Non-Supersymmetric Attractors

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Irvine, June'06

Outline

1. Motivation & Introduction

2. What is an attractor?
   When does an attractor exist?

3. a) Spherically symmetric attractors
   b) Rotating attractors

4. Microscopic & Macroscopic Entropy

5. Conclusions

Collaborators:

I) K. Goldstein, N. Iizuka, R. Jena, S.P.T., hep-th/0507093

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III) K. Goldstein, G. Mandal, R. Jena, S.P.T., hep-th/0512138

IV) A. Dabholkar, A Sen, S.P.T., in prep.

V) D. Astefanesei, K. Golstein, R. Jena, A. Sen, S. P. T.

Some Related References:

1) Ferrara, Gibbons, Kallosh, hep-th/9702103

2) Denef, hep-th/0005049, …

3) A. Sen, hep-th/0506177 …

4) Kallosh, …

5) Ferrara et. al.,

6) Kraus and Larsen, 0506173, 0508218.

Some Related References Cont'd

7) Ooguri, Vafa, Verlinde, hepth/0502211

8) Gukov, Saraikin, Vafa, hepth/0509109, hep-th/0505204

Non-Supersymmetric Attractors

Motivations:

1. Black Hole Physics: Entropy, etc

2. Landscape. Vacuum Selection?

Interesting Parallels with Flux Compactifications.

II. What is an Attractor?

- 4 Dim. Gravity, Gauge fields F_i , Scalars, ϕ_i
- Family of Extremal Black Hole Solutions .
- The near horizon region of these black holes is universal, determined only by the charges.

(Extremal Black Holes have minimum mass for given charge)

Attractor:

- Scalars take fixed values at horizon. Independent of Asymp. Values (but dependent on charges).

- Resulting near-horizon geometry of form, $AdS_2 \times S^2$ also independent of asymptotic values of moduli.

