String Landscape + the Swampland

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Suppose $L_{\text{QFT}}$ is a consistent QFT.

Since string theory is the only consistent quantum theory of gravity known to date we will interpret this question as follows:
We conjecture a few criteria which distinguish the String Landscape from the Swampland. We will not be able to prove these conjectures; we will find many non-trivial examples supporting these conjectures. These conjectures are about the space of flat directions $M \times X$, as well as the nature of gauge groups, gauge coupling constants, and of light fields.
The Conjectures

0. $M_X \leftrightarrow \langle \phi \rangle$
   "No adjustable parameters"

1. $\phi \in M_X$  "string landscape"

$L = \ldots + |\nabla \phi|^2$

$\text{Vol}(M_X) = \text{finite}$
   (more precisely $M^\Lambda_X : (m \gg \Lambda)$)

- Uniform bound
  (depending only on $d$)

- Choices for gauge groups
- Gauge coupling constants cannot be too weak; gravity is the weakest force.

- Diameter of \( \text{diam} (M_x) = \infty \)

and pts at \( \infty \) correspond to appearance of a tower of light particles with \( m : \)

\[ m(d) \sim e^{-\alpha d} \]
6. $R \ (d \to \infty) < 0$  
   (scalar curvature)

7. $\pi_1(\overline{M}_x) = \emptyset$
   
   Non-trivial cycle

Can't happen:

We provide evidence for these conjectures and show they can all be violated if we deouple gravity, (for example by taking $
\to \int R 
\to \text{gravity decoupled}$

lower dim. $d \to \infty$

$M_{pl}$
Based on

hep-th/0509212
Title: The String Landscape and the Swampland
Authors: Cumrun Vafa

hep-th/0601001
Title: The String Landscape, Black Holes and Gravity as the Weakest Force
Authors: Nima Arkani-Hamed

hep-th/0605264
Title: On the Geometry of the String Landscape and the Swampland
Authors: Hirosi Ooguri

(related work: Douglas et al., Kachru et al., Li et al., Motl et al.)
\[- M \leftrightarrow \langle \phi \rangle \]

is an obvious fact in string theory, but it did not have to be the case. Note that QFT's without gravity do have adjustable parameters.

**Finiteness of Vol.**

Examples of finiteness:

Let \( X = T^2 \times \ldots \)

Then \( \phi \in M_{T^2}^{\text{cplx}} \times \ldots \)

\[
\text{Vol} (\phi) = \int \frac{d^2 \tau}{\tau_2^2} < \infty
\]

fundamental domain
Other examples

\[ X = CY \text{ 3-fold} \]

\[ \phi \in M^{cpl}_{CY} \]

There are arguments (Todorov) (Douglas et al.)

\[ \text{Vol}(M^{cpl}_{CY}) < \infty \]

More subtle applications:

Kähler moduli

Dualities

\[ \begin{align*}
T\text{-duality} & \quad \text{(mirror symmetry)} \\
S\text{-duality} &
\end{align*} \]

Vol \( (M^{cpl}) \) finite

E.g., \( \tau \) (coupling constant) in Type IIB

\( (SL(2,\mathbb{R}) \rightarrow \text{Vol}(\tau) = \text{finite} \)
This is a "gravity" effect. If we decoupled "gravity" by taking $X =$ non-compact then the volume is not finite,

\[ \text{(e.g. } \text{Vol} (\phi \in \text{QFT}) = \infty) \]
\[ \phi \in \mathbb{R}^n \]
2 - \((\dim M_x) = \text{bounded}\)

This is meant as a uniform bound. (fixed, for \(\dim X = \text{fixed}\))

Example: \(X : CY^3\)

\((\dim_\sigma M_x) = h^{1,1} + h^{1,2}\)

The conjecture in this case suggests \(h^{1,1}, h^{2,1}\) bounded.

(This has been conjectured by Douglas)

It is not known whether this mathematical conjecture is true or false. Currently there
is no counterexample (i.e., no sequence of CY with increasing hodge \( \ast \)'s).

