The Higgs in the Sky:

production of gravitational waves
during a first order phase transition

work done in collaboration with

C. Delaunay, G. Servant
to appear soon

see also

C.G., G. Servant, J. Wells


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GW and the ElectroWeak Phase Transition

GW interact very weakly and are not absorbed

direct probe of physical process of the very early universe

possible cosmological sources:
  inflation, vibrations of topological defects, excitations of xdim modes, 1st order phase transitions...

ElectroWeak Phase Transition (if 1st order)
typical freq. ~ (size of the bubble)$^{-1}$ ~ (fraction of the horizon size)$^{-1}$

@ $T = 100$ GeV,

\[ H = \sqrt{\frac{8\pi^3}{45}} \frac{T^2}{M_{Pl}} \sim 10^{-15} \text{ GeV} \]

redshifted freq.

\[ f \sim \# \frac{2 \cdot 10^{-4} \text{ eV}}{100 \text{ GeV}} 10^{-15} \text{ GeV} \sim \# 10^{-5} \text{ Hz} \]

~ today ~

The GW spectrum from a 1st order electroweak PT is peaked around the milliHertz frequency
Why should you be excited about mHZ freq.?

\[ \Omega_{\text{GW}} h^2 \]

- we can except to learn something about the EW phase transition from GW experiments

- test of the dynamics of the phase transition (quite important to analyze models of EW baryogenesis!)

- reconstruction of the Higgs potential / study of new models of EW symmetry breaking (little Higgs, gauge-Higgs, composite Higgs, Higgsless...)

complementary to collider informations
A not so new subject...

Early 90’s, M. Turner and his students studied the production of GW produced by bubble collisions. Not much attention since the LEP data excluded a 1st order phase transition within the SM.

‘01–’02: Kosowsky et al. and Dolgov et al. computed the production of GW from turbulence ⇒ stronger signal. Application to the (N)MSSM where a 1st order phase transition is still plausible.

In this talk:

⇒ Model-independent analysis for detectability of GW from 1st order phase transitions

⇒ GW from $H^6$ induced 1st order EW phase transition

Kosowsky, Turner, Watkins’92
Kamionkowski, Kosowsky, Turner ‘94
Kosowsky, Mack, Kahniashvili’02
Dolgov, Grassi, Nicolis’02
Caprini, Durrer ’06

Delaunay, Grojean, Servant, Wells

Grojean, Servant
A two parameter problem...

A 1st order phase transition proceeds by nucleation of bubbles

kinetic energy of bubbles is transferred to GW either by

- bubble collisions
- injection of energy into the plasma fluid
  (creating a homogeneous, isotropic, fully developed and stationary turbulent regime).

Need to move large mass rapidly ⇒ detonation regime: bubble walls propagate faster than the speed of sound

The GW background is controlled by two quantities

\[ \alpha \sim \frac{\text{false vacuum energy density}}{\text{plasma thermal energy density}} \]

\[ \beta \sim \text{rate of time variation of the nucleation rate } \Gamma \left( \Gamma = \Gamma_0 e^{-\beta t} \right) \]
\[ \sim (\text{duration of transition})^{-1} \]

The stronger is the transition, the larger is \( \alpha \) and the smaller is \( \beta \)
Estimate of the GW energy density

energy density at the emission time

\[ \rho_{GW}^* \sim \frac{E_{GW}^*}{\text{volume}} \sim \frac{\mathcal{P}_{GW}^* \times \beta^{-1}}{(\text{bubble wall speed} \times \beta^{-1})^3} \]

quadrupole formula

for power of GW emission

\[ \mathcal{P}_{GW}^* = \frac{1}{5M_{Pl}^2} \left\langle \left( \dddot{Q}_{ij}^{TT} \right)^2 \right\rangle \]

\[ \dddot{Q}_{ij}^{TT} \sim \frac{\text{mass of system in motion} \times (\text{size of system})^2}{(\text{time scale of system})^3} \sim \frac{\text{kinetic energy}}{\text{time scale of system}} \]

\[ E_{\text{kin}} \sim \kappa \alpha \rho_{\text{rad}} \times \text{volume} \]