- The near-horizon region has enhanced symmetry $SO(2,1) \times SO(3)$

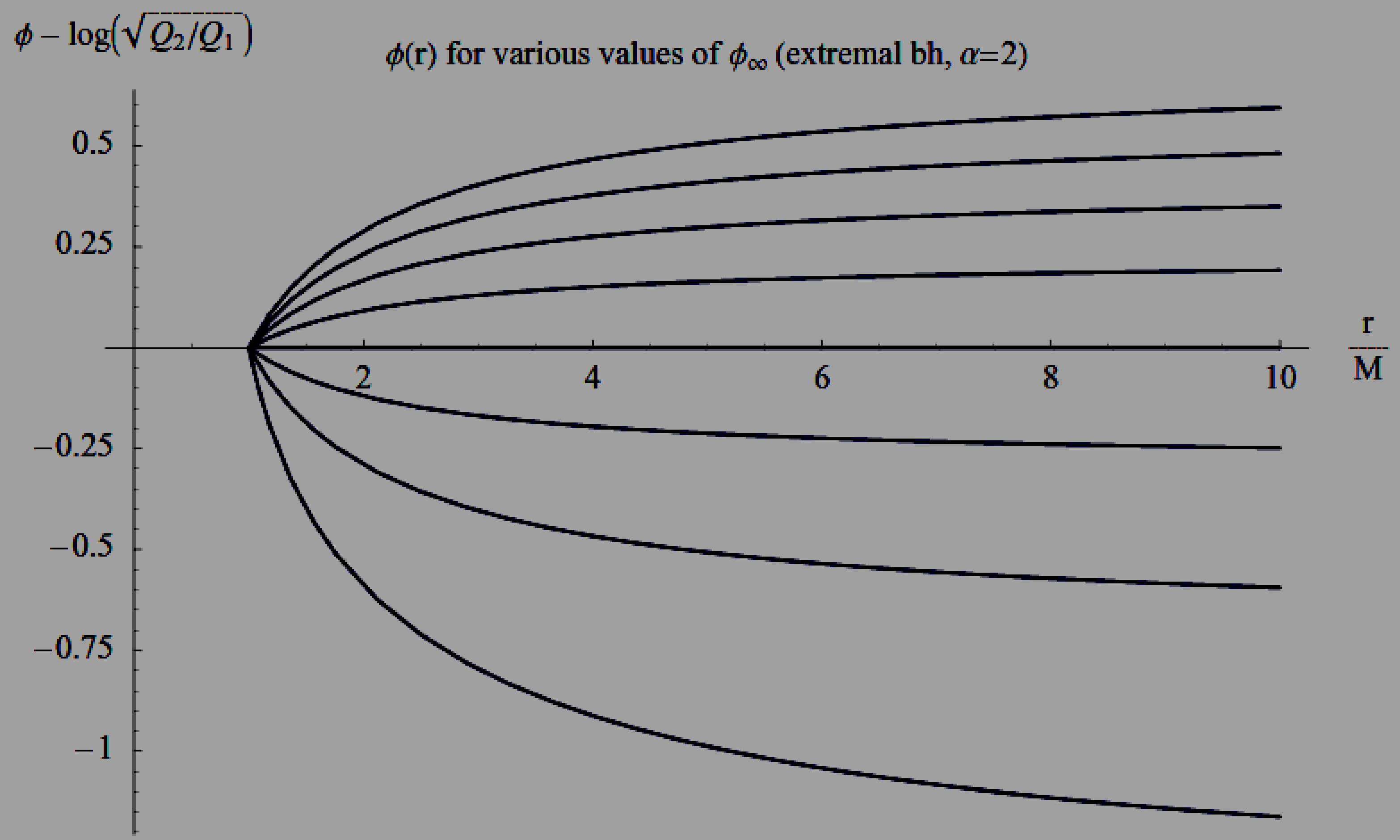


Figure 1: Attractor behaviour for the case $\gamma = 1$; $\alpha, -\tilde{\alpha} = 2$

- So far Mainly explored in Supersymmetric Cases.

    Kallosh, Ferrara, Strominger .

    Denef ,Gibbons, …,

    Ooguri, Strominger, Vafa, …

- N=2 Supersymmetry.

- Black Holes BPS, preserve N=1 Susy.

Aim of the Talk

- In this talk we shall ask whether there are non-supersymmetric attractors.
- And what we can learn from them.
- The Black Holes we shall be interested in are Extremal and break supersymmetry.

Conditions for an Attractor (Not Necessarily Supersymmetric):

Spherically symmetric, Non-Rotating Black Holes:

$$
\begin{aligned}
S \;=\; & \int d^4x\sqrt{-G}\Big(R - 2g_{ij}\partial\phi^i\partial\phi^j \\
& - f_{ab}(\phi^i)F^a_{\mu\nu}F^{b\ \mu\nu} \\
& - \frac{1}{2}\tilde{f}_{ab}(\phi^i)F^a_{\mu\nu}F^b_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\Big)
\end{aligned}
$$

Conditions for an Attractor

$$V_{eff}(\phi^i) \equiv f^{ab}(Q_{ea} - \tilde{f}_{ac}Q^c_m)(Q_{eb} - \tilde{f}_{bd}Q^d_m)$$
$$+f_{ab}Q^a_m Q^b_m$$

Q_{ea}, Q^a_m : are electric and magnetic charges of black hole.

$f_{ab}, \tilde{f}_{ab}$: depend on the moduli fields

Result:

- There is an attractor provided V_{eff} has a minimum :
- 1) $\partial_i V_{eff}(\phi_{i0}) = 0$

  2) $\partial_i \partial_j V_{eff}(\phi_{i0}) > 0$
- The attractor values are ϕ_{i0}
- Entropy: $S = \pi V_{eff}(\phi_{io})$