Again this conjecture is if we decouple gravity.

The decoupling of gravity can be done by taking

\[ X : \text{ NM-compact} \]

In this case 3 counterexamples:

\[ X = \mathbb{C}^2 / \mathbb{Z}_n \]

\[ Z_n : (z_1, z_2) \rightarrow (\omega^m z_1, \omega^{-m} z_2) \]

\[ \omega^n = 1 \]
This gives rise to $A_{n-1}$ singularity which can be resolved:

This finiteness of $\dim (M_x)$ has a counterpart conjecture involving a on the $\dim (M_x)$ at least in certain cases. For example if we consider $\mathcal{N}=2$ supersymmetric compactifications to $d=4$, we always seem to end up with scalar field. (CY case: $h_2 \geq 1$).
Conjecture 2 \implies \dim G < \infty

But it seems there are more restrictions on $G$.

Examples:

$d=10 \implies G = \text{E}_8 \times \text{E}_8$

$N=1$ susy \quad G = \text{Spin}(32)/\mathbb{Z}_2$

\text{landscape}

\text{swampland}
In other words, even though $E_8 \times U(1)$ and $U(1)^{496}$ are semiclassically consistent choices for gauge group $G$, but they are not realized in string theory (nor in its non-perturbative completions, M- or F- theories). We conjecture that these choices are actually inconsistent as a full quantum theory.

There are similar restrictions in other dimensions. For example

\[ d=4 \quad \text{locally} \quad \frac{C^2}{Z_n} \Rightarrow G = U(n) \]

but in the context of a compact
theory $\mathbb{C}^{2}/\mathbb{Z}_n \subset K3$

$\Rightarrow \text{rank } (G) = 20$

In addition, not all rank = 20 gauge groups are realized.

( $G$ realized $\iff$ root lattice $\subset$ self-dual even integral lattice)
Again these restrictions on $G$ seem correlated with having a dynamical gravity, in particular for $X$ non-compact no restrictions known.

(Example: $\mathbb{C}^2/\mathbb{Z}_n \rightarrow U(n) \mathbb{V}_n$)
Strength of gauge coupling const.

Consider a U(1) gauge theory with coupling constant $g$

Claim: There is a light charged particle

$m \leq g M_{\text{Planck}}$

$g \geq \frac{m}{M_{\text{Pl}}}$
Checks

1) Heterotic strings:
\[ M_s \sim g M_{\text{Planck}} \]
exists light states, strings, and \( \Lambda > M_s \), effective gauge theory breaks down

2) KK reductions
\[ g M_{\text{Pl}} \sim \frac{1}{R} \]
\( \Lambda > \frac{1}{R} \), effective gauge theory breaks down
clearly the condition is vacuous when $M_{\text{pl}} \to \infty$

$$\left( g \gtrsim \frac{m}{M_{\text{pl}}} \sim 0 \right)$$

One way to motivate this conjecture:

(Unless protected by some charge):

* Stable particles $\ll \infty$

$$\left( \frac{M}{Q} \right)_{\text{min}} < 1$$
Extremal Black holes

If \( \frac{M}{Q} > 1 \Rightarrow M > Q \)

(\(\leftrightarrow\) M \(\leftrightarrow\) M) = attractive force

\(\Rightarrow\) bound state
\(M' < 2M \quad Q' = 2Q\)
If \( \frac{M'}{Q'} > 1 \) \( \rightarrow \) continue \( M'', Q'' \)

\[ \sum \] by many particles

\[ \frac{M}{Q} \]

So \( X \) stable \( < \infty \)

\( \exists \left( \frac{M}{Q} < 1 \right) \rightarrow \)

\( (M/Q)_{\text{min}} < 1 \)

Non-trivial example:

Non-susy BH in heterotic string compactifications.
\[ -\text{Diam.}(M_x) = \infty \]

\[ \text{Diam} = \max_{p, q \in M_x} [d(p, q)] \]

\[ \text{Diam} = \infty \]

fix \( p \); look at all \( q \)
there are points at \( \infty \)
Moreover pts. at $\infty$ correspond to the regions with an $\infty$ tower of light particles:

as $d \to \infty$, for some $\alpha$.