\[ \kappa : \text{fraction of vacuum energy which goes into kinetic energy of bulk motion of the fluid (as opposed to heating)} \]

\[ \Omega_{GW}^* = \frac{\rho_{GW}^*}{\rho_{\text{tot}}} \quad \text{with} \quad \rho_{\text{tot}} = (1 + \alpha)\rho_{\text{rad}} \quad \quad H_*^2 = \frac{\rho_{\text{tot}}}{M_{Pl}^2} \]

\[ \Omega_{GW}^* = \kappa^2 v_b^3 \frac{\alpha^2}{(1 + \alpha)^2} \frac{H_*^2}{\beta^2} \]

\[ v_b \quad \text{bubble wall speed} \]
Detonation Regime

- **bubble wall speed**
  
  $$v_b = \frac{1}{\sqrt{3}} + \frac{\sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha} \leq 1$$
  
  (weak phase transition)

- **fraction of vacuum energy into kinetic energy**
  
  $$\kappa \sim \frac{1}{1 + 0.715\alpha} \left(0.715\alpha + \frac{4}{27} \sqrt{\frac{3\alpha}{2}}\right)$$
  
  (turbulent regime)

- **characteristic velocity of the eddies in the plasma**
  
  $$u_s = \sqrt{\frac{\kappa\alpha}{4/3 + \kappa\alpha}} \leq v_b$$
  
  (strong phase transition)

  Kamionkowski, Kosowsky, Turner '94

  Steinhardt'82

  Nicolis '02
GW energy density today

\[ \Omega_{GW} = \left( \frac{a_*}{a_0} \right)^4 \left( \frac{H_*}{H_0} \right)^2 \]

\[ \Omega_{GW}^* \sim 2 \times 10^{-5} h^{-2} \left( \frac{100}{g_*} \right)^{1/3} \Omega_{GW} \]

\[ H_0 \sim h \times 2 \times 10^{-42} \text{ GeV} \]

\[ \Omega_{GW} \]

had to be quite big to get an observable signal today

\[ \Omega_{GW}^* \geq 10^{-6} \text{ for LIGO/LISA} \]

\[ \Omega_{GW}^* \geq 10^{-12} - 10^{-9} \text{ for BBO} \]
How to compute $\alpha$ and $\beta/H$

the numerical computation of $\alpha$ of $\beta/H$ is quite involved

need to compute the nucleation temperature

find critical bubbles

$$S_3(T) = 4\pi \int dr \ r^2 \left[ \frac{1}{2} \left( \frac{d\phi_b}{dr} \right)^2 + V(\phi_b, T) \right]$$

Overshooting-undershooting method

to search for the bounce solution

$$\frac{d^2 \phi_b}{dr^2} + \frac{2}{r} \frac{d\phi_b}{dr} - \frac{\partial V}{\partial \phi_b} = 0$$

$$\left. \frac{d\phi_b}{dr} \right|_{r=0} = 0 \ , \ \phi_b\big|_{r=\infty} = 0$$

Nucleation occurs when the probability

for the nucleation of 1 bubble

per 1 horizon volume is $\sim O(1)$

$\Rightarrow$ translates into \[ S_3(T_*)/T_* \simeq 140 \]
How to compute $\alpha$ and $\beta/H$

once you know the nucleation temperature

$\alpha$ is the energy difference between the false and the true vacuum

$\beta/H$ is the related to the derivative of the euclidean action

\[
\Gamma \propto e^{-S_3(T)/T}
\]

the peak frequency of the GW signal (at the time of emission) is

\[
\frac{f^*_H}{H^*_T} = T \frac{d}{dT} \left( \frac{S_3}{T} \right) \sim \frac{S_3}{T} \sim \ln \frac{m_{Pl}}{T^*_T}
\]

typically, $\beta/H \sim \frac{f^*_H}{H^*_T} \sim 200$
The Higgs in the Sky

Complete Spectrum: Collision

\[ \Omega_{GW} h^2 \sim 10^{-6} \frac{\kappa^2 H_*^2}{\beta^2 (1 + \alpha)^2} \frac{v_b^3}{0.24 + v_b^3} \left( \frac{100}{g_*} \right)^{1/3} \]

\[ f_{\text{peak}} \approx 5 \times 10^{-6} \frac{\beta}{H_*} \frac{T_*}{100 \text{ GeV}} \left( \frac{g_*}{100} \right)^{1/6} \text{ Hz} \]

\[ \Omega_{\text{peak}} h^2 \]

\[ \alpha = 0.4, \beta/H_* = 200, T = 1 \text{ TeV}, g_* = 100 \]