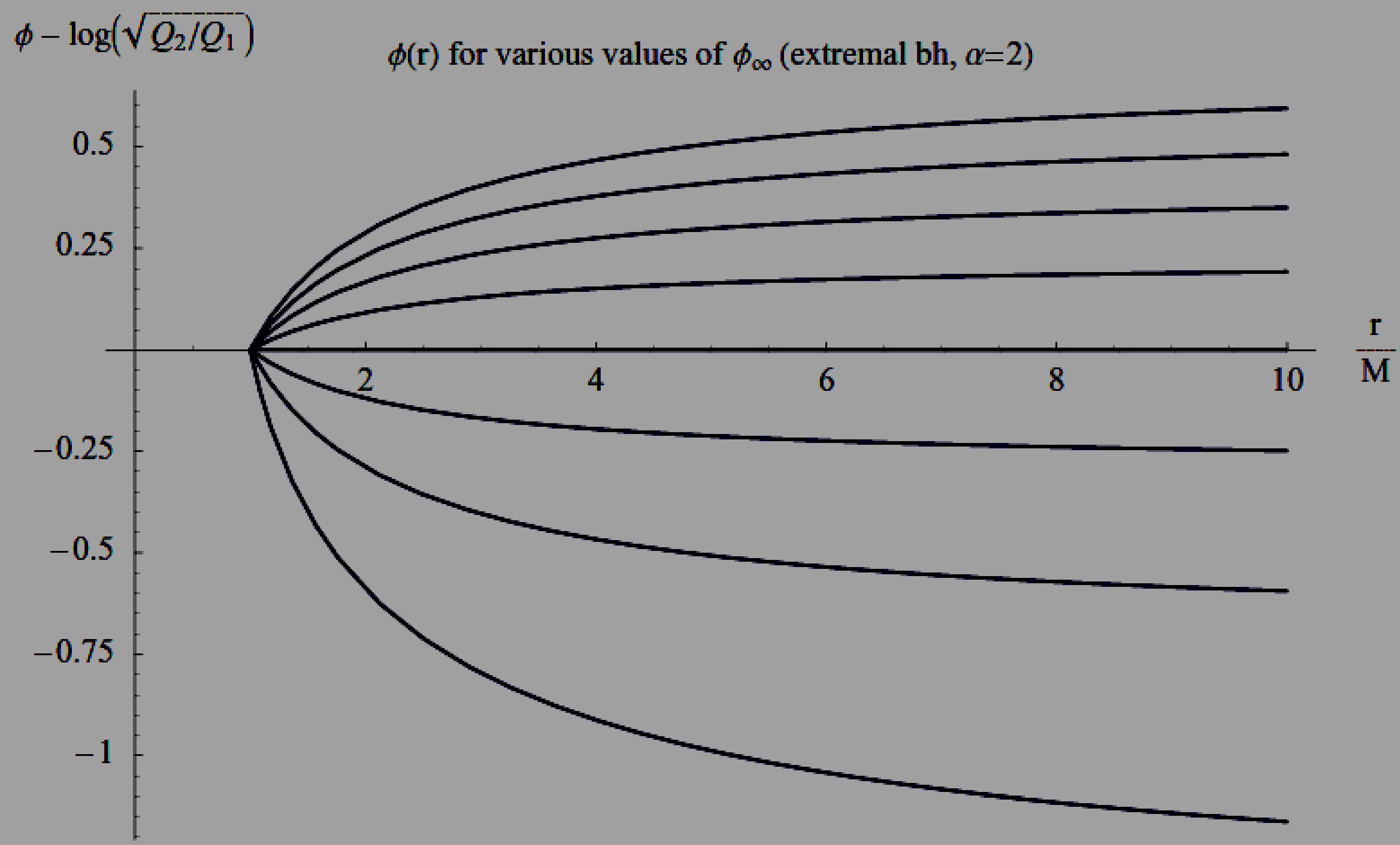


Figure 1: Attractor behaviour for the case $\gamma = 1$; $\alpha, -\tilde{\alpha} = 2$

Analysis:

The essential complication is that the equations of motion are non-linear second order equations. Difficult to solve exactly.

Attractor Solution:

- Scalars take attractor value at infinty and are kept constant.

- Resulting solution: Extremal Reissner Nordstrom Black Hole

Small Parameter: $\varepsilon = \phi_{\infty} - \phi_0$

Equations are second order but Linear in perturbation theory.