Examples: $T^2$ compactification

$\mathcal{L}$:

$$d = \int \frac{d\tau_2}{\tau_2} \sim \ln \frac{1}{\Lambda}$$

$\Lambda \sim e^{-d}$
Another example:

\[ M - \text{theory on } S^1_R : \]

\[ \ldots \rightarrow \sim \rightarrow \infty \]

\[ \uparrow \sim \uparrow \]

\[ \sim \rightarrow \to \infty \]


tensionless strings

counter examples if we decouple gravity:

\[ \text{diam} (M_x) \neq \infty \]

\[ M_x = G/H \]

\[ \text{e.g. } G = SU(2) ; H = U(1) \]
Also in field theory even if diam($M_x$) = $\infty$

\[ \Rightarrow \text{extra light modes:} \]

e.g. $N = 4$ YM

$G = SU(2)$

$M_x = \mathbb{R}^6 / \mathbb{Z}_2$ \ni \Phi^i

$p \in M_x$ at $\infty$ correspond to $W^+$ infinitely massive, no extra light particles appear.
$6 - R(d=\infty) < 0.$

Note: $d=\infty$ \{clash\} $\text{vol} = \text{finite}$

$\rightarrow R \bigg|_{\infty} < 0$ is almost a requirement.

Example: $G(2) \setminus \mathbb{G}/H$

that appear in string theory have $R<0$.

Condition violated for field theory w/o gravity for $N=2$ gauge theories $R > 0$.
\[ \pi_1(M_x) = \emptyset \]

Intuition: Duality group is generated by gauge symmetry.

\[ M_x = \frac{T_x}{P} \hookrightarrow \text{duality group} \]

\[ p \in T_x \quad \text{fixed by } g \in \Gamma \rightarrow g \in \text{gauge group at } p \]
Example:

$\mathbb{Z}_3 \rightarrow \mathbb{Z}_2$ (S)

$\text{SL}(2,\mathbb{Z})$ generated by S, ST

If $\Gamma$ generated by elements with fixed pts $\Rightarrow \pi_1(M_x) = \emptyset$

(Another intuition: compactification to 1+1 $\Rightarrow$ global charge

$\Rightarrow$ not allowed in a quantum gravity)
Again, counterexample can be constructed if gravity is decoupled:

\[ \text{eg. Axiom: } \mathbb{Z} (\text{Axion}) = \mathbb{Z} \]

(Suggests axiom is always accompanied by an extra field when coupled to gravity)
Conclusion

Dynamical gravity puts restrictions on allowed theories.

Mathematically:

\[ \text{compact vs. non-compact} \]

\[ \Rightarrow \text{more restrictive} \]

In particular:

\[ \text{Vol}(X) < \infty \Rightarrow \text{Vol}(M_x) < \infty \]

Also:

\[ \dim(M_x) < \infty \quad \text{(uniform)} \]

\[ X = \text{singular} \Rightarrow G: \text{restrictive} \]
We have also seen finiteness have other consequences:

\[ \left( \frac{M}{Q} \right)_{\text{min}} < 1 \]

(gravitational is the weakest force)

\( \Rightarrow \exists \) scale

\( \Lambda \sim g \Lambda \) MeV

\( E > \Lambda \)

Effective field theory breaks down
\[- \text{diam} \left( M_x \right) = \infty \]

(points at \( \infty \)) \( \Rightarrow \) (Tower of \( m \to 0 \))

\[- R \bigg|_{d \to \infty} < 0 \]

\[- \prod_1 \left( M_x \right) = \emptyset \]

It would be important to:

\[ \begin{cases} 
- \text{Try to find counterexamples} \\
\text{or} \\
- \text{Try to understand why they are true.} \\
- \text{Broaden the list of conjectures.} 
\end{cases} \]