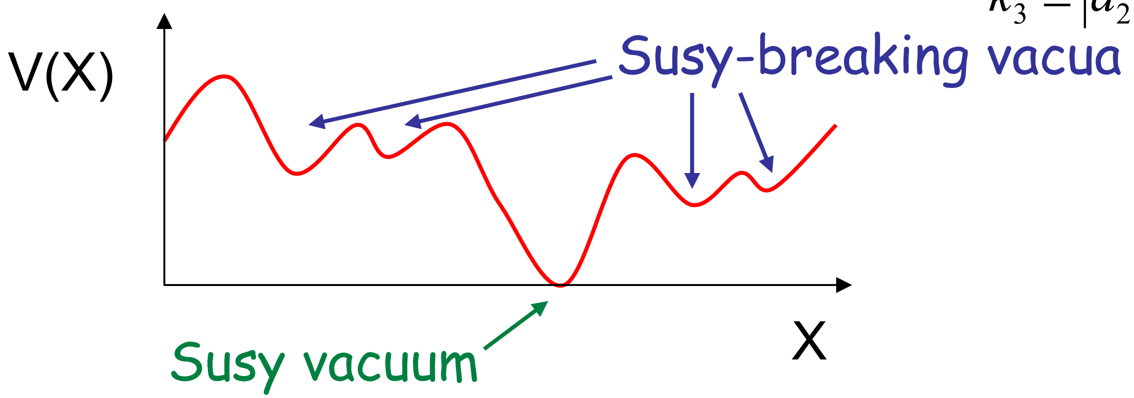
Distribution of susy scale

Denef, Douglas
Dine, O'Neil, Sun

$$W = \sum_n \frac{a_n}{n} X^n \qquad K = \sum_{p,q} \frac{c_{pq}}{(p+1)(q+1)} X^{(p+1)} X^{*(q+1)}$$

$$V = \frac{|\partial_X W|^2}{\partial_X \partial_{X^*} K} = |a_1|^2 + \left(k_1 X + k_2 X^2 + \text{h.c.}\right) + k_3 |X|^2 + \ldots$$

$$k_1 \equiv a_1^* (a_2 - a_1 c_{10})$$
$$k_2 \equiv a_1^* (a_3 - a_1 c_{20}) - c_{10} k_1$$
$$k_3 \equiv |a_2|^2 - |a_1|^2 c_{11} - \left(c_{10}^* k_1 + \text{h.c.}\right)$$



3 conditions on complex parameters to have a local minimum (k_1=0), stable ($|k_3|>2|k_2|$) with susy breaking at M_S ($|a_1|=M_S$)

$$dN \propto dM_S^6$$

If susy vacua dominate and strong dynamics occur:

$$dN \propto d \ln M_S$$

RECAP: Supersymmetry & Naturalness

EW BREAKING

After LEP: a % tuning on soft terms

Problem of criticality: { dynamics? / chance ? / statistics ? }

Talks by Nomura, Dermisek, Toro, Okumura, Kitano, Falkowski, Shirman, Maekawa

DARK MATTER

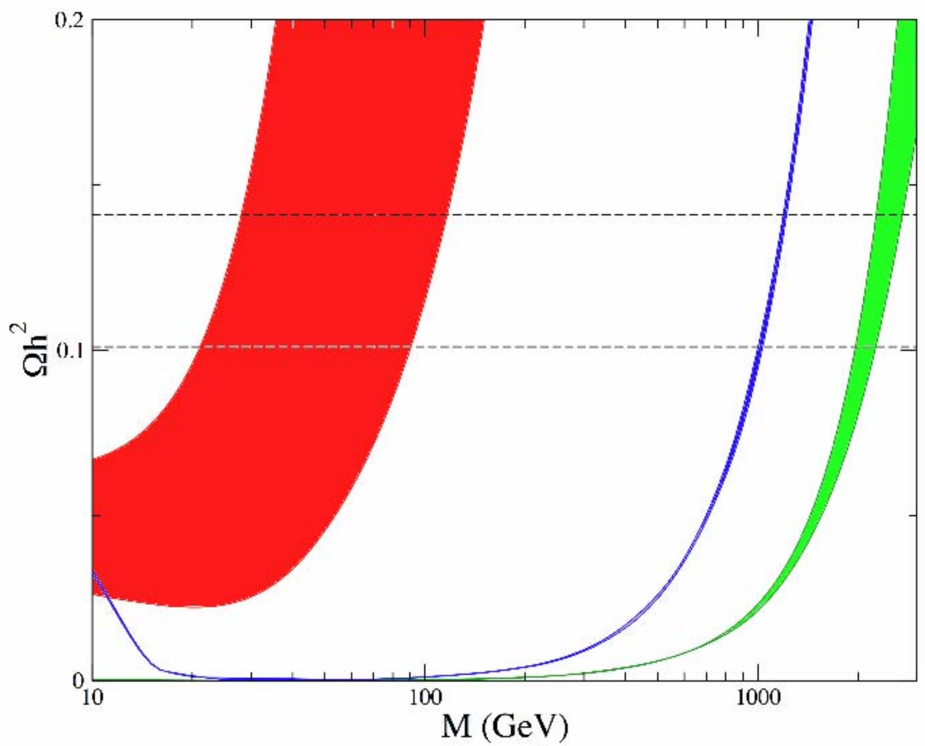
Quantitative difference after LEP & WMAP:

$\Omega_{DM}h^2 = 0.127\,^{+0.007}_{-0.013}$

For $M_S > M_Z$: χ is almost pure state

B-ino: annihilation through sleptons (too slow without coannihilation): $\tilde{m}_e < 115$ GeV at 95% CL (LEP: $\tilde{m}_e > 100$ GeV)

H-ino, W-ino: annihilation through gauge bosons (too fast)



DM is possible in "special" regions:

- coannihilation
- Higgs resonance
- "Well-tempered"

or non-thermal

Both M_Z and Ω_{DM} can be reproduced by low-energy supersymmetry, but with "atypical" parameter choices.

Unlucky circumstances or dynamical explanation?

Statistics? (always assumed when tuning is discussed?)

RECAP: Supersymmetry & Environmental Selection

Use of anthropic principle controversial

- Symmetry principles have been very successful
- Lack of predictive power

However:

- Failure of dynamical explanation for CC
- Landscape in string theory
- Predictions are possible: probabilistic (CC, axion)
  change of perspective (Split Susy)

Near-criticality of susy?