$\phi \approx 1/r, const.$

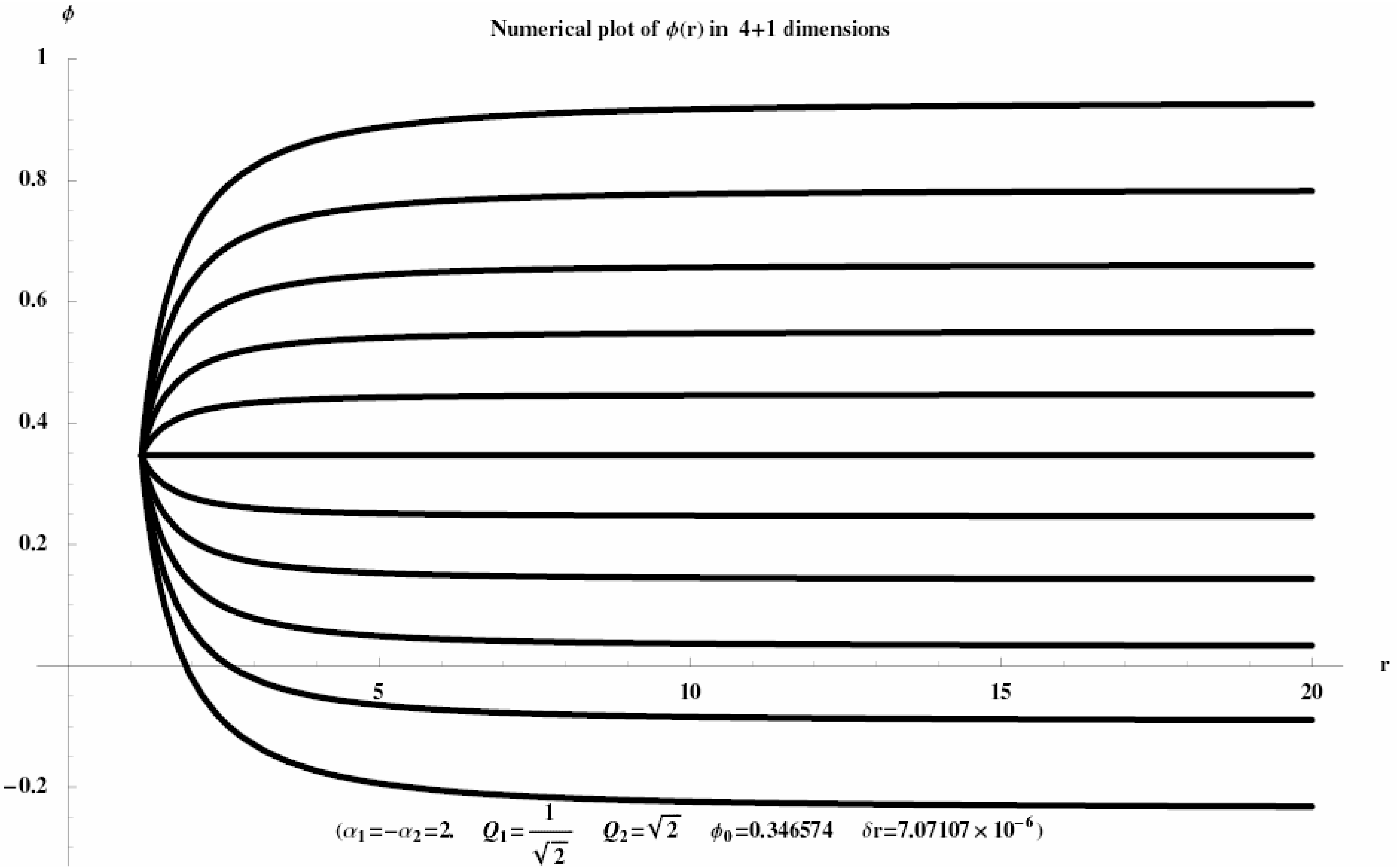
Generalisations:

Attractor Mechanism works, with similar conditions on the effective potential in:

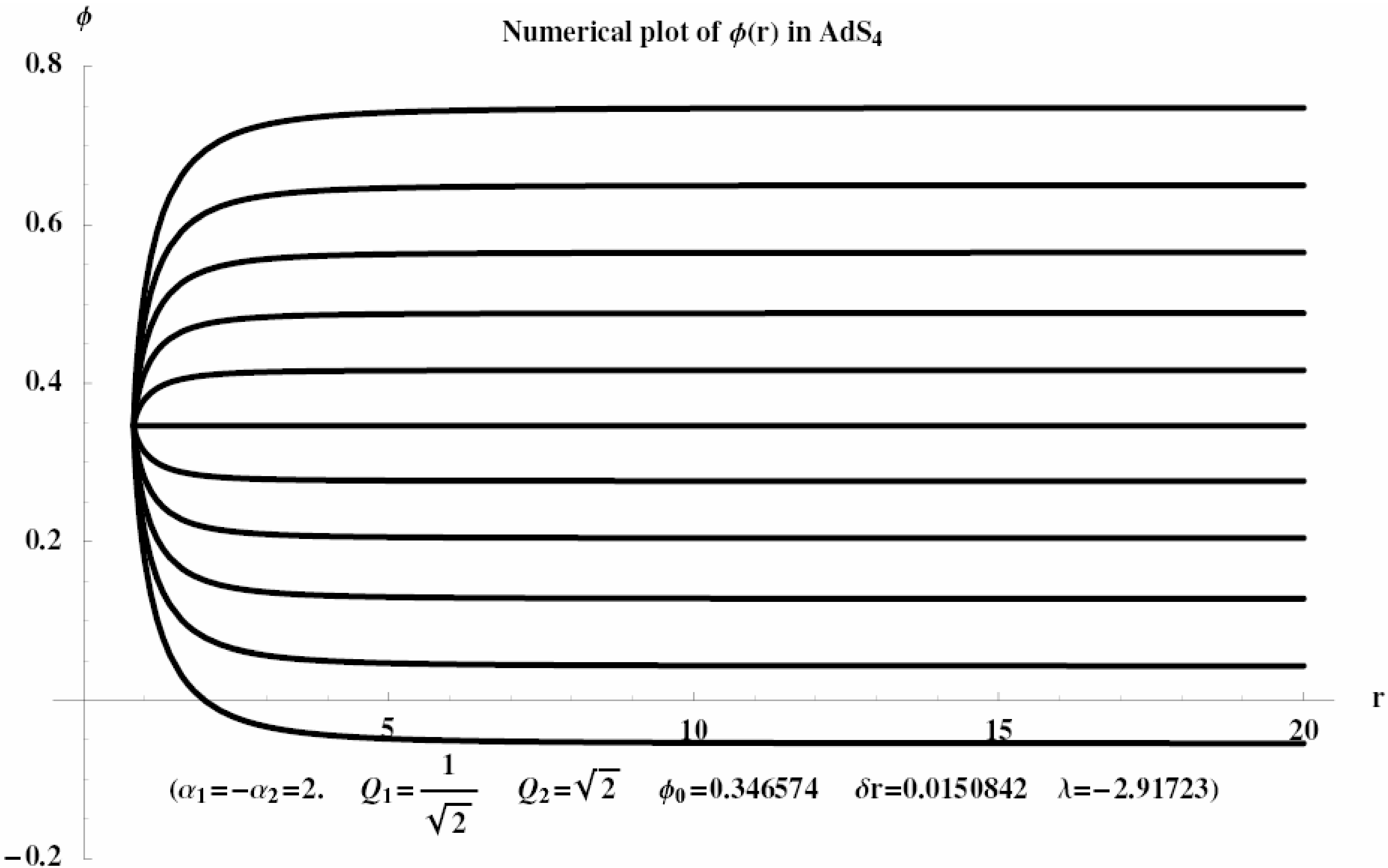
1. Higher Dimensions

2. Anti-DeSitter Space

3. De-Sitter Space



Numerical plot of $\phi(r)$ in 4+1 dimensions
ϕ
1
0.8
0.6
0.4
0.2
−0.2
r
5
10
15
20
$(\alpha_1=-\alpha_2=2. \quad Q_1=\frac{1}{\sqrt{2}} \quad Q_2=\sqrt{2} \quad \phi_0=0.346574 \quad \delta r=7.07107\times10^{-6})$



ϕ
Numerical plot of ϕ(r) in AdS4
0.8
0.6
0.4
0.2
−0.2
r
5
10
15
20
(α1=−α2=2. Q1=1/√2 Q2=√2 ϕ0=0.346574 δr=0.0150842 λ=−2.91723)

Related work: Entropy Function

A. Sen

- Basic idea : By extremising the Entropy function one obtains the attractor values for the moduli and geometry.
- Entropy is the extremum value of the function.
- Focus on the near-horizon region and use it's enhanced symmetries.

Entropy Function Cont'd:

- Advantage: Higher derivative corrections can be included.

- Limitation: Does not tell which extrema can be obtained starting from infinity. Nor does it distinguish between stable and unstable attractors.

- For two-derivative actions, in calculating the attractor values, it agrees with the results from V_{eff}

Rotating Attractors

Motivation:

- How general is this phenomenon?
- Consider cases where supersymmetry is slightly broken.

II) Rotating Attractors

- We find similar results for rotating extremal black holes as well.
- Near horizon geometry :

$$SO(2,1) \times U(1)$$

Results:

- Generically all moduli and metric components have fixed functional form at horizon.
- Entropy is determined by charges alone.
- Analysis should generalise to higher dimensions and also for horizons with any compact topology.

Example: Type II on Calabi Yau Three-fold

$$V(\phi) = e^{K}(|DW|^2 + |W|^2)$$

W :Superpotential determined by the charges

K : Kahler potential

This is analogous to the potential in flux compactifications.

$$V = e^K[|D_i W|^2 - 3|W|^2]$$

W : Determined by the fluxes.

N=2 Susy Case Cont'd:

Susy Attractor:

$$D_i W = 0 \Rightarrow \partial_i V = 0$$

$$\frac{\partial^2 V}{\partial \phi^2} > 0$$

Entropy ~ $e^K |W|^2$

Non-Susy Attractor

Condition 1: $\partial_i V = 0$

(But $D_i W \neq 0$)

Condition 2: $\partial_{ij} V > 0$

(with suitable generalisations)

Result

- Depending on charges one gets either a susy attractor or an non-susy extremum of the effective potential.
- Sometimes the non-susy extremum is an attractor.
- Entropy in the non-susy case is obtained by appropriately continuing the susy formula.