Kamionkowski, Kosowsky, Turner '94
Complete Spectrum: Turbulence

standard Navier-Stokes turbulence, with an ordinary Kolmogorov energy spectrum

$$\Omega_{GW} h^2 \sim 10^{-4} u^5 v^2 \beta^2 \frac{H^2}{g^*} \left( \frac{100}{g^*} \right)^{1/3}$$

$$f_{peak} \approx 3 \cdot 10^{-6} \frac{u_s}{v_b} \frac{\beta}{H_*} \frac{T_*}{100 \text{ GeV}} \left( \frac{g^*}{100} \right)^{1/6} \text{ Hz}$$

$$\Omega_{GW} h^2 \sim 10^{-4} u^5 v^2 \beta^2 \frac{H^2}{g^*} \left( \frac{100}{g^*} \right)^{1/3}$$
Complete Spectrum: MHD Turbulence

MHD effects due to the presence of large magnetic fields produced before or during the phase transition

\[ \Omega_{GW} h^2 \sim 10^{-4} u_s v_b H_*^2 \left( \frac{100}{g_*} \right)^{1/3} \]

\[ \Omega_{GW}^{\text{peak}} h^2 \sim 3 \cdot 10^{-6} \frac{u_s}{v_b} \frac{\beta}{H_*} \frac{T_*}{100 \text{ GeV}} \left( \frac{g_*}{100} \right)^{1/6} \text{ Hz} \]

\[ f_{\text{peak}} \approx 3 \cdot 10^{-6} \frac{u_s}{v_b} \frac{\beta}{H_*} \frac{T_*}{100 \text{ GeV}} \left( \frac{g_*}{100} \right)^{1/6} \]

\( \alpha = 0.4, \beta/H_* = 200, T = 1 \text{ TeV}, g_* = 100 \)
Scanning the $(\alpha, \beta/H)$ plane

We always have $f_{\text{turb peak}} \leq f_{\text{coll peak}}$

2 peaks are separated
slope change only

- turb. peak is observable
- coll. peak is observable
Observability of a 1\textsuperscript{st} order PT at Lisa
The galactic astrophysical background can be removed since in the galactic plane (sources are removed one by one: lot of work!)

An extra galactic astrophysical background can affect the sensitivity of BBO

while this extra galactic background doesn’t affect LISA nor LIGO, it will affect BBO
Observability of a 1st order PT at BBO

\[ \frac{\beta}{H} \]

\[ \text{BBO} \quad \text{T} = 50 \text{ GeV} \]

\[ \alpha \]

\[ 10^5 \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10^1 \]

\[ 10 \]

\[ 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]
Observability of a 1st order PT at BBO

\[ \beta/H \]

BBO

\[ T = 50 \text{ GeV} \]

\[ 10^5 \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10 \]

\[ 1 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ \alpha \]

\[ \beta/H \]

BBO

\[ T = 100 \text{ GeV} \]

\[ 10^5 \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10 \]

\[ 1 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ \alpha \]

\[ \beta/H \]

BBO

\[ T = 500 \text{ GeV} \]

\[ 10^5 \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10 \]

\[ 1 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ \alpha \]

\[ \beta/H \]

BBO

\[ T = 1 \text{ TeV} \]

\[ 10^5 \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10 \]

\[ 1 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ \alpha \]
Observability of a 1\textsuperscript{st} order PT at BBO

\begin{align*}
\beta/H & \quad BBO \quad T = 10 \text{ TeV} \\
& \quad \alpha \\
\beta/H & \quad BBO \quad T = 1 \text{ PeV} \\
& \quad \alpha \\
\beta/H & \quad BBO \quad T = 100 \text{ TeV} \\
& \quad \alpha \\
\beta/H & \quad BBO \quad T = 10 \text{ PeV} \\
& \quad \alpha 
\end{align*}
Observability of a 1st order PT at LIGO

A phase transition at $T \sim 10^7$ GeV could be observed at LIGO.
Inflation vs. Phase Transition Signal

\[ \Omega_{\text{GW}} h^2 \]

If the scale of inflation is high enough, its GW signal can be observable at BBO unless it is screened by GW background produced by a 1st order phase transition.