Type IIA at Large Volume

I) No D6 Branes:

D4 brane charge p^a

D0 brane charge q_0

$$D \equiv D_{abc} p^a p^b p^c$$

Susy solution exists when $q_0/D > 0$

Non-susy attractor exists when

$$q_0/D < 0$$

Susy entropy:

$$S = 2\pi\sqrt{Dq_0},$$

Non-susy entropy:

$$S = 2\pi\sqrt{-Dq_0},$$

Non-supersymmetric Attractors

With D6 brane charge:

•P0 D6 branes with p^a units of D4 brane charge along the Σ^a cycle and q0 units of D0 brane charge.

•Supersymmetric attractor exists if

$$\frac{4D}{q_0} > (p^0)^2$$

It has entropy,

$$S = \pi\sqrt{4Dq_0 - (p^0)^2 q_0^2}.$$

A Non-susy solution exists when

$$\frac{4D}{q_0} < (p^0)^2$$

$$S = \pi\sqrt{(p^0)^2 q_0^2 - 4Dq_0}.$$

- However, there are $N_v - 1$ zero modes.
- The leading corrections along these directions are cubic:

$$V_{eff}(x^a) = V_{eff}(x_0^a) + \lambda D_{abc}\alpha^a\alpha^b\alpha^c,$$

- Thus non-positive.
- As a result extremum is not an attractor.

Microstate counting and Attractors

The microstate counting for many non-susy extremal black holes agrees with their Beckenstein Hawking entropy.

e.g. MSW string has the same left and right moving central charge (for large Q).

$$S_{BH} = 2\pi\sqrt{|Dq_0|}$$

Why?

Microscopic calculation and supergravity calculations are controlled in different regions of moduli space:

Microscopic: $g_s Q \ll 1$

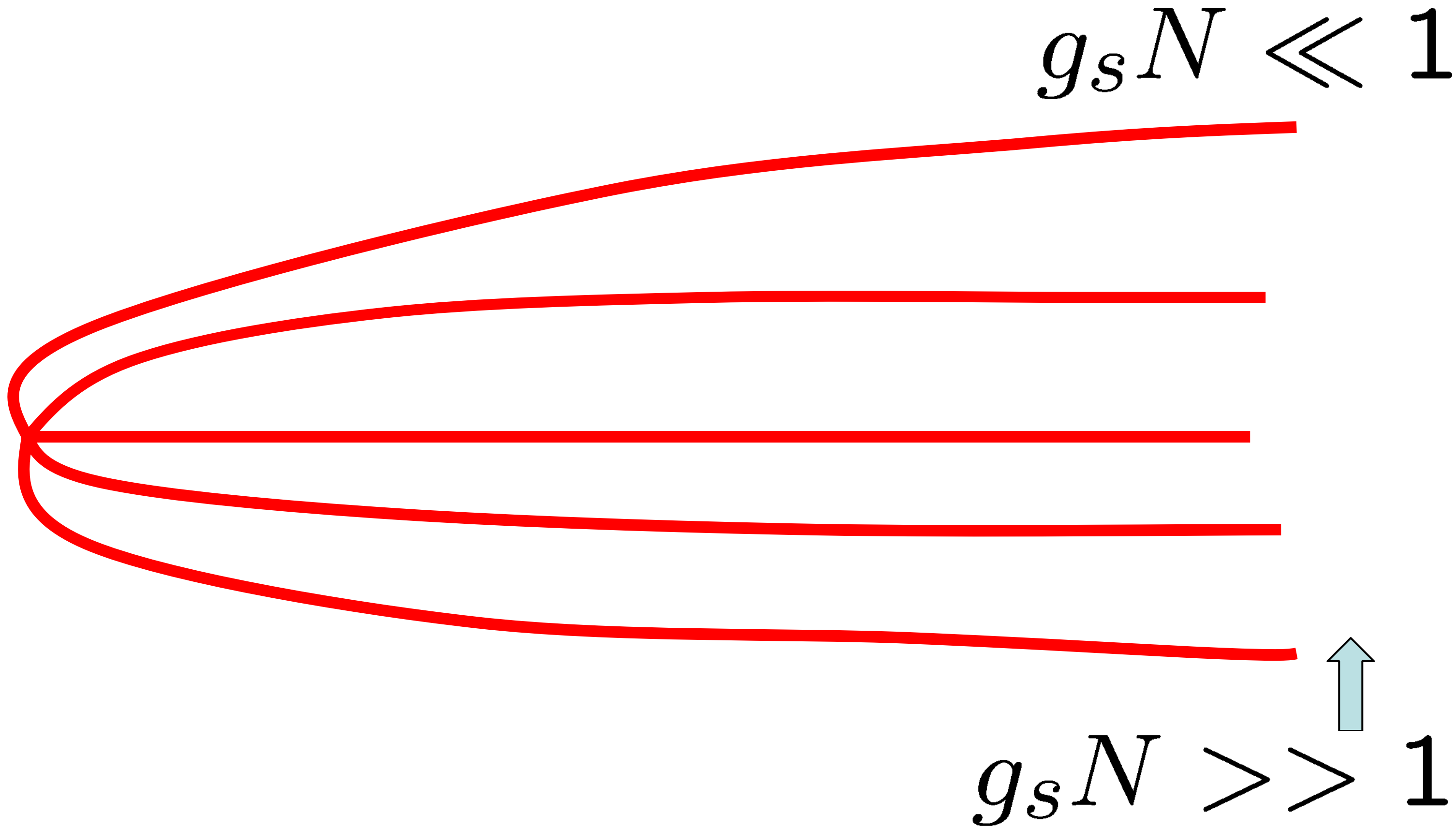
Macroscopic: $g_s Q \gg 1$

Perhaps the Attractor phenomenon can provide an explanation:

- Suppose the $g_s Q \ll 1$ region and the $g_s Q \gg 1$ region both lie in the same basin of attraction. That is they both flow to the same attractor.

- Then the entropy in the two cases must also agree.

- In effect the attractor mechansim provides an argument for the non-renormalisation of the entropy.



$g_s N \ll 1$
$g_s N >> 1$

- Typically when $g_s Q \ll 1$, at asymptotic infinity, there will be some region where the supergravity description breaks down.
- Still, as long as the attractor geometry is unchanged the entropy will remain unchanged.
- Higher derivative corrections can be included, if need be, using the entropy function of Sen.

Some Assumptions:

1. No phase transitions: i.e., the same basin of attraction

2. Assume that extremal black holes correspond to states with minimum mass for given charge.

Comments:

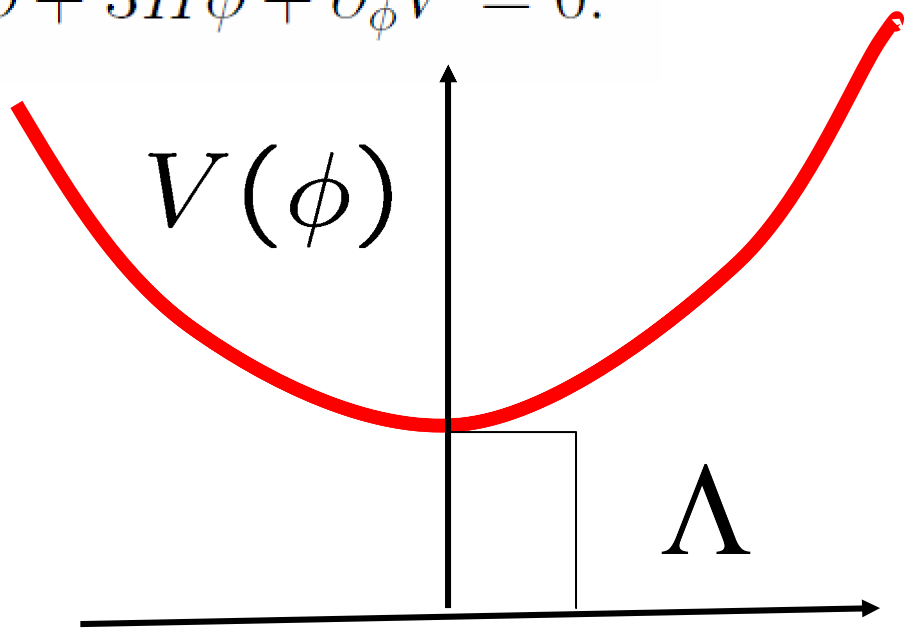
Argument applies to susy black holes too. For total no. of states not an index.

Some comments related to Cosmology

- A dynamical system is drawn to an attractor at late times regardless of initial conditions.
- For black holes the attractor behaviour is non-generic.

Cosmology in an expanding universe:

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V = 0.$$



Friction term means system will settle to bottom of potential. This is the attractor.