WMAP: B-mode polarization imposes \( \Omega_{\text{GW}} h^2 \leq 10^{-15} - 10^{-14} \)
Inflation vs. Phase Transition Signal

Observability of the peaks at BBO from a PT at $T=5$ TeV
Inflation vs. Phase Transition Signal

\[ \frac{\beta}{H} \]

vs.

\[ \alpha \]

$log_{10}$ scales:

- $10^3$
- $10^4$
- $10^5$

Curves:
- Blue: Inflation
- Red: Phase Transition
- Yellow: $T=5$ TeV
A phase transition up to $T = 1$ PeV can totally mask a GW signal from inflation at BBO.
What can we learn from GW in the EW symmetry breaking sector of a particular model?
EW Phase Transition in the Standard Model

In the SM, a $1^{st}$ order phase transition could occur due to thermally generated cubic Higgs interactions:

$$ V(\phi, T) \approx \frac{1}{2} \left( -\mu_h^2 + cT^2 \right) \phi^2 + \frac{\lambda}{4} \phi^4 - ET\phi^3 $$

In the SM:

$$ \sum_i \approx \sum_{W,Z} \quad \rightarrow \quad \text{not enough} \quad \quad \quad \quad \quad \quad \quad E = \frac{4m_W^3 + 2m_Z^3}{12\pi v_0^3} \sim 6 \cdot 10^{-3} $$

$$ \frac{\langle \phi(T_c) \rangle}{T_c} \geq 1 \quad \quad \quad \quad m_h \leq 47 \text{ GeV} $$
A first order EW phase transition from $H^6$

does not rely on a thermally generated negative Higgs cubic interaction

instead, we add a non-renormalizable $\phi^6$ interaction in the Higgs potential

$$V(\phi) = \mu_h^2 |\Phi|^2 - \lambda |\Phi|^4 + \frac{|\Phi|^6}{\Lambda^2}$$

Can induce a strong 1\textsuperscript{st} order phase transition if

$$\Lambda \sim 1 \text{ TeV}$$

$$\langle \phi^2(T_c) \rangle = \frac{3}{2} v_0^2 - \frac{m_h^2 \Lambda^2}{2v_0^2}$$

and

$$T_c^2 = \frac{\Lambda^4 m_h^4 + 2\Lambda^2 m_h^2 v_0^4 - 3 v_0^8}{16c\Lambda^2 v_0^4}$$

Little Higgs theories integrating out a singlet coupled to the Higgs
A strongly 1st order PT at large Higgs mass

The blue region corresponds to a first order phase transition
Testing the $H^6$ interaction @ colliders

The $h^6$ interaction generates large deviations to the Higgs self-couplings:

$$\mathcal{L} = \frac{m_H^2}{2} H^2 + \frac{\mu}{3!} H^3 + \frac{\eta}{4!} H^4 + \ldots$$

where

$$\mu = 3 \frac{m_H^2}{v_0} + 6 \frac{v_0^3}{\Lambda^2}$$

$$\eta = 3 \frac{m_H^2}{v_0^2} + 36 \frac{v_0^3}{\Lambda^2}$$

The dotted lines delimit the region for a strong 1rst order phase transition.

Even deviations of order 1 are difficult to see at LHC/ILC.
Testing the H$^6$ interaction in the sky

Delaunay, Grojean, Servant, to appear
Conclusions

LHC/ILC will tell us about the EWSB sector

GW experiments might also bring interesting and complementary pieces of information

We might well be learning something about the Higgs by looking at the sky