Aside on the scalar perturbation equation for attractors:

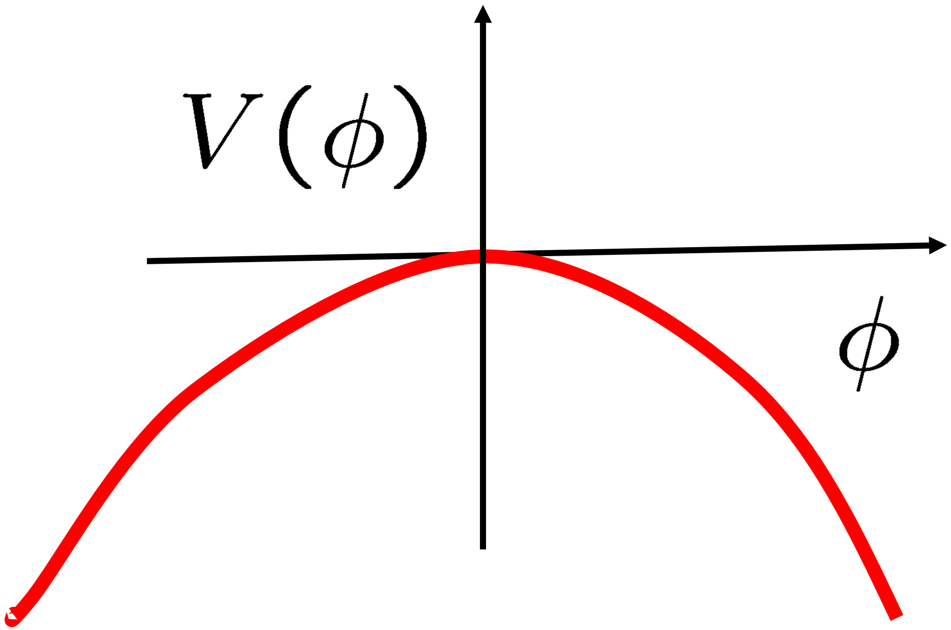
$$e^{-t} \equiv \frac{r - r_H}{r_H}.$$

$$t \to \infty, \text{ as } (r - r_H) \to 0.$$

$$\delta\ddot{\phi} - \delta\dot{\phi} - \frac{m^2}{r_H^2}\delta\phi = 0.$$

Anti-friction term

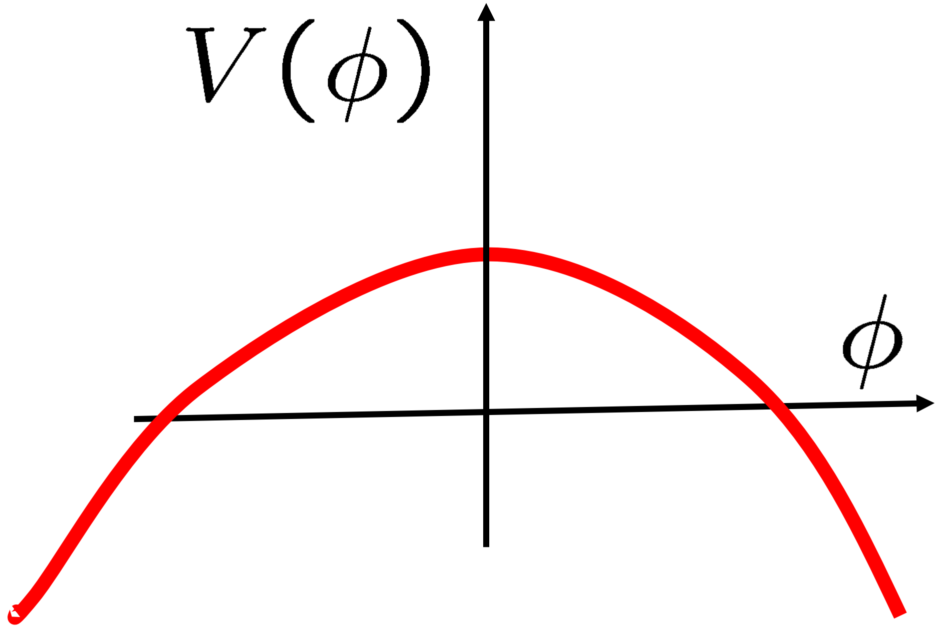
Upside down sqaure well



Anti-friction aids the motion.
Potential pushes it down.

Note here $V(\phi) = -V_{eff}$

Analogue of the black hole attractor is early time behaviour in deSitter space of a negative mass scalar.



$$ds^2 = -\frac{dt^2}{t^2} + t^2 dx_i^2,$$

Requiring $t \to 0$ to be singularity free, means only one of the two solutions is allowed. For $m^2 < 0$ this solution vanishes leading to attractor behavior.

Can we make something of this attractor?

Conclusions

1) Non-suspersymmetric extremal black holes show attractor behaviour. The phenomenon is quite general.

2) Includes examples in String Theory.

3) Interesting implications for microstate counting of black hole entropy .

And maybe in future for cosmology.

From Strings to LHC

January 2007 4th-11th

Dabholim Beach Goa

See you there!